2023 Virginia Mathematics *Standards of Learning –* Overview of Revisions
August 2023

This document provides an overview of the revisions to the 2016 Mathematics *Standards of Learning*, including a summary of the content embedded in each content strand, a comparison chart of the 2016 Mathematics SOL and the 2023 Mathematics SOL, and summary charts documenting the changes.

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Kindergarten Mathematics *Standards of Learning* - 2023 Overview of Revisions

This overview includes a summary of the content embedded in five content strands.

Number and Number Sense: *Developing a sense of quantity allows us to see relationships between numbers, think flexibly about numbers, and notice patterns that can emerge as we work with numbers to quantify, measure, and make decisions in life.*
- Use flexible counting strategies to determine and describe quantities up to 100
- Identify, represent, and compare quantities up to 30

Computation and Estimation: *The operations of addition and subtraction are used to represent and solve many different types of problems.*
- Model and solve single-step contextual problems using addition and subtraction with whole numbers within 10

Measurement and Geometry: *Analyzing and describing geometric objects, the relationships and structures among them, or the space that they occupy can be used to classify, quantify, measure, or count one or more attributes.*
- Reason mathematically by making direct comparisons between two objects or events using the attributes of length, height, weight, volume, and time
- Identify, describe, name, compare, and construct plane figures (circles, triangles, squares, and rectangles)
- Describe the units of time represented in a calendar

Probability and Statistics: *The world can be investigated through posing questions and collecting, representing, analyzing, and interpreting data to describe and predict events and real-world phenomena.*
- Apply the data cycle (pose questions; collect or acquire data; organize and represent data; and analyze data and communicate results) with a focus on object graphs and picture graphs

Patterns, Functions, and Algebra: *Relationships can be described, and generalizations can be made using patterns and relations.*
- Identify, describe, extend, and create simple repeating patterns using various representations

Comparison of Kindergarten Mathematics *Standards of Learning* – 2016 to 2023

2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Number and Number Sense**	2023 *Standards of Learning* Knowledge and Skills (KS) **Number and Number Sense (NS)**
K.1 The student will **a) tell how many are in a given set of 20 or fewer objects by counting orally; and** • Count orally to tell how many are in a given set containing 20 or fewer concrete objects, using one-to-one correspondence, and identify the corresponding numeral. (a)	**K.NS.1 The student will utilize flexible counting strategies to determine and describe quantities up to 100.** a) Use one-to-one correspondence to determine how many are in a given set containing 30 or fewer concrete objects (e.g., cubes, pennies, balls), and describe the last number named as the total number of objects counted. b) Recognize and explain that the number of objects remains the same regardless of the arrangement or the order in which the objects are counted. c) Represent forward counting by ones using a variety of tools, including five-frames, ten-frames, and number paths (a prelude to number lines). d) Count forward orally by ones from 0 to 100. e) Count forward orally by ones, within 100, starting at any given number. f) Count backward orally by ones when given any number between 1 and 20. g) State the number after, without counting, when given any number between 0 and 30. h) State the number before, without counting, when given any number between 1 and 20. i) Use objects, drawings, words, or numbers to compose and decompose numbers 11-19 into a ten and some ones.

2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Number and Number Sense**	2023 *Standards of Learning* Knowledge and Skills (KS) **Number and Number Sense (NS)**
	j) Group a collection of up to 100 objects (e.g., counters, pennies, cubes) into sets of ten and count by tens to determine the total (e.g., there are 3 groups of ten and 6 leftovers, 36 total objects).

K.1 The student will
b) read, write, and represent numbers from 0 through 20.
 	
● Read, write, and represent numbers from 0-20 to include:
   o construct a set of objects that corresponds to a given numeral, including an empty set;
   o read and write the numerals from 0 through 20;
   o identify written numerals from 0 through 20 represented in random order;
   o identify the numeral that corresponds to the total number of objects in a given set of 20 or fewer concrete objects; and
   o write a numeral that corresponds to a set of 20 or fewer concrete objects. (b)
	K.NS.2 The student will identify, represent, and compare quantities up to 30.
	
	a) Read, write, and identify the numerals 0 through 30.
	b) Construct a set of objects that corresponds to a given numeral within 30, including an empty set.
	c) Determine and write the numeral that corresponds to the total number of objects in a given set of 30 or fewer concrete objects or pictorial models.
	d) Given a set of up to 30 objects, construct another set which has more, fewer, or the same number of objects using concrete or pictorial models.
	e) Given a numeral up to 30, construct a set which has more, fewer, or the same number of objects using concrete or pictorial models.
	f) Compare two sets containing up to 30 concrete objects or pictorial models, using the terms more, fewer, or the same as (equal to).
	g) Compare numbers up to 30, to the benchmarks of 5 and 10 using various models (e.g., five frames, ten frames, number paths [a prelude to number lines], beaded racks, hands) using the terms greater than, less than, or the same as (equal to).

