Rabbits: Comparing Linear, Quadratic, and Exponential Functions

Strand: Functions

Topic: Comparing linear, quadratic, and exponential functions

Primary SOL: A.F.2 The student will investigate, analyze, and compare characteristics of functions, including quadratic and exponential functions, and model quadratic and exponential relationships.

h) Compare and contrast the key characteristics of linear functions ($f(x) = x$), quadratic functions ($f(x) = x^2$), and exponential functions ($f(x) = b^x$) using tables and graphs.

Related SOL: A.F.1.f-h; A.F.2.c-g

Materials

- Graphing utility
- Rabbits worksheet (included)

Vocabulary

linear, quadratic, exponential

Note about the activity:
*As this activity is about graphing functions from data, and comparing various functions, it is assumed that this activity is used **after** students have familiarity with linear, quadratic, and exponential functions. With modifications, this activity could be used as an introduction to various function-types.*

Teacher Resources:
Learn Desmos: Regressions (YouTube)
Desmos: How to find an exponential regression equation (YouTube)
These two videos may be helpful in using the regression curves with Desmos.

Student/Teacher Actions: What should students be doing? What should teachers be doing?

1. Every student should have a copy of the included paper response sheet. It is encouraged to use ***one (1) graphing utility per two (2) students*** to encourage conversation.
2. The writing and prediction steps are essential as students are developing conceptual understanding. Encourage students to compare their thinking with other groups. The written summaries can serve as notes to reference in the future.
3. As students are asked to sketch their prediction of the graphs, encourage students to explain their sketches to their partner *before* they use the graphing utility to

check their ideas. Discuss the importance of what should be precise and what can be a rough sketch.

Assessment

- **Questions**
  - What similarities exist between linear and exponential functions? How do they differ?
  - What similarities exist between exponential functions and quadratic functions?
  - How do linear and exponential functions differ from quadratic functions?
- **Journal/writing prompts**
  - Describe the distinguishing characteristics that can help to determine which type of function is most likely when provided a graphical representation of the data.
  - Describe the distinguishing characteristics that can help to determine which type of function is most likely when provided a tabular representation of the data.
- **Other Assessments**
  - Create a Venn diagram to compare and contrast linear, quadratic, and exponential functions.

Extensions and Connections (for all students)

- Quadratic: Explore the jumping abilities of various animals and model their jumps. Which animal jumps proportionally the farthest relative to its body size?
- Exponential: Research what happened to the actual rabbit population. Did it plateau at any point? What were the causes? How could a piecewise function model the situation?
- Linear: Explore the speed of various animals. How long can each animal sustain its top speed? Which is preferable: a quick sprint or a more sustained run?

Strategies for Differentiation

- The conversation about the exponential growth doubling every six months may need considerable support for many students. The constants $h_o, r,$ and d may need significant support for students to discover.

Note: The following pages are intended for classroom use for students as a visual aid to learning.

Rabbits

Name ____________________________________ Date_________________________

Materials for each group

- Graphing Utility

In this activity we are going to discuss three stories about rabbits. With each story, we will try to model the situation using the appropriate function.

Linear: $f(x) = ax + b$ Quadratic: $g(x) = ax^2 + bx + c$ Exponential: $h(x) = h_o \cdot r^{\frac{x}{d}}$

h_0 indicates the initial amount

r indicates the rate of growth

d indicates the frequency the rate is repeating

Using Desmos for curve fitting:

Linear: $y_1 \sim ax_1 + b$ Quadratic: $y_1 \sim ax_1^2 + bx_1 + c$ Exponential: $y_1 \sim a \cdot r^{\frac{x_1}{d}}$

Before we begin, use your previous knowledge to draw a rough sketch of each function-type below.

Story 1: How far? How high?

Sources to Investigate:
Protect My Paws: https://protectmypaws.com/how-high-can-rabbits-jump

Rabbits are known to be able to jump up to 4 feet high and a distance of up to 9 feet.

If this is the case, we can predict three points. [Consider x_1 as the horizontal distance from the starting point and y_1 as the vertical distance from the ground.] ***Justify each value you add to the table.***

x_1	y_1
	4
9	

Write:
What shape would the path of the rabbit take? Describe and draw a sketch of what you predict the graph will look like.

