

Angles in Polygons

Strand:	Polygons and Circles
Topic:	Exploring angles in polygons
Primary SOL:	G.PC.2 The student will verify relationships and solve problems involving the number of sides and angles of convex polygons.
Related SOL:	G.PC.1

Materials

- Polygon Names activity sheet (attached)
- Angles in Polygons Companion sheet (attached)
- Angles in Polygons activity sheet (attached)
- Angles in Polygons Modified activity sheet (attached)
- Dynamic geometry software (such as Geogebra Geometry, <https://www.geogebra.org/geometry>) or straight edges and protractors

Vocabulary

area, array, concave polygon, convex polygon, diagonal, exterior angle, factor, interior angle, irregular, n-gon, polygon, regular polygon, equiangular, vertex, supplementary

Student/Teacher Actions: What should students be doing? What should teachers be doing?

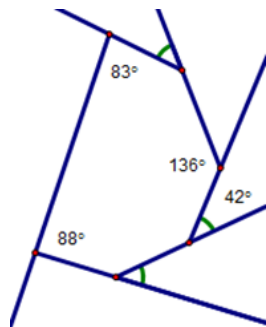
1. Have students work in pairs to complete the Polygon Names activity sheet. They should justify their answers to another pair. Review answers to ensure that students have the correct names. The “Numerical Prefixes Table” can be used to support the students’ initial work or to verify their answers.
2. Before starting the Angles in Polygons activity, students will need to understand:
 - a. that n is used to represent the number of sides of a polygon and therefore, a n -gon represents a polygon with an unknown number of sides;
 - b. that interior and exterior angles of a polygon form a straight line and are therefore supplementary;
 - c. the difference between a convex polygon and a concave polygon;
 - d. that a diagonal of a polygon is a line segment that joins two vertices of the polygon which are not already joined by an edge of the polygon; and
 - e. that a regular polygon is equilateral (all side lengths are equal) and equiangular (all interior angle measures are equal).
3. Provide the Angles in Polygons activity sheet to students. This activity can be completed using dynamic geometry software (such as Geogebra Geometry, <https://www.geogebra.org/geometry>) or straight edges and protractors. Support students through the first row of the table (triangle) – see the guidance provided below. Once this row is completed, students should clearly understand that the interior angles of any triangle add to 180° .

- a. Draw $\triangle ABC$, \overrightarrow{AB} , \overrightarrow{BC} , and \overrightarrow{CA} .
 - i. Determine the sum of the measures of the interior angles of $\triangle ABC$ ($\angle 1$, $\angle 2$, and $\angle 3$ in the image provided in the table). Write your answer in the table. Will this be true for any triangle?
 - ii. Determine the sum of the measures of the exterior angles of $\triangle ABC$ ($\angle 4$, $\angle 5$, and $\angle 6$ in the image provided in the table). Write your answer in the table. Will this be true for any triangle?
 - iii. Consider a regular triangle.
 1. Determine the measure of each interior angle of a regular triangle. Write your answer in the table.
 2. Determine the measure of each exterior angle of a regular triangle. Write your answer in the table.
4. Provide the Angles in Polygons Companion sheet to students. Have students work in pairs to complete the Angles in Polygons activity sheet. Each student should record their own findings. Have students discuss their findings with their partners. Discuss the findings as a whole group.

Assessment

- **Questions**

- What is incorrect with the diagram below?



- How are an interior and an exterior angle at the same vertex related?
 - How can you find the sum of the measures of the interior angles of a convex 102-gon without using the formula?
 - If four of the angles of a pentagon measure 80° , 90° , 100° , and 110° , what is the measure of the fifth angle? Explain your reasoning.
 - Which is greater, the measure of an exterior angle of a regular triangle or the measure of an interior angle of a regular pentagon? Explain your reasoning.
- **Journal/writing prompts**
 - Complete a journal entry summarizing your investigations.
 - Write directions for how to find the measure of an interior angle of a regular polygon.
 - If you are given the measure of an exterior (or interior) angle of a regular polygon, explain how to determine how many sides the polygon has.

- Describe a practical situation that uses angles of polygons.
- **Other Assessments**
 - Have students explain and demonstrate why a given regular polygon can or cannot be used to tessellate a plane.
 - Have groups of students create “fragments” of regular polygons made from found materials (e.g., paper, fabric, craft foam, plastic) that are missing some of the angles and sides. Have groups swap fragments and determine how many sides the regular polygons had by measuring the interior angles.

