

Simplify and Determine Equivalent Rational Exponents of Whole Numbers

Strand:	Expressions and Operations
Topic:	Simplifying square roots of whole numbers
Primary SOL:	A.EO.4 The student will simplify and determine equivalent radical expressions involving square roots of whole numbers and cube roots of integers. d) Generate equivalent numerical expressions and justify their equivalency for radicals using rational exponents, limited to rational exponents of $\frac{1}{2}$ and $\frac{1}{3}$ (e.g., $\sqrt{5} = 5^{\frac{1}{2}}$; $\sqrt[3]{8} = 8^{\frac{1}{3}}$; $(2^3)^{\frac{1}{3}} = 2$).
Related SOL:	7.NS.1, 8.PFA.1, A.EO.1a, A.EO.1b

Materials

- Calculator
- Instructional Page
- Activity Sheet

Vocabulary

radical expression, radicand, square root, whole numbers, integer, cube root, radical exponent

Student/Teacher Actions: What should students be doing? What should teachers be doing?

1. Using the Instructional Page, students should work with a partner or in a small group to review the concepts of exponent rules. Upon completion of Part I of the activity, one representative from the group or partnership should check their answers. While students are working, the teacher should be circulating to answer questions and ensure that students have strong background knowledge on this concept.
2. Part II should be primarily a teacher-led section; however, scaffold this information and allow students to participate and claim ownership to the new idea. For instance, ask students to work with their partners or small groups and using their knowledge of exponent rules, determine the value of p in the following $(8^p)^3 = 8$ and the value of q in the following $4^q \cdot 4^q = 4$. Their exponential rule background should enable students to understand that the exponents are multiplied for the first example and the exponents are added in the second example. Students should utilize the space given, titled Part II, on the instruction page to record their ideas and conclusions.
3. After students have had some time to work with the given examples, the teacher should demonstrate the values of p and q with teacher led instruction. The teacher should demonstrate the algebraic equivalence by showing:

$$(8^p)^3 = 8$$

$$8^{3p} = 8$$

(Using just the exponents) $3p=1$ (as the exponent of a whole number is 1), therefore $p=\frac{1}{3}$

The teacher should demonstrate the second example in a similar fashion.

$$4^q \cdot 4^q = 4$$

$$4^{2q} = 4$$

(Using just the exponents) $2q=1$, therefore $q=\frac{1}{2}$

Now that students are aware of fractional exponents, have students work with a partner to determine what is equivalent to rational exponents. Give students a few examples to consider, for example $16^{\frac{1}{2}} = 4$ or $27^{\frac{1}{3}} = 3$. Circulate to monitor progress, answer questions, and select student work to display using a demonstration tool (e.g., document camera, digital display). Guide students as needed and ensure that all students have concluded that the rational exponent of $\frac{1}{2}$ is equivalent to the square root and the rational exponent of $\frac{1}{3}$ is equivalent to the cube root.

4. The final section gives students an opportunity to simplify square roots, cube roots and determine equivalent rational exponents individually. The teacher should circulate to answer questions. Upon completion of this section, students can compare responses with another student and discuss discrepancies before submitting their work.

Assessment

- **Questions**

- Explain the equivalence between $5^{\frac{1}{3}} \cdot 5^{\frac{1}{3}} \cdot 5^{\frac{1}{3}}$ and $(5^{\frac{1}{3}})^3$. Demonstrate your understanding by utilizing the rules of exponents.

- **Journal/writing prompts**

- In your own words, explain the relationship(s) between the numbers 64, $\sqrt{64}$ and $64^{\frac{1}{3}}$.

- **Other Assessments**

- Exit tickets could be issued asking students to simplify an expression like $128^{\frac{1}{2}}$.

Extensions and Connections (for all students)

- Questions that apply the Pythagorean Theorem (taught in Mathematics 8) could be posed with the expectation that solutions be expressed in simplest radical form rather than rounding a decimal answer.
- This lesson could extend into one where Algebra 1 students simplify the square root of a monomial expression. Questions could be asked where students are given the area of a square and are asked to determine a side length.
- Students have background knowledge concerning prime factorization, perfect squares, and square roots. Instruction on this content begins in Grade 5 and continues through Grade 8, where students learn to determine the two consecutive integers between

Mathematics Instructional Plan – Algebra 1

which a square root lies. Algebra I students can apply the simplification of a square root when solving quadratic equations later in the course.

- Square roots are applied in many formulas used in high school science courses. Teachers may prefer exact, simplified values rather than rounded decimal approximations.
- Provide students with opportunities to work with equivalent rational exponents and radical expressions, for example provide students with problems involving $(9n^3)^{\frac{1}{2}}$.

Strategies for Differentiation

- Provide students with graph paper.
- Provide a graphing utility for students to check their simplified radical against its equivalent value.
- Allow for collaboration on the “individual practice” portion of the activity sheet.

Note: The following pages are intended for classroom use for students as an instructional aid to learning.

Student Activity Page

I. Review/Background Information: “Product” Rule & “Power to a Power” Rule of Exponents

Product Rule: Given the following chart, fill in the missing information.

	Expanded Form	Exponential Form
$3^1 \cdot 3^5$		
$3^2 \cdot 3^3$		
$3^1 \cdot 3^3$		
$3^2 \cdot 3^1$		
$3^1 \cdot 3^1$		

Power to a Power Rule: Given the following chart, fill in the missing information.

	Expanded Form	Exponential Form
$(4^3)^4$		
$(4^3)^3$		
$(4^3)^2$		
$(4^2)^2$		
$(4^2)^1$		

II. Determining the value of p and q , given $(8^p)^3 = 8$ and given $4^q \cdot 4^q = 4$.

Use this space provided to record your thoughts, ideas, and conclusions.

III. Individual Practice

Simplify each square root to its simplest form using the method of your choice.

Unsimplified	Simplified (with radical)	Simplified (with rational exponents)
$\sqrt{125}$		
$\sqrt[3]{250}$		
$\sqrt{300}$		
	$4\sqrt{3}$	
	$6\sqrt{2}$	
$\sqrt[3]{192}$		
		$6(3)^{\frac{1}{3}}$
	$5\sqrt[3]{6}$	