

Mathematical Analysis *Standards of Learning* - 2023 Overview of Revisions

This overview includes a summary of the content embedded in three content strands.

Characteristics of Functions

- Identify and analyze the properties of polynomial, rational, piecewise, absolute value, radical, and step functions and sketch the graphs of the functions
- Determine the limit of a function if it exists
- Analyze and describe the continuity of functions

Functional Relationships

- Analyze compositions of functions to determine and verify inverses of functions
- Analyze the characteristics of exponential and logarithmic functions, and sketch the graphs of the functions
- Analyze and represent sequences and finite series

Analytic Geometry

- Identify and analyze the properties of conic sections, and sketch a graph given an equation
- Use parametric equations to model and solve problems in context
- Perform operations with vectors in the coordinate plane
- Investigate and identify the characteristics of the graphs of polar equations
- Use matrices to organize data and add and subtract matrices, multiply matrices, multiply matrices by a scalar, and use matrices to solve systems of equations

Comparison of Mathematical Analysis *Standards of Learning* – 2016 to 2023

2016 <i>Standards of Learning</i> Essential Knowledge and Skills (EKS) Functions	2023 <i>Standards of Learning</i> Knowledge and Skills (KS) Characteristics of Functions (CF)
<p>MA.1 The student will investigate and identify the properties of polynomial, rational, piecewise, and step functions and sketch the graphs of the functions.</p> <ul style="list-style-type: none"> ● Identify a polynomial, rational, piecewise, and step function, given an equation or graph. ● Given a graph or equation of a polynomial, rational, piecewise, or step function, identify: <ul style="list-style-type: none"> ○ domain and range; ○ zeros; ○ intercepts; ○ symmetry; ○ asymptotes (horizontal, vertical, and oblique/slant); ○ points of discontinuity; ○ intervals for which the function is increasing, decreasing or constant; ○ end behavior; and ○ relative and/or absolute maximum and minimum points. ● Sketch the graph of a polynomial, rational, piecewise, or step function. ● Investigate and verify characteristics of a polynomial, rational, piecewise, and step function, using a graphing utility. ● Rationalize the denominator of a rational function. 	<p>MA.CF.1 The student will identify and analyze the properties of polynomial, rational, piecewise-defined, absolute value, radical, and step functions and sketch the graphs of the functions.</p> <ul style="list-style-type: none"> a) Use mathematical reasoning to identify polynomial, rational, piecewise-defined, absolute value, radical, and step functions, given an equation or graph. b) Given multiple representations of a polynomial, rational, piecewise-defined, absolute value, radical, and step function, analyze: <ul style="list-style-type: none"> i) domain and range; ii) roots (including complex roots); iii) intercepts; iv) symmetry (including even and odd functions); v) asymptotes (horizontal, vertical, and oblique/slant); vi) points of discontinuity; vii) intervals for which the function is increasing, decreasing or constant; viii) end behavior; and ix) relative and/or absolute maximum and minimum points. c) Sketch the graph of a polynomial, rational, piecewise-defined, absolute value, radical, and step function.

<p align="center">2016 Standards of Learning Essential Knowledge and Skills (EKS) Functions</p>	<p align="center">2023 Standards of Learning Knowledge and Skills (KS) Characteristics of Functions (CF)</p>
<p>MA.2 The student will investigate and identify the characteristics of exponential and logarithmic functions to graph the function, solve equations, and solve practical problems.</p> <ul style="list-style-type: none"> ● Identify exponential functions from an equation or a graph. ● Identify logarithmic functions from an equation or a graph. ● Define e, and know its approximate value. ● Convert between equations written in logarithmic and exponential form. ● Identify common and natural logarithms, given an equation or practical situation. ● Use laws of exponents and logarithms to solve equations and simplify expressions. ● Model practical problems, using exponential and logarithmic functions. ● Graph exponential and logarithmic functions and identify asymptotes, end behavior, intercepts, domain, and range. 	<p>[Moved to MA.FR.2]</p>
<p>MA.3 The student will apply compositions of functions and inverses of functions to practical situations and investigate and verify the domain and range of resulting functions.</p> <ul style="list-style-type: none"> ● Determine the composition of functions algebraically and graphically. ● Determine the inverse of a function algebraically and graphically. ● Determine the domain and range of composite functions algebraically and graphically. 	<p>[Moved to MA.FR.1]</p>