2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Number and Number Sense**	2023 *Standards of Learning* Knowledge and Skills (KS) **Number and Number Sense (NS)**
K.2 The student, given no more than three sets, each set containing 10 or fewer concrete objects, will **a) compare and describe one set as having more, fewer, or the same number of objects as the other set(s); and** **b) compare and order sets from least to greatest and greatest to least** • Compare and describe no more than three sets of 10 or fewer objects, using the terms *more, fewer,* and *the same*. (a) • Given a set of objects, construct a second set which has more, fewer, or the same number of objects. (a) • Compare and order three or fewer sets, each set containing 10 or fewer concrete objects, from least to greatest and greatest to least. (b)	**[Included in K.NS.2]**
K.3 The student will **a) count forward orally by ones from 0 to 100;** **b) count backward orally by ones when given any number between 1 and 10;** **c) identify the number after, without counting, when given any number between 0 and100 and identify the number before, without counting, when given any number between 1 and 10; and** **d) count forward by tens to determine the total number of objects to 100.** • Count forward orally by ones from 0 to 100. (a) • Count backward orally by ones when given any number between 1 and 10. (b)	**[Included in K.NS.1]**

2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Number and Number Sense**	2023 *Standards of Learning* Knowledge and Skills (KS) **Number and Number Sense (NS)**
• Identify the number after, without counting, when given any number between 0 and 100. (c) • Identify the number before, without counting, when given any number between 1 and 10. (c) • Count forward orally by tens, starting at 0, to determine the total number of objects up to 100. (d)	
K.4 The student will **a) recognize and describe with fluency part-whole relationships for numbers up to 5; and** **b) investigate and describe part-whole relationships for numbers up to 10.** • Recognize and describe with fluency part-whole relationships for numbers up to 5 in a variety of configurations. (a) • Investigate and describe part-whole relationships for numbers up to 10 using a variety of configurations. (b)	**[Included in K.CE.1]**
K.5 The student will investigate fractions by representing and solving practical problems involving equal sharing with two sharers. • Share a whole equally with two sharers, when given a practical situation. • Represent fair shares concretely or pictorially, when given a practical situation. • Describe shares as equal pieces or parts of the whole (e.g., halves), when given a practical situation.	**[Included in Grade 1]**

2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Computation and Estimation**	2023 *Standards of Learning* Knowledge and Skills (KS) **Computation and Estimation (CE)**
K.6 The student will model and solve single-step story and picture problems with sums to 10 and differences within 10, using concrete objects. ● Model and solve various types of story and picture problems using 10 or fewer concrete objects. (Types of problems should include joining, separating, and part-part-whole scenarios.)	**K.CE.1 The student will model and solve single-step contextual problems using addition and subtraction with whole numbers within 10.** a) Use objects, drawings, words, or numbers to compose and decompose numbers less than or equal to 5 in multiple ways. b) Recognize and describe with fluency part-part-whole relationships for numbers up to 5 in a variety of configurations. c) Model and identify the number that makes 5 when added to a given number less than or equal to 5. d) Use objects, drawings, words, or numbers to compose and decompose numbers less than or equal to 10 in multiple ways. e) Model and identify the number that makes 10 when added to a given number less than or equal to 10. f) Model and solve single-step contextual problems (join, separate, and part-part-whole) using 10 or fewer concrete objects.

2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Measurement and Geometry**	2023 *Standards of Learning* Knowledge and Skills (KS) **Measurement and Geometry (MG)**
K.7 The student will recognize the attributes of a penny, nickel, dime, and quarter and identify the number of pennies equivalent to a nickel, a dime, and a quarter.	**[Attributes included in K.PS.1]** **[Equivalencies of nickels and dimes moved to Grade 1]**