Extensions and Connections (for all students)

- Use virtual manipulatives (found online) to demonstrate interior and exterior angles of polygons.
- Invite an artist or architect to the class to discuss the use of polygons and transformations in art or architecture.

Strategies for Differentiation

- Have students use auditory instructions to assist with the directions for the dynamic geometry software.
- Have students use 3-D models of polygons.
- Have students use a reflective transparent math geometry tool to explore and construct reflections for tessellations.
- Have students create a math glossary or folded graphic organizer with definitions and examples.
- Modify the table by using the modified Angles in Polygons activity sheet or eliminating rows or columns from the full activity sheet.

Note: The following pages are intended for classroom use for students as a visual aid to learning.

Polygon Names

Name _____ Date _____

Complete the table using the terms from the word bank in the “Name of Polygon” column.

Word Bank

undecagon	hexagon	quadrilateral	triangle
octagon	pentagon	nonagon	decagon
dodecagon	heptagon		

Number of Sides	Name of Polygon	Justification
3		
4		
5		
6		
7		
8		
9		
10		
11		
12		
<i>n</i>		

Numerical Prefixes Table

Number	Latin		Greek	
	Prefix	Examples	Prefix	Examples
3	Tri-	<u>Triad</u> – group of 3 people or things <u>Trilogy</u> – series of 3 books/movies	Tri-	<u>Triathlon</u> – race consisting of 3 sports (swimming, biking, running) <u>Tripod</u> – 3-legged stand
4	Quad- (Quart-)	<u>Quadrant</u> – 4 parts of a plane <u>Quarter</u> – each of 4 equal parts	Tetra-	<u>Tetrarch</u> – one of 4 joint rulers <u>Tetrachord</u> – series of 4 musical notes
5	Quint- (Quin-)	<u>Quintet</u> – a group containing 5 members <u>Quinate</u> – composed of groups of 5, such as leaves	Penta-	<u>Pentathlon</u> – race consisting of 5 sports (fencing, shooting, swimming, horse riding, and running) <u>Pentagram</u> – 5-pointed star
6	Sext-	<u>Sextuplets</u> – combination of 6 of a kind <u>Sextant</u> – dental term; dental arches divided into 6 sections	Hexa-	<u>Hexapods</u> – 6-legged insects <u>Hexameter</u> – in poetry, a line of verse consisting of 6 metrical feet
7	Sept-	<u>September</u> – The old Roman calendar started in March, originally making September the 7 th month	Hept-	
8	Oct-	<u>October</u> - The old Roman calendar started in March, originally making October the 8 th month. <u>Octopus</u> – sea creature with 8 legs	Octa-	<u>Octave</u> – In a traditional scale, like C major, the distance from each octave is 8 notes. <u>Octane</u> - a straight chain alkane composed of 8 carbon atoms
9	Non-	<u>Nonapeptide</u> – A peptide containing 9 amino acids <u>Nonan</u> – occurring on the 9 th day	Ennea-	<u>Enneagram</u> – a personality test with 9 possible personality types
10	Deci-	<u>Decimal</u> – the number system which uses 10 digits (0-9) <u>Decimate</u> – in history, to kill one in every 10 of a group as punishment	Deca-	<u>Decade</u> – 10 years <u>Decameter</u> – a length measurement equaling 10 meters

Angles in Polygons Companion

This activity can be completed using dynamic geometry software (such as Geogebra Geometry, <https://www.geogebra.org/geometry>) or straight edges and protractors. Use the guidance below to complete the “Angles in Polygons” activity sheet.

We want to be able to find the sum of the measures of the interior angles of any convex polygon.

Quadrilateral

1. Draw quadrilateral ABCD, \overrightarrow{AB} , \overrightarrow{BC} , \overrightarrow{CD} , and \overrightarrow{DA} .
2. Draw as many diagonals as you can starting at point A. How many triangles does this create? Write your answer in the table.
3. Based on your answer to question 2 and using what you have learned about the sum of the interior angles of a triangle, what should be the sum of the interior angles of a quadrilateral? Verify this by measuring the interior angles. Write your answer in the table.
4. Determine the sum of the measure of the exterior angles of your quadrilateral. Write your answer in the table.

Pentagon

5. Draw pentagon ABCDE, \overrightarrow{AB} , \overrightarrow{BC} , \overrightarrow{CD} , \overrightarrow{DE} , and \overrightarrow{EA} .
6. Draw as many diagonals as you can starting at point A. How many triangles does this create? Write your answer in the table.
7. Based on your answer to question 2 and using what we have learned about the sum of the interior angles of a triangle, what should be the sum of the interior angles of a pentagon? Verify this by measuring the interior angles. Write your answer in the table.
8. Determine the sum of the measure of the exterior angles of your pentagon. Write your answer in the table.