<p align="center">2016 Standards of Learning Essential Knowledge and Skills (EKS) Functions</p>	<p align="center">2023 Standards of Learning Knowledge and Skills (KS) Characteristics of Functions (CF)</p>
<ul style="list-style-type: none"> ● Determine the domain and range of the inverse of a function algebraically and graphically. 	
<p>MA.4 The student will determine the limit of an algebraic function, if it exists, as the variable approaches either a finite number or infinity.</p> <ul style="list-style-type: none"> ● Verify estimates about the limit of a function using a graphing utility. ● Determine the limit of a function algebraically and verify with a graphing utility. ● Determine the limit of a function numerically and verify with a graphing utility. ● Use limit notation when describing end behavior of a function. 	<p>MA.CF.2 The student will determine the limit of a function if it exists.</p> <ol style="list-style-type: none"> Verify estimates about the limit of a function using graphing technology. Determine the limit of a function algebraically and verify with graphing technology. Determine the limit of a function numerically and verify with graphing technology. Use proper limit notation, including when describing the end behavior of a function. Communicate mathematically using proper limit notation. As the variable approaches a finite number, <ol style="list-style-type: none"> determine the limit of a function numerically by direct substitution; determine the limit of a function using algebraic manipulation; estimate the limit of a function using a table; and determine the limit of a function from a given graph. As the variable approaches positive or negative infinity, analyze the limit of a function to describe the end behavior.
<p>MA.5 The student will investigate and describe the continuity of functions.</p>	<p>MA.CF.3 The student will analyze and describe the continuity of functions.</p>

<p align="center">2016 Standards of Learning Essential Knowledge and Skills (EKS) Functions</p>	<p align="center">2023 Standards of Learning Knowledge and Skills (KS) Characteristics of Functions (CF)</p>
<ul style="list-style-type: none"> ● Describe continuity of a function. ● Investigate the continuity of functions including absolute value, step, rational, and piecewise functions, using graphical and algebraic methods. ● Classify types of discontinuity. ● Prove continuity at a point, using the definition of limits. 	<ol style="list-style-type: none"> a) Describe continuity of a function. b) Use mathematical notation to communicate and describe the continuity of functions including polynomial, rational, piecewise, absolute value, radical, and step function, using graphical and algebraic methods. c) Prove continuity at a point, using the definition. d) Classify types of discontinuity based on which condition of continuity is violated.

<p align="center">2016 Standards of Learning Essential Knowledge and Skills (EKS) Equations</p>	<p align="center">2023 Standards of Learning Knowledge and Skills (KS) Functional Relationships (FR)</p>
<p>[Moved from MA.3]</p>	<p>MA.FR.1 The student will analyze compositions of functions to determine and verify inverses of functions.</p> <ol style="list-style-type: none"> a) Construct the composition of functions algebraically and graphically. b) Determine the domain and range of composite functions algebraically and graphically. c) Develop the inverse of a function algebraically and graphically. d) Compare the domain and range of the inverse of a function with the original function, both algebraically and graphically. e) Use mathematical reasoning to generalize and communicate the criteria for an inverse function to exist.

