

Multiplying Two-Digit Numbers: Bridging from the Concrete to the Symbolic

Strand:	Computation and Estimation
Topic:	Multiplying two-digit by two-digit whole numbers using concrete and symbolic representations
Primary 2023 SOL:	4.CE.2 The student will estimate, represent, solve, and justify solutions to single-step and multistep problems, including those in context, using multiplication with whole numbers, and single-step problems, including those in context, using division with whole numbers; and recall with automaticity the multiplication facts through 12×12 and the corresponding division facts. g) Apply strategies (e.g., rounding, place value, properties of multiplication and/or addition) and algorithms, including the standard algorithm, to estimate and determine the product of two whole numbers when given: i. a two-digit factor and a one-digit factor;* ii. a three-digit factor and a one-digit factor;* or iii. a two-digit factor and a two-digit factor.* * On the state assessment, items measuring this objective are assessed without the use of a calculator.

Materials

- Base-10 materials for ones (units), rods (tens), and flats (hundreds)
- Grid paper
- Multiplication Models activity sheet (attached)

Vocabulary

area model, array model, factor, multiply, partial products, product, symbolic

Student/Teacher Actions: What should students be doing? What should teachers be doing?

1. Using base-10 blocks, demonstrate an array to show the product of 5×6 . Then, ask the students to use base-10 blocks to show the product of 5×16 . Choose a student to model a solution for the class, using the blocks. Explain to students that today they will be exploring the area of a rectangle whose dimensions are larger factors. The Multiplication Models activity sheet is a helpful document to use for this lesson.
 - a. Lead a discussion to brainstorm ways of using the area model to find the product of 13×34 . Have students share their suggestions.
 - b. Demonstrate how to build a 13-by-34 rectangle with base-10 blocks. Begin by representing 13 with one rod and three units to correspond to the length of the rectangle. Then represent 34 with three rods and four units placed to correspond to the width of the rectangle. Remind students that they were able

to find the product of 5×6 by completing a rectangle that was five units wide and six units long and that the same process can be used to find the product of two-digit factors.

Ask, “What blocks can be used to complete the 13-by-34 rectangle, using the fewest number of blocks?” Have them start with the largest blocks, the flat. Model placing the three flats in the rectangle and point out that the sides of the flats are adjacent to the rods that represent the factors.

Ask, “How many rods will fit in the remaining area so that the length of the rod is adjacent to the flat or another rod.?” Next, ask, “How many units will fit in the remaining area so that the unit piece is adjacent to the end of a rod or another unit piece?” At this point, students should have created a rectangle containing three flats, 13 rods, and 12 units. Ask them to determine the area of the rectangle. After trading in units for rods and rods for flats, students should have a final value of four flats, four rods, and two units, and the value is 442 units.

2. Have students work in pairs to find the product of 32×25 , using the same process with the base-10 materials to find the product. When the groups have finished building their array model and they have determined the product, ask a group to use a demonstration tool or the base-10 blocks at the front of the room to review the process and how they determined the product.
3. Drawing the base-10 pieces is tedious, and the goal is to support the students as they learn an efficient way to multiply that builds from their understanding of the operation of multiplication developed with the base-10 materials. Bridge from the base-10 model to a model that uses the area of a rectangle and decomposition of numbers to bridge from the proportional representation to a symbolic process of multiplication.
4. Let students know they are going to learn another way to find 32×25 . As you introduce the new model, tell students you want them to take notes and you will remind them to record the important information as it is discussed. Set up a graphic organizer by drawing a rectangle. Show two rows by two columns, because the factors are both two-digit numbers as shown below but at first without any numbers.
5. Ask, “What base-10 blocks do you need for 25?” This should be two rods and five units. Write the value of 25 above the chart and ask the students, “What base-10 blocks did you use for 25?” Make the connection that, because 25 is made with rods and units, it can be treated as $20 + 5$, which is just decomposing into tens and ones, or writing the number in expanded form. On the graphic organizer write $20 + 5$, as shown below. Then write the value of 32 on the side. Ask, “How should we decompose the 32?” Write $30 + 2$, as shown below. Remind students to record this in their notebooks.

	20	$+$	5
30			
+			
2			

- Let the students know that they are going to use this open area model to help keep track of the numbers as they multiply. Facilitate a guided class discussion to find the partial products. Start by writing 30×20 in the upper-left area, and ask students for the partial product (600), then record the 600.

Continue the same way with 30×5 , 2×20 , and 2×5 . You are using the distributive property of multiplication; however, students do not have to remember the name of the property. Ask, “How does this look the same as your base-10 block model? What are we going to do with these partial products that we have?” Once students add the partial products together, ask “Is this the same product or answer for 32×25 as your base-10 blocks model shows?” Pose questions that highlight how place value is a part of this process.

	$20 + 5$	
30	$30 \times 20 = 600$	$30 \times 5 = 150$
$+$		
2	$2 \times 20 = 40$	$2 \times 5 = 10.$

- Revisit 13×34 and ask the students to use the open area model to find the product. Ask, “How should we draw the open area?” Then ask what they should write along the top and along the left side. Give students time to work on finding the product. Walk around the room to pose questions if students are having trouble getting started or completing the task. Identify one or two groups to explain their work on the board or using a demonstration tool. Then ask students to think about the base-10 block model and the open area model to identify the connection between concrete and pictorial representations.
- Instruct students to complete the two problems— 12×23 and 36×47 —problem 2 on the Multiplication Models activity sheet (attached). Use the base-10 materials to build the array, and then on the back of the sheet represent the problem with an open area model. Complete the missing information in the table and use the open area model to show the connection between the model for multiplying and symbolic method using numbers. When students have finished, ask them to respond to the questions at the bottom of the activity sheet. Circulate around the room to identify any misconceptions or address questions. Make note of students who will need additional support.
- Close the lesson by asking for volunteer pairs to show their work for the last two problems. Then, facilitate a discussion based on the questions and highlight the partial products connections between the open area model strategy and the procedure for multiplication.