Hexagon

9. Try to determine rules or patterns to complete the hexagon row. Conduct the same process for a hexagon as you did for quadrilateral and pentagon above to verify your answers.

n-gon

10. Use the rules or patterns that you have determined to complete the *n*-gon row (the last row on page 2) which will create formulas that can be used for this process.

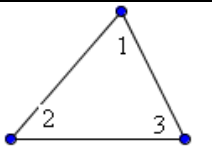
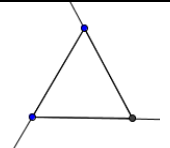
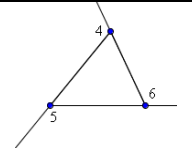
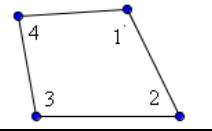
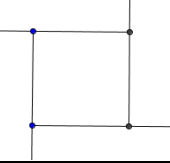
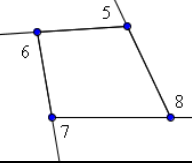
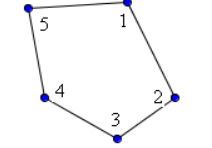
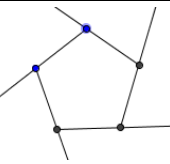
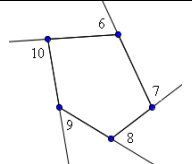
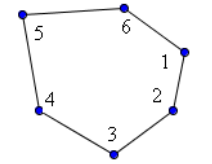
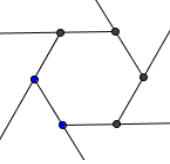
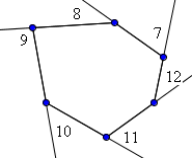
Remaining polygons

11. Use your formulas from the *n*-gon row to complete the rows for the remaining polygons.

Name: _____

Date: _____

Angles in Polygons (page 1)

n -gon	Formula →	$n - 2$	*	$(n - 2)180$	*	$\frac{(n - 2)180}{n}$	*	360	$\frac{360}{n}$
Name of Polygon	n (# of sides)	# of Δ s	Convex n -gon	Sum of Interior Angles	Regular n -gon	Each Interior Angle of Regular n -gon	Convex n -gon	Sum of Exterior Angles	Each Exterior Angle of Regular n -gon
	3								
	4								
	5								
	6								

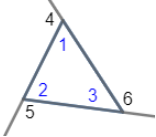
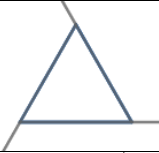
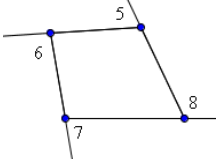
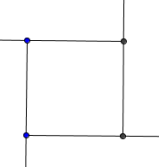
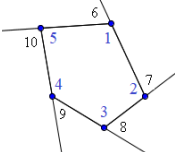
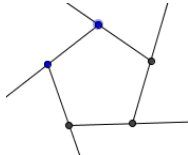
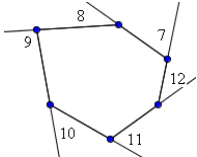
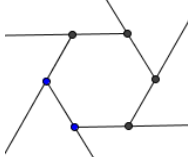
Angles in Polygons (page 2)

Name of polygon	n (# of sides)	# of Δ s	Convex n -gon	Sum of the interior angles	Regular n -gon	Each interior angle of regular n -gon	Convex n -gon	Sum of the exterior angles	Each Exterior Angle of Regular n -gon
	7		*		*		*		
	8		*		*		*		
	9		*		*		*		
	10		*		*		*		
	11		*		*		*		
	12		*		*		*		
	15		*		*		*		
	23		*		*		*		
	100		*		*		*		
	n (formula)		*		*		*		

Angles in Polygons (modified)

Name _____

Date _____

Name of Polygon	n (# of sides)	# of Δ s	Convex n -gon	Sum of Interior Angles	Sum of Exterior Angles	Regular n -gon	Each Interior Angle of Regular n -gon	Each Exterior Angle of Regular n -gon
Triangle	3	1		180°	360°		60°	120°
Quadrilateral	4							
Pentagon	5							
Hexagon	6							
n -gon	n (formula)		*			*		
Decagon	10		*			*		
15-gon	15		*			*		
23-gon	23		*			*		