Multiple Madness: Common Multiples, Least Common Multiples, and Fractions

Strand: Computation and Estimation

Topic: Determining multiples, common multiples, and least common multiple

Primary SOL: **5.CE.2 The student will estimate, represent, solve, and justify solutions to single-step and multistep problems, including those in context, using addition and subtraction of fractions with like and unlike denominators (with and without models), and solve single-step contextual problems involving multiplication of a whole number and a proper fraction, with models.**

a) Determine the least common multiple of two numbers to find the least common denominator for two fractions.

Materials

- Number Cards for Multiple Madness (attached)
- Multiple Madness Hundreds Chart (attached)
- Multiple Madness Recording Sheet (attached)
- Game markers (e.g., colored chips, colored cubes) in two different colors
- Large hundred chart for display (optional)
- Making Connections Recording Sheet (attached)

Vocabulary

common multiples, least common multiple (LCM), multiples, addition, fraction, numerator, denominator, simplest form, least common denominator

Student/Teacher Actions: What should students be doing? What should teachers be doing?

Note: Prepare for this activity by creating a set of Number Cards for Multiple Madness for each pair of students.

1. Ask students whether the word *multiple* makes them think of a word that teachers and students use sometimes in mathematics class. Listen for multiplication, then let students know that *multiple* is the root word of the word *multiplication*. Write the number 6 on the board, and ask students to volunteer a number they think is a multiple of 6. Write *Multiple of 6* and *Not a Multiple of 6* on the board, and let the students know you will record their number in the appropriate list. When there are several numbers in the two lists, stop.
    a. Now the teacher will call several additional numbers and ask students which list the number goes in. Call out numbers that are multiples of 6 and numbers that are not multiples of 6. If 6 is not already in the multiples of 6 list, be sure to call that one out and clarify with students why 6 is a multiple of itself (i.e., 1 x 6 = 6). After you have added several more numbers to both lists, ask students to look at the two lists and talk with a shoulder partner to discuss how you can decide whether a number goes in the "not a multiple of 6" list. Then have a couple of

partners share. Listen for the words that students use, and highlight important ideas that are shared, such as: There isn't a number you can multiply by six to get this product; if you skip count by sixes starting from 0, you will not say this number; if you divide the number by six, there will be a remainder, etc. Support students in understanding the connection in these different ways.

b. Let students know that they are going to explore multiples of different numbers and find *common multiples* of two different numbers. Have students discuss the meaning of the word *common* and the terms *common multiples* and *least common multiple.*

2. Pair students and give each pair a pile of two different colored game markers and the Multiples Madness Hundreds Chart. Model play of the Multiples Madness Game as follows. (A large hundreds chart can be used to display multiples for the class.)
    a. Tell students they will work in pairs. Using the Multiples Madness Hundreds Chart, have students cover all the multiples of 6, starting at 6, through 100. Walk around the room and ask probing questions to students who are having trouble placing markers on the correct numbers. Next, have the students tell you which numbers to cover on the class hundreds chart to show all multiples of 6. Then ask: *"How did you know which numbers to cover?" "What patterns do you notice on the hundreds chart when you examine the covered multiples of 6 and uncovered numbers that are not multiples of 6?"*
    b. Next, ask student pairs to use the other colored marker and cover all multiples of 4, starting at 4, through 100. Monitor students as they place the game markers, as was done with multiples of 6. Have students tell you which numbers to cover on the class hundreds chart to show all multiples of 4. Then ask: *"How did you know which numbers to cover?" "What patterns do you notice on the hundreds chart when you examine the covered multiples of 4 and uncovered numbers that are not multiples of 4?"*
    c. Ask: *"What numbers are covered by both colors of game markers?" "Why do those numbers have both colors of game markers?" "What would be a mathematical term we could use to describe the numbers that are covered by both colors of game markers?"* Listen for the term *common multiple* or something that means common multiple. Clarify with the students so they understand that the multiples the two numbers have in common are called common multiples. Then point to 12 and ask, *"What is special about that number?"* Since it is the first number that has both colors of marker, then 12 is the *least common multiple.* Ask students whether they think there is a greatest common multiple. Through class discussion, clarify that we could keep adding hundreds charts for the 200s and beyond to keep showing multiples confirming that there is no *greatest common multiple*.
3. Let the students know that you are going to share another way to find common multiples and the least common multiple. Complete the following list on the board. Let students know that you are going to list the first 10 multiples and use the three dots to show that the pattern continues. It helps to use colors or underlining to show the

common multiples. Then identify the least common multiple. Use the full phrase and the abbreviation, LCM, together for this lesson and for future work in class. Now write the numbers 4 and 6 on the board, one above the other so that you can list multiples horizontally. List multiples, common multiples, and least common multiple as follows:

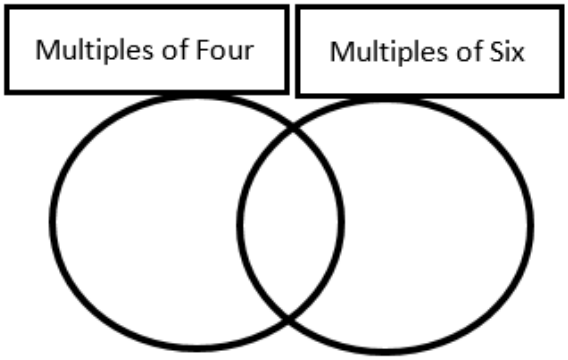
Multiples of 4: 4, 8, 12, 16, 20, 24, 28, 32, 36, 40, …

Multiples of 6: 6, 12, 18, 24, 30, 36, 42, 48, 54, 60, …

Common multiples of 4 and 6: 12, 24, 36, …

The least common multiple (LCM) is 12.

You may also want to show how to find the common multiples with a Venn diagram:



4. Give pairs of students the two numbers, 5 and 10, and ask them to use one of the methods to identify multiples of 5 and 10 and the least common multiple of 5 and 10. Use a second method to check their work. When they are done, have different pairs compare what they found. Tell students that if their answers are different from another pair, they need to justify their answer. Debrief as a whole group. Ask: *"Why is one of the numbers, 10, the least common multiple?" "When given two numbers to find the least common multiple, will one of those numbers always be the least common multiple?"*
5. Distribute the Multiple Madness Recording Sheet. Students continue to work in pairs, and each pair will need a set of Number Cards for Multiple Madness, a Multiples Madness Hundreds Chart, and two piles of colored game markers in two different colors. Model for students how to play the game:
    - Each player gathers a pile of colored game markers in a color different from that of their opponent.
    - Player 1 draws a number card from the pile and identifies the multiples of that number by marking them on the hundreds chart with their colored game markers. Player 1 also records the multiples of the number below the hundreds chart.
    - Player 2 draws a number card and repeats the same steps.
    - If a player draws a previously drawn number, the player places the card back in the pile, shuffles the pile, and draws again.
    - Once players have identified and recorded the multiples of their numbers, they compare multiples and fill in a Venn diagram graphic organizer on the recording sheet. They also find the least common multiple (i.e., the smallest number in the overlapping parts of the two circles).

6. Ask students to use the horizontal listing strategy to find the first ten multiples for 8 and 9, identify the common multiples, and then identify the least common multiple. When students are finished, facilitate a class discussion that develops why 72, which is also the product of 8 and 9, is the least common multiple, but the least common multiple of 6 and 4 is not 24.
7. Discuss with the class what they have discovered about the multiples of the pairs of numbers. Ask students specifically how knowing multiples of a number can help them. Ask, *"Have there been any other things we have done in math where knowing the multiples of a number would have helped?"*
8. Tell students that they will be using this process to help them find common denominators for fractions that have unlike denominators. Have students talk with their partner about fractions. Ask questions to access students' prior knowledge. *("What are fractions?" "What is the numerator? What does it represent?" What is the denominator? What does it represent?" "What does the algorithm tell us about combining fractions?", etc.)* Students should have prior knowledge of adding and subtracting fractions with like denominators.
9. Give students the Making Connections to Fractions Recording Sheet. **(Students are not asked to solve the problem. The focus is on the relationship between multiples and finding a common denominator.)** Read the first problem to the students. If we want to answer a question such as, "How much more cake did Sam eat than his brother?" we need to have a common denominator. Tell students, "When using the standard algorithm to solve fraction problems with unlike denominators, you must find a common denominator." In partners, have students discuss how finding common multiples could help us to find a least common denominator between fractions with unlike denominators. Have volunteers share their ideas.
10. Ask students to work with their partner to find the least common denominator for the problem.
11. Have a volunteer share the common denominator they found and justify how they found their answer.
12. Working with their partner, have them find the common denominator they would use for the remaining problems on their Making Connections Recording Sheet.

Assessment

- **Questions**
    - Can two numbers always be multiplied together to find the least common multiple of the numbers? Provide examples of numbers to support your answer.
    - Why do you think that one of the rules for Multiple Madness is to put the cards back in the pile and shuffle, if both players draw the same number?
    - How can understanding multiples help you with other mathematical concepts? Give an example.
    - Is there a limit to the number of multiples a number can have?
    - What is the difference between factors and multiples?

  - How do least common multiples help us solve fraction problems with unlike denominators?
  - Why is it important to have common denominators when solving addition and subtraction problems?

- **Journal/writing prompts**
  - Explain how this mathematical concept about least common multiples could relate to real-life situations.
  - What is the least common multiple for 10 and 8? Justify your answer.
  - Explain the connection between least common multiples and fractions with unlike denominators.

- **Other Assessments**
  - Complete an exit ticket explaining why 12 would be the least common denominator for $\frac{1}{3} + \frac{1}{4}$.
  - Have students solve the problems on the Making Connections Recording Sheet.