Which of the three function types would best model the situation? ***Justify your choice of a function.*** *What do we know in advance about any of the constants?*

Use your graphing utility to plot the points from your table. Find the appropriate regression curve to model the situation. Draw a sketch of your graph here. [Make sure you label the units on each axis and any important coordinates.]

We have now modeled a rabbit jumping on level ground. What would happen if the rabbit jumped off a small hill?

Write:

*If the rabbit is jumping from a starting point that is 3 feet higher than where it will land, how do you think the horizontal distance traveled will compare to a jump on level ground? **Justify your answer.***

(Write and sketch) What part of the function will change? Rewrite your function and graph it using your graphing utility. Draw a sketch of BOTH curves here.

What is the horizontal distance traveled by the rabbit when starting from this initial, increased height?

Story 2: How many?

European rabbits were not native to Australia. In 1959, Thomas Austin introduced 13 rabbits into the wild for hunting. However, with no natural predators, the population began to grow quickly. The population of rabbits was found to double every six months.

Sources to Investigate:
National Museum of Australia: https://www.nma.gov.au/defining-moments/resources/rabbits-introduced
National Geographic: https://education.nationalgeographic.org/resource/how-european-rabbits-took-over-australia/

Complete the chart to show what it looks like when rabbits double every six months.

Months	Years	Rabbits
0	0	13
6	0.5	
12	1	
18	1.5	
24	2	

Write:

What shape would the growth of the rabbit population have on a graph? Describe and draw a sketch (above) of what you predict the graph will look like.

Which of the three function types would best model the situation? ***Justify your choice of a function.*** *What do we know in advance about any of the constants?*

Use your graphing utility to plot the points from your table. Find the appropriate regression curve to model the situation. Draw a sketch of your graph here. [Make sure you label the units on each axis and any important coordinates.]

We have now modeled the initial population growth for the rabbits. But their growth continued for several decades.

Write:

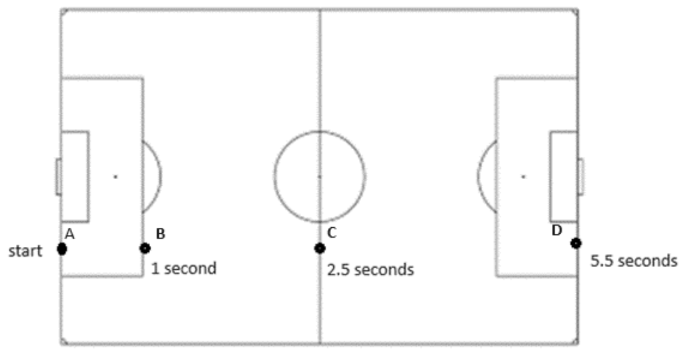
How many rabbits were present 10 years after they were introduced to Australia? What about after 20 years?

Assuming the pattern continued, how many rabbits would be present today?

Optional: Research the rabbit proof fences that were attempted to contain the growing populations of rabbits.

Story 3: How fast?

Rabbits are fast! Javier was finishing soccer practice when he saw a rabbit sprinting the length of the soccer field. The rabbit started at point A. It took one second to travel from the end line through the penalty box (18 yards) to point B; a total of 2.5 seconds to reach midfield (60 yards) at point C; and 5.5 seconds to finish the run across the length of the field ending at point D.



Document the distance the rabbit traveled in the table below.

Time (sec) x	Distance (yds) y
0	
1	
2.5	
5.5	

Write:

What would be the shape of the graph of the rabbit's distance traveled over time? Describe and draw a sketch of what you predict the graph will look like.

*Which of the three function-types would best model the situation? **Justify your choice of a function.** What do we know in advance about any of the constants?*

Use your graphing utility to plot the points from your table. Find the appropriate regression curve to model the situation. Draw a sketch of your graph here. [Make sure you label the units on each axis and any important coordinates.]

Now that we have modeled the path of the rabbit across the soccer field, we could find how far the rabbit traveled at different times.

Write:

How far would the rabbit travel in 10 seconds? ***Justify your answer.***

How far would the rabbit travel in 30 seconds? ***Justify your answer.***

We can also explore the speed of the rabbit.

Write:

From your function, what is the speed of the rabbit? What are the units of this speed?

Convert this speed into ft/sec. Convert this speed into mph.