<p align="center">2016 Standards of Learning Essential Knowledge and Skills (EKS) Equations</p>	<p align="center">2023 Standards of Learning Knowledge and Skills (KS) Functional Relationships (FR)</p>
<p>[Moved from MA.2]</p>	<p>MA.FR.2 The student will analyze the characteristics of exponential and logarithmic functions, and sketch the graphs of the functions.</p> <ul style="list-style-type: none"> a) Generalize characteristics of exponential and logarithmic functions from an equation or a graph. b) Define e and estimate its value. c) Convert between equations written in logarithmic and exponential form. d) Use laws of exponents and properties of logarithms to solve equations and simplify expressions. e) Represent contextual problems, using exponential and logarithmic functions, to include common and natural logarithms. f) Sketch the graph of exponential and logarithmic functions and identify asymptotes, end behavior, intercepts, domain, and range.
<p>[Moved from MA.13 and AII.5]</p>	<p>MA.FR.3 The student will analyze sequences and finite series, and model and solve problems in context using sequences and series.</p> <ul style="list-style-type: none"> a) Use and interpret the notation: \sum, n, n^{th}, and a_n. b) Derive the formulas associated with arithmetic and geometric sequences and series. c) Determine the nth term, a_n, for an arithmetic or geometric sequence. d) Determine the sum, S_n, if it exists, of an arithmetic or geometric series. e) Model and solve problems in context, using sequences and series.

<p align="center">2016 Standards of Learning Essential Knowledge and Skills (EKS) Equations</p>	<p align="center">2023 Standards of Learning Knowledge and Skills (KS) Functional Relationships (FR)</p>
	<p>f) Distinguish between a convergent and divergent series.</p> <p>g) Describe convergent series in relation to the concept of a limit.</p>

<p align="center">2016 Standards of Learning Essential Knowledge and Skills (EKS) Analytic Geometry</p>	<p align="center">2023 Standards of Learning Knowledge and Skills (KS) Analytic Geometry (AG)</p>
<p>MA.6 The student will investigate, graph, and identify the properties of conic sections from equations in vertex and standard form.</p> <ul style="list-style-type: none"> ● Given a translation or rotation matrix, determine an equation for the transformed function or conic section. ● Investigate and verify graphs of transformed conic sections, using a graphing utility. ● Graph conic sections from equations written in vertex or standard form using transformations. ● Identify properties of conic sections. 	<p>MA.AG.1 The student will identify and analyze the properties of conic sections and sketch a graph given an equation.</p> <ul style="list-style-type: none"> a) Given a translation or rotation matrix, determine an equation for the transformed function or conic section. b) Convert between standard and general forms of conic equations by completing the square. c) Graph conic sections from equations written in general or standard form using transformations. d) Identify characteristics of conic sections including center, vertices, axes, symmetry, foci, directrix, eccentricity, and asymptotes. e) Represent applications of conic sections.
<p>[Moved from MA.10]</p>	<p>MA.AG.2 The student will use parametric equations to model and solve problems in context.</p> <ul style="list-style-type: none"> a) Graph and analyze parametric equations and use the graph to determine solutions. b) Use parametric equations to model contextual problems, including motion over time.

<p style="text-align: center;">2016 Standards of Learning Essential Knowledge and Skills (EKS) Analytic Geometry</p>	<p style="text-align: center;">2023 Standards of Learning Knowledge and Skills (KS) Analytic Geometry (AG)</p>
<p>MA.7 The student will perform operations with vectors in the coordinate plane and solve practical problems using vectors.</p> <ul style="list-style-type: none"> ● Use vector notation. ● Perform the operations of addition, subtraction, scalar multiplication, and inner (dot) product on vectors. ● Graph vectors and resultant vectors. ● Express complex numbers in vector notation. ● Identify properties of vector addition, scalar multiplication, and dot product. ● Determine the components of a vector. ● Determine the norm (magnitude) of a vector. ● Use vectors in simple geometric proofs. ● Solve problems, including practical problems, using vectors. 	<p>MA.AG.3 The student will perform operations with vectors in the coordinate plane.</p> <ul style="list-style-type: none"> a) Use vector notation. b) Perform the operations of addition, subtraction, and scalar multiplication, graphically and algebraically on vectors. c) Find the dot (inner) product of two vectors and use it to determine the angle between two vectors. d) Determine if two vectors are orthogonal. e) Express complex numbers in vector notation. f) Verify properties of the dot product. g) Determine the components of a vector. h) Determine the norm (magnitude) of a vector. i) Find a unit vector in the same direction of a given vector. j) Apply vectors to problems in context.
<p>MA.8 The student will identify, create, and solve practical problems involving triangles.</p> <ul style="list-style-type: none"> ● Solve and create problems, including practical problems, using trigonometric functions. ● Solve and create problems, including practical problems, using the Pythagorean Theorem. ● Solve and create problems, including practical problems, using the Law of Sines and the Law of Cosines. ● Solve problems, including practical problems, where triangles are formed from vectors. 	<p>[Included in Trigonometry]</p>

