Whole-number Exponents and Perfect Squares

Strand: Number and Number Sense

Topic: Investigating positive exponents and perfect squares

Primary SOL: **6.NS.3 The student will recognize and represent patterns with whole number exponents and perfect squares.**

Students will demonstrate the following Knowledge and Skills:

a) Recognize and represent patterns with bases and exponents that are whole numbers.

b) Recognize and represent patterns of perfect squares not to exceed 20^2, by using concrete and pictorial models.

c) Justify if a number between 0 and 400 is a perfect square through modeling or mathematical reasoning.

d) Recognize and represent powers of 10 with whole number exponents by examining patterns in place value.

Materials

- Cubes (enough for 36 per student or student pair)
- Perfect Square Grids activity sheet (attached)
- Perfect Squares Notes (attached)
- Recognize Powers of 10 activity sheet (attached)
- Multiplication Chart (attached)
- Graph Paper
- Perfect Squares Chart activity sheet (attached)
- Markers, crayons, or colored pencils in at least nine different colors

Vocabulary

powers, exponents, base, square root, perfect square, exponential notation, expanded notation

Student/Teacher Actions: What should students be doing? What should teachers be doing?

1. Give each student or student pair 36 cubes. Tell them that the area of a square is 36 cubes but no other information. Only knowing the area, can you make a square and determine the length and width?
2. Now ask students to use only 22 of their cubes. Can you make a perfect square using all 22 cubes? Why/Why not?
3. Distribute the Perfect Square Grids activity sheet and markers, crayons, or colored pencils so that each student has nine different colors. For each grid on the handout, have students shade the boxes along the diagonal from bottom left to top right, using a different color for each grid. Have students write down their observations about each grid (e.g., number of blocks along the diagonal, numbers of blocks on the vertical and horizontal, total number of blocks, way to calculate the total number of blocks).

4. Question students about their observations. They should notice that each grid forms a square. Ask why they are classified as squares. (Width and length are the same.) Also, they should notice that whatever number of blocks are along the diagonal corresponds with the number of blocks in the vertical rows and in the horizontal rows.
5. Give students graph paper and ask them to draw the smallest square that they can. Then ask them to draw several additional squares on the recording sheet. Have a classroom discussion having students prove a number is a perfect square. Then have students prove why some numbers are not perfect squares.
6. Distribute the multiplication charts and display a large version of the chart. Ask students how each of the grids can be modeled on the multiplication chart. Ask whether they can use the multiplication chart to depict other "square relationships" that are not on the grids. Explain that each square was derived from multiplying a number by itself, producing a "perfect square."
7. Ask students to describe the connection between perfect squares and the area model. Discuss with students the patterns they see as they identify perfect squares using the Multiplication Chart. Ask students what observations they see between the Multiplication Chart and the Perfect Square Grids. Facilitate a discussion with students as to why some numbers are considered perfect squares (e.g., 25, 36) and why others (e.g., 15, 18) are not. This difference can be modeled using grid paper or square tiles. Help students recognize that the areas of these numbers are determined by their factors. However, the area does not create a perfect square; instead, a rectangular model is formed.
8. Distribute Perfect Squares Notes. At this point, have students incorporate their notes by explaining/reviewing how to rewrite $9 \cdot 9$, written in expanded form as "9 x 9,", in exponential form as 9^2. Have them complete their notes by naming *x* as the base and *n* as the exponent (which is exponential form or exponential notation). Tell them how to read it ("nine squared" or "nine to the second power").
9. Distribute the Perfect Squares Chart, and have students complete it.
10. Distribute the Recognize Powers of 10 activity sheet. Students will recognize powers of 10 with whole-number exponents by examining patterns in place value. Model and discuss how to complete the table. Students must find the missing values to complete the table using the patterns that they observe. Once students have completed the table, they should provide a response to the two prompts given. Discuss students' findings as a whole group.
11. Like the Recognize Powers of 10 activity sheet, have students expand and compute using bases and exponents that are whole numbers to determine any patterns that exist.