Extensions and Connections (for all students)

- Have students create a way to help other students know the difference between factor and multiple.
- Have students identify an example where the product of the two numbers is the least common multiple; an example where one of the numbers is the least common multiple of two numbers; and an example where the least common multiple is neither of the numbers nor the product of the two numbers.
- Draw a three-circle Venn diagram to show multiples of 4, 6, and 8. Describe how you would find the common multiples and LCM for these three numbers.
- Solve the following problem. "The principal has some tickets for the fall festival and some apples to give away. She does not have enough for everyone, and you overheard her tell the teachers that she plans to give every sixth person in line an apple, and every 10th person in line a free ticket to the fall festival. If there are 100 people in line, what ordinal positions should you stand in to make sure you get both a free ticket and an apple?"

Strategies for Differentiation

- Allow students to use a calculator or multiplication chart for finding multiples.
- Create anchor charts of Venn diagrams to display in the room.
- Use word wall cards to help students struggling with vocabulary.
- Have students create a vocabulary section in their mathematics notebooks to help with challenging vocabulary words. Let students draw pictures to illustrate the terms.

Note: The following pages are intended for classroom use for students as a visual aid to learning.

Number Cards for Multiple Madness

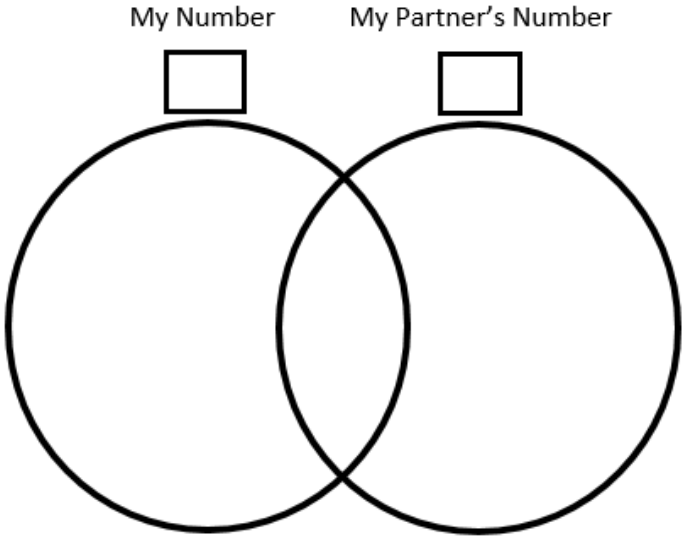
Copy the cards on card stock, and cut out.

2	3	4
5	6	7
8	9	10
11	12	12
3	4	5
6	7	8
9	10	11

Multiple Madness Hundreds Chart

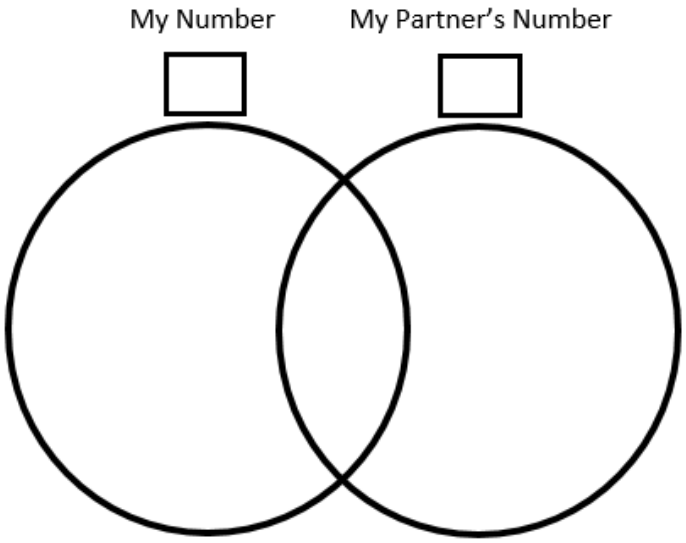
1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Multiple Madness Recording Sheet



What is your least common multiple (LCM)? How do you know?

Multiple Madness Recording Sheet



What is your least common multiple (LCM)? How do you know?

Making Connections Recording Sheet

Name:_____ Date:_____

1. Sam ate $\frac{1}{4}$ of a cake. His brother ate $\frac{1}{12}$ of the cake. What common denominator could be used to determine how much more cake Sam ate than his brother?

2. Ailin completed $\frac{1}{6}$ of her homework at school and $\frac{1}{8}$ of her homework on the bus. What common denominator could be used to determine how much homework has Ailin completed?

3. Myasiah ran $\frac{1}{2}$ mile on Wednesday and $\frac{3}{4}$ mile on Thursday. What common denominator could be used to determine how much farther Myasiah ran on Thursday?

4. Carlos ate $\frac{3}{8}$ of a pizza. His friend Randy ate $\frac{1}{12}$ of the pizza. What common denominator could be used to determine how much the boys ate in all?

5. Carson gave Mary $\frac{1}{6}$ of his candy. He gave Brady $\frac{1}{4}$. of his candy. What common denominator could be used to determine how much candy Carson had left?