<p style="text-align: center;">2016 Standards of Learning Essential Knowledge and Skills (EKS) Analytic Geometry</p>	<p style="text-align: center;">2023 Standards of Learning Knowledge and Skills (KS) Analytic Geometry (AG)</p>
<p>MA.9 The student will investigate and identify the characteristics of the graphs of polar equations.</p> <ul style="list-style-type: none"> ● Classify polar equations (rose, cardioid, limaçon, lemniscate, spiral, and circle), given the graph or the equation. ● Determine the effects of changes in the parameters of polar equations on the graph, using a graphing utility. ● Convert between complex numbers written in rectangular form and polar form. ● Determine and verify the intersection of the graphs of two polar equations, using a graphing utility. 	<p>MA.AG.4 The student will investigate and identify the characteristics of the graphs of polar equations.</p> <ul style="list-style-type: none"> a) Classify polar equations (rose, cardioid, limaçon, lemniscate, spiral, and circle), given the graph or the equation. b) Determine the effects of changes in the parameters of polar equations on the graph, using graphing technology. c) Convert between polar and rectangular forms of coordinates. d) Convert between complex numbers written in rectangular form and polar form. e) Convert equations between polar and rectangular forms. f) Determine and verify the intersection of the graphs of two polar equations, using graphing technology.
<p>MA.10 The student will use parametric equations to model and solve practical problems.</p> <ul style="list-style-type: none"> ● Graph parametric equations. ● Use parametric equations to model practical problems, including motion over time. ● Determine solutions to parametric equations graphically. ● Use a graphing utility to graph and analyze parametric equations. 	<p>[Moved to MA.AG.2]</p>
<p>MA.11 The student will use matrices to organize data and will add and subtract matrices, multiply matrices, multiply matrices by a scalar, and use matrices to solve systems of equations.</p> <ul style="list-style-type: none"> ● Multiply matrices by a scalar. ● Add, subtract, and multiply matrices. ● Model problems with a system of no more than three linear equations. 	<p>MA.AG.5 The student will use matrices to organize data and will add and subtract matrices, multiply matrices, multiply matrices by a scalar, and use matrices to solve systems of equations.</p> <ul style="list-style-type: none"> a) Multiply matrices by a scalar. b) Add, subtract, and multiply matrices. c) Represent problems with a system of no more than three linear equations.

<p align="center">2016 Standards of Learning Essential Knowledge and Skills (EKS) Analytic Geometry</p>	<p align="center">2023 Standards of Learning Knowledge and Skills (KS) Analytic Geometry (AG)</p>
<ul style="list-style-type: none"> ● Express a system of linear equations as a matrix equation. ● Solve a system of equations using matrices. ● Determine the inverse of a two-by-two or three-by-three matrix using paper and pencil. ● Verify two matrices are inverses using matrix multiplication. ● Verify the commutative and associative properties for matrix addition and multiplication. 	<ul style="list-style-type: none"> d) Express a system of linear equations as a matrix equation. e) Solve a system of equations using matrices. f) Determine the inverse of a two-by-two or three-by-three matrix using paper and pencil. g) Verify two matrices are inverses using matrix multiplication. h) Verify the commutative and associative properties for matrix addition and multiplication.