Assessment

- **Questions**
  - How can the area model be used to relate perfect squares to multiplication?
  - Using square tiles or grid paper, model how we know the numbers 6 and 12 are *not* perfect squares. Explain and justify your reasoning.
  - Using square tiles or grid paper, model how we know the numbers 16 and 49 are perfect squares. Explain and justify your reasoning.
  - Is zero to the zero power (0^0) a perfect square?
  - Why is any real number other than zero raised to the zero power equivalent to 1?

- **Journal/writing prompts**
  - Derrick stated, "The number 225 is not a perfect square because it is an odd number." Sarah stated, "You are incorrect. The number 225 is a perfect square. In fact, there can be even and odd numbers that are perfect squares." Explain who is correct in this scenario and justify your reasoning using a concrete representation, pictorial representation, examples, and/or counterexamples.
  - Suppose you are given 10^2, 10^3, and 10^4. Provide the next three terms in the pattern and explain how you arrived at your answer.

- **Other Assessments**
  - Have students create a scenario where they must justify their identification of a perfect square using an area model, a concrete representation, or an algorithm.
  - Have students create a commercial about perfect squares, representing patterns with whole-number exponents, and/or powers of 10.

Extensions and Connections (for all students)

- The formula for area of a square is Area = s^2. How does this relate to perfect squares?
- Provide other numerical examples of positive exponents with powers greater than 2.
- Assist students in recognizing how the exponent is related to powers of 10.

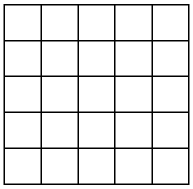
Strategies for Differentiation

- Some students may need to use the Multiplication Chart throughout the lesson to complete the perfect squares grids.
- Some students may need to number the blocks to see a connection between the grid and the Multiplication Chart.
- Pre-teach/review necessary vocabulary for students, as needed.
- Provide two or three examples in the Expanded Form column of the Recognize Powers of 10 activity sheet, if necessary, for students.
- Work with a peer on the Perfect Square Grids activity sheet to make their observations and discoveries orally.

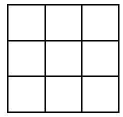
Note: The following pages are intended for classroom use for students as a visual aid to learning.

Perfect Square Grids

Name ________________________________ **Date** ______________________



Observations



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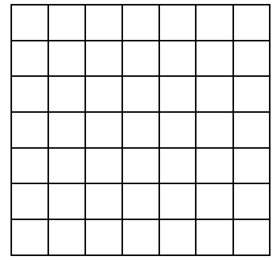

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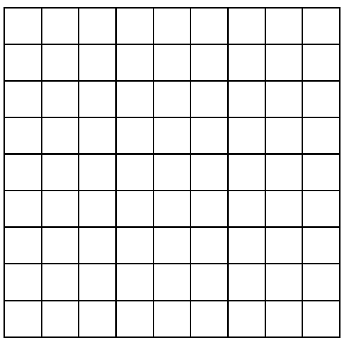

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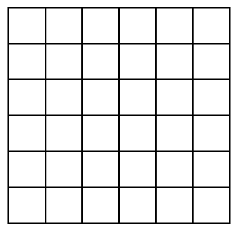

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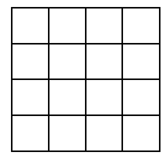

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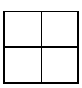

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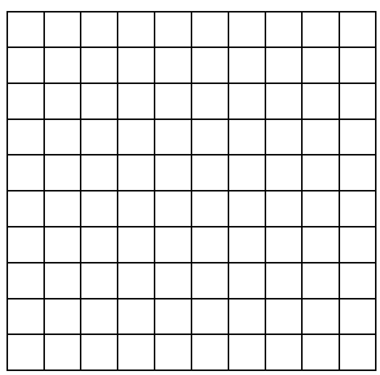

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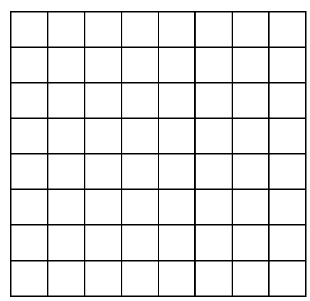