Assessment

- **Questions**
 - How would you finish solving a problem that resulted in 12 flats, six vertical rods, six horizontal rods, and eight units? Explain your thinking and state the final product.
 - Could we use the open area model to multiply larger numbers (e.g., more than two-digit factors)? If so, show and explain how.
- **Journal/writing prompts**
 - Write a letter to your teacher explaining what you discovered about finding the product using the open area model.
 - Write a list of steps to follow for the open area model from start to final product.
- **Other Assessments**
 - On an exit ticket, have students explain how they would solve 34×18 .
 - On an exit ticket, have students explain another strategy they might use to solve two-digit-by-two-digit multiplication problems.

Extensions and Connections (for all students)

- Distribute index cards, and have each student create a study card illustrating the open-area model. Have them put the two-digit number times the two-digit number multiplication problem on one side of the card and illustrate the model on the reverse side. Have them include any helpful notes on this side as well.
- Write a word problem that requires two-digit multiplication and solve it using a method of your choice.
- Explore different ways to partition double-digit numbers in double-digit division.

Strategies for Differentiation

- Create an anchor chart that describes how to multiply. This anchor chart should include visuals.
- Help students understand the array model, which leads to the open area model, by providing additional practice with the place value blocks and multiplying a single digit times a double digit.
- Supply students with grid paper so they can line up their numbers in the correct place value. This will help them make connections from open-area model symbolic/abstract procedure.
- Have students share their strategies on poster paper. Display around the room so students can see and compare different strategies.

Note: The following pages are intended for classroom use for students as a visual aid to learning.

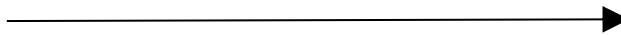
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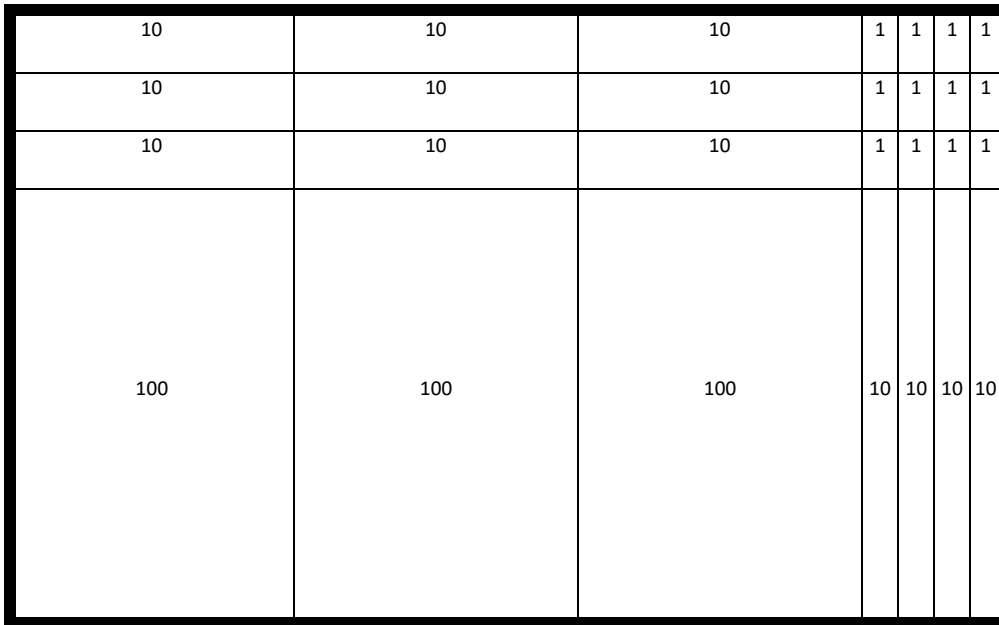
Multiplication Models

Name _____ Date _____

Represent the problem $13 \times 34 = \underline{\quad}$ as shown below. Use trading to find the product.

- Represent 13 vertically. (The height of the rectangle equals the first factor.)
- Represent 34 horizontally. (The width of the rectangle equals the second factor.)
- Fill in the rectangle with as few base-10 blocks as possible.
- The sum of the value of these blocks after trading is the product.

Width is 3 rods and 4 units = 34 



Height is 1 rod and 3 units = 13

Use this strategy to find the following partial products. Complete the table.

Problems	Value of Flats	Value of Vertical Rods	Value of Horizontal Rods	Value of Units	Product
13×34	3×100 300	4×10 40	9×10 90	12×1 12	442
32×25					

3. Challenge Problem: Use the open-area model to find 237×45 . Show your work.

4. Use the strategy you just discovered to find the following product mentally. You can write down the partial products if you need help keeping track. After you complete the problem mentally, use the open-area model to check your work.

$$31 \times 22 = \underline{\hspace{2cm}}$$