<p align="center">2016 Standards of Learning Essential Knowledge and Skills (EKS) Discrete Mathematics</p>	<p align="center">2023 Standards of Learning Knowledge and Skills (KS)</p>
<p>MA.12 The student will expand binomials having positive integral exponents.</p> <ul style="list-style-type: none"> ● Expand binomials having positive integral exponents. ● Use the Binomial Theorem, the formula for combinations, and Pascal's Triangle to expand binomials. 	<p>[Moved to Discrete Mathematics]</p>
<p>MA.13 The student will determine the sum of finite and infinite convergent series.</p> <ul style="list-style-type: none"> ● Use and interpret the notation: \sum, n, n^{th}, and a_n. ● Derive the formulas associated with arithmetic and geometric sequences and series. ● Given the formula, determine the nth term, a_n, for an arithmetic or geometric sequence. ● Given the formula, determine the sum, S_n, if it exists, of an arithmetic or geometric series. 	<p>[Moved to MA.FR.3]</p>

<p style="text-align: center;">2016 <i>Standards of Learning</i> Essential Knowledge and Skills (EKS) Discrete Mathematics</p>	<p style="text-align: center;">2023 <i>Standards of Learning</i> Knowledge and Skills (KS)</p>
<ul style="list-style-type: none"> ● Model and solve problems, using sequence and series information. ● Distinguish between a convergent and divergent series. ● Discuss convergent series in relation to the concept of a limit. 	
<p>MA.14 The student will use mathematical induction to prove formulas and mathematical statements.</p> <ul style="list-style-type: none"> ● Compare inductive and deductive reasoning. ● Prove formulas and mathematical statements, using mathematical induction. 	<p>[Moved to Discrete Mathematics]</p>

2023 Mathematical Analysis SOL – Summary of Changes

Mathematical Analysis (2016 SOL to 2023 SOL Numbering)	Parameter Changes/Clarifications (2023 SOL)
<ul style="list-style-type: none"> ● MA.1 → MA.CF.1 ● MA.2 → MA.FR.2 ● MA.3 → MA.FR.1 ● MA.4 → MA.CF.2 ● MA.5 → MA.CF.3 ● MA.6 → MA.AG.1 ● MA.7 → MA.AG.3 ● MA.8 → [Included in Trigonometry] ● MA.9 → MA.AG.4 ● MA.10 → MA.AG.2 ● MA.11 → MA.AG.5 ● MA.12 → [Moved to Discrete Mathematics] ● MA.13 → MA.FR.3 ● MA.14 → [Moved to Discrete Mathematics] 	<ul style="list-style-type: none"> ● MA.CF.2 [KS] - More specific parameters defined for determining and estimating the limit of a function and using limits to describe end behavior ● MA.CF.3 [KS] - Continuity of functions addresses polynomial and radical functions; classify types of discontinuity based on which condition of continuity is violated ● MA.FR.1 [KS] - Generalize and communicate the criteria for an inverse function to exist ● MA.AG.1 [KS] - Convert between standard and general forms of conic equations by completing the square; characteristics of conics to be identified are specified; model applications of conics

Deletions from Mathematical Analysis (2016 SOL)	Additions to Mathematical Analysis (2023 SOL)
<ul style="list-style-type: none"> ● MA.1 [EKS] Rationalize the denominator of a rational function [Deleted] ● MA.6 [EKS] Given a translation or rotation matrix, determine an equation for a transformed function or conic section [Deleted] ● MA.8 - Identify, create, and solve practical problems involving triangles [included in Trigonometry] ● MA.12 - Expand binomials having positive integral exponents [included in Discrete Mathematics] ● MA.13 [EKS] - Determine the sum of infinite convergent series [Deleted] ● MA.14 - Use mathematical induction to prove formulas and mathematical statements [Included in Discrete Mathematics] 	<ul style="list-style-type: none"> ● MA.CF.1 [KS] - Investigate and identify the properties of absolute value and radical functions given an equation or graph and sketch the graphs of the functions ● MA.AG.3 [KS] - Determine if two vectors are orthogonal

KEY: CF = Characteristics of Functions; FR = Functional Relationships; AG= Analytic Geometry; EKS =Essential Knowledge and Skills (2016); KS = Knowledge and Skills (2023); US = Understanding the Standard