Observations

Perfect Squares Notes

Name ____________________________________ Date ______________________

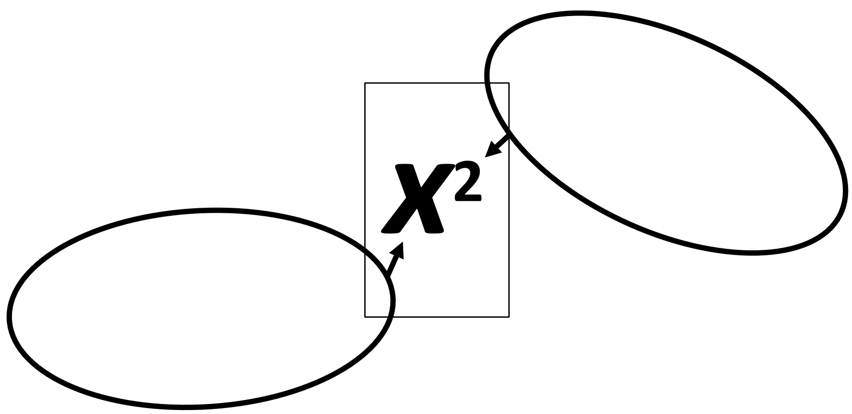
How can I write 9 · 9 in a condensed form? _______

This can be read as "_____________________" or "________________________."

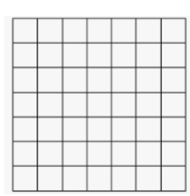
How can I write 4 · 4 in a condensed form? _______

This can be read as "_____________________" or "________________________."

What are the parts called?



Use the diagram below to write a perfect square.



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Recognize Powers of 10

Name ________________________________ **Date** ______________________

Determine the missing values to complete the table by examining patterns in place value.

Exponential Notation	Expanded Form	Value
10^0		1
10^1		10
10^2		100
10^3		__________________
_____		10,000
10^5		__________________
10^6		__________________
_____		__________________
10^8		__________________
_____		__________________
_____		10,000,000,000
10^{11}		__________________
_____		__________________

Explain the patterns in place value you observe.

Based on this pattern, what is the value of 10^{15}? Explain your reasoning.

Prove that any real number other than zero raised to the zero power is equal to 1.

Multiplication Chart

	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**
1	1	2	3	4	5	6	7	8	9	10	11	12
2	2	4	6	8	10	12	14	16	18	20	22	24
3	3	6	9	12	15	18	21	24	27	30	33	36
4	4	8	12	16	20	24	28	32	36	40	44	48
5	5	10	15	20	25	30	35	40	45	50	55	60
6	6	12	18	24	30	36	42	48	54	60	66	72
7	7	14	21	28	35	42	49	56	63	70	77	84
8	8	16	24	32	40	48	56	64	72	80	88	96
9	9	18	27	36	45	54	63	72	81	90	99	108
10	10	20	30	40	50	60	70	80	90	100	110	120
11	11	22	33	44	55	66	77	88	99	110	121	132
12	12	24	36	48	60	72	84	96	108	120	132	144

Perfect Squares

Side dimensions and area	Illustration
1 x 1 = 1	
2 x 2 =	
3 x 3 =	
4 x 4 =	
5 x 5 =	
15 x 15 =	
18 x 18 =	

Some numbers are called perfect squares. For example, 100 is a perfect square. Can you prove that 100 is a perfect square? Use a model or mathematical reasoning to justify your answer.

However, 10 is not a perfect square. Prove that 10 is not a perfect square. Use a model or mathematical reasoning to justify your answer.

Perfect Squares

n	n^2
0	
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	
11	
12	
13	
14	
15	
16	
17	
18	
19	
20	

What is the relationship between a geometric square and a perfect square? You may use an illustration or mathematical reasoning to prove your answer.

Do you see a relationship between the multiplication and a perfect square? If so, explain the relationship.

Fill in the missing number in the pattern:

- 36, 49, 64, _____, 100, 121, 144, _____, 196, 225
- 225, 256, _____, 324, 361, ______
- 9, 16, ______, 36, ______, 64, 81, 100, ______, _______, 169, 196
- Create your own pattern using perfect squares:

  _______, _______, _______,________, ________, ________

In your own words, explain why these numbers are called perfect squares.