2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Measurement and Geometry**	2023 *Standards of Learning* Knowledge and Skills (KS) **Measurement and Geometry (MG)**
● Describe the attributes (e.g., color, relative size) of a penny, nickel, dime, and quarter. ● Identify a penny, nickel, dime, and quarter. ● Identify the number of pennies equivalent to a nickel, a dime, and a quarter (i.e., a nickel has the same value as five pennies).	
K.8 The student will investigate the passage of time by reading and interpreting a calendar. ● Name the twelve months of the year. ● Name the seven days in a week. ● Determine the day before and after a given day (e.g., yesterday, today, tomorrow).	**K.MG.3 The student will describe the units of time represented in a calendar.** a) Identify a calendar as a tool used to measure time. b) Name the days of the week and state that there are seven days in one week. c) Determine the day before and after a given day (e.g., yesterday, today, tomorrow). d) Name the twelve months of the year and state that there are twelve months in one year. e) Distinguish between days of the week and months of the year.

K.9 The student will compare two objects or events, using direct comparisons, according to one or more of the following attributes: length (longer, shorter), height (taller, shorter), weight (heavier, lighter), temperature (hotter, colder), volume (more, less), and time (longer, shorter).
 	
● Compare and describe lengths of two objects as longer or shorter, using direct comparison (e.g., the bus is longer than the car).
● Compare and describe heights of two objects (as taller or shorter), using direct comparison.
	K.MG.1 The student will reason mathematically by making direct comparisons between two objects or events using the attributes of length, height, weight, volume, and time.
	
	a) Use direct comparisons to compare, describe, and justify the:
	  i) lengths of two objects using the terms longer or shorter;
	  ii) heights of two objects using the terms taller or shorter;
	  iii) weights of two objects using the terms heavier or lighter;
	  iv) volumes of two containers using the terms more or less; and

2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Measurement and Geometry**	2023 *Standards of Learning* Knowledge and Skills (KS) **Measurement and Geometry (MG)**
• Compare and describe weights of two objects (as heavier or lighter), using direct comparison. • Compare and describe temperatures of two objects or environment (as hotter or colder), using direct comparison. • Compare and describe volumes of two containers (as more or less), using direct comparison. • Compare and describe the amount of time spent on two events (as longer or shorter), using direct comparison.	v) amount of time spent on two events using the terms longer or shorter.
K.10 The student will a) **identify and describe plane figures (circle, triangle, square, and rectangle);** b) **compare the size (smaller, larger) and shape of plane figures (circle, triangle, square, and rectangle); and** c) **describe the location of one object relative to another (above, below, next to) and identify representations of plane figures (circle, triangle, square, and rectangle) regardless of their positions and orientations in space.** • Identify a circle, triangle, square, and rectangle. (a) • Describe the characteristics of triangles, squares, and rectangles, including number of sides and number of vertices. (a) • Describe a circle using terms such as *round* and *curved*. (a) • Compare and group plane figures (circle, triangle, square, and rectangle) according to their relative sizes (smaller, larger). (b) • Compare and group plane figures (circle, triangle, square, and rectangle) according to their shapes. (b)	**K.MG.2 The student will identify, describe, name, compare, and construct plane figures (circles, triangles, squares, and rectangles).** a) Identify and name concrete and pictorial representations of circles, triangles, squares, and rectangles regardless of their orientation in space. b) Describe triangles, squares, and rectangles to include the number of sides and number of vertices. c) Describe a circle using terms such as round and curved. d) Distinguish between examples and nonexamples of identified plane figures (circles, triangles, squares, and rectangles). e) Compare and contrast two plane figures using characteristics to describe similarities and differences. f) Construct plane figures (circles, triangles, squares, and rectangles) using a variety of materials (e.g., straws, sticks, pipe cleaners).

2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Measurement and Geometry**	2023 *Standards of Learning* Knowledge and Skills (KS) **Measurement and Geometry (MG)**
● Distinguish between examples and nonexamples of identified plane figures (circle, triangle, square, and rectangle). (b) ● Identify pictorial representations of a circle, triangle, square, and rectangle, regardless of their position and orientation in space. (c) ● Describe the location of one object relative to another, using the terms *above*, *below*, and *next to*. (c)	

2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Probability and Statistics**	2023 *Standards of Learning* Knowledge and Skills (KS) **Probability and Statistics (PS)**
K.11 The student will **a) collect, organize, and represent data; and** **b) read and interpret data in object graphs, picture graphs, and** **tables.** ● Collect data on categories identified by the teacher and/or student (e.g., number of siblings, types/numbers of pets, types of flowers in the garden). Data points, collected by students, should be limited to 16 or fewer for no more than four categories. (a) ● Represent data by arranging concrete objects into organized groups to form a simple object graph. (a) ● Represent gathered data, using pictures to form a simple picture graph (e.g., a picture graph of the weather for a month). (a) ● Represent gathered data in tables (vertically or horizontally). (a)	**K.PS.1 The student will apply the data cycle (pose questions; collect or acquire data; organize and represent data; and analyze data and communicate results) with a focus on object graphs and picture graphs.** a) Sort and classify concrete objects into appropriate subsets (categories) based on one attribute (e.g., size, shape, color, thickness). b) Describe and label attributes (e.g., size, color, shape) of a set of objects (e.g., coins, counters, buttons) that has been sorted. c) Pose questions, given a predetermined context, that require the collection of data (limited to 25 or fewer data points for no more than four categories).

2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Probability and Statistics**	2023 *Standards of Learning* Knowledge and Skills (KS) **Probability and Statistics (PS)**

• Answer questions related to the gathered data displayed in object graphs, picture graphs, and tables:
  o read the graph to determine the categories of data and the data as a whole (e.g., the total number of responses) and its parts (e.g., five people are wearing sneakers); and
  o interpret the data that represents numerical relationships, including categories with the greatest, the least, or the same. (b)
	d) Determine the data needed to answer a posed question, and collect the data using various methods (e.g., counting objects, drawing pictures).
	e) Organize and represent a data set (vertically or horizontally) by sorting concrete objects into organized groups to form a simple object graph.
	f) Organize and represent a data set (vertically or horizontally) using pictures to form a simple picture graph.
	g) Analyze data represented in object graphs and picture graphs and communicate results:
	  i) ask and answer questions about the data represented in object graphs and picture graphs (e.g., how many in each category, which categories have the greatest, least, or the same amount of data); and
	  ii) draw conclusions about the data and make predictions based on the data.

K.12 The student will sort and classify objects according to one attribute. • Identify the attributes of an object (e.g., color, size, shape, thickness). • Sort objects into appropriate groups (categories) based on one attribute (e.g., size – large bears and small bears). • Classify sets of objects into groups (categories) of one attribute. • Label attributes of a set of objects that has been sorted. • Name multiple ways to sort a set of objects.	**[Included in K.PS.1]**

2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Patterns, Functions, and Algebra**	2023 *Standards of Learning* Knowledge and Skills (KS) **Patterns, Functions, and Algebra (PFA)**
K.13 The student will identify, describe, extend, create, and transfer repeating patterns. ● Identify and describe the core (the part of the sequence that repeats) found in repeating patterns of common objects, sounds, movements, and pictures. ● Extend a repeating pattern by adding at least two complete repetitions of the core to the pattern. ● Create a repeating pattern. ● Compare similarities and differences between patterns. ● Transfer a repeating pattern from one representation to another.	**K.PFA.1 The student will identify, describe, extend, and create simple repeating patterns using various representations.** a) Identify and describe the core found in repeating patterns. b) Extend a repeating pattern by adding at least two complete repetitions of the core to the pattern. c) Create and describe a repeating pattern using objects, colors, sounds, movements, or pictures.

2023 Kindergarten Mathematics SOL – Summary of Changes

Kindergarten (2016 SOL to 2023 SOL Numbering)	Parameter Changes/Clarifications (2023 SOL)
- K.1a → K.NS.1 - K.1b → K.NS.2 - K.2a → K.NS.2 - K.2b → [Included in Grade 1] - K.3a,b,d → K.NS.1 - K.3c → K.NS.1 - K.4a-b → K.CE.1 - K.5 → [Included in Grade 1] - K.6 → K.CE.1 - K.7 → [Coin attributes embedded in K.PS.1; Equivalencies moved to Grade 1] - K.8 → K.MG.3 - K.9 → K.MG.1 - K.10a-c → K.MG.2 - K.11a-b → K.PS.1 - K.12 → K.PS.1 - K.13 → K.PFA.1	- K.NS.1 - Describe the last number named when counting a set as the total number of objects counted - K.NS.1a - Tell how many in a set increased from 20 to 30 objects - K.NS.1b - Recognize and explain that the number of objects remains the same regardless of the arrangement or the order in which the objects are counted - K.NS.1c - Represent forward counting by ones using a variety of tools, including five-frames, ten-frames, and number paths - K.NS.1f - Count backward increased from 10 to 20; count forward orally to 100 from any given number, previously limited to count orally from 0 to 100 - K.NS.1g-h - State the number after, decreased from between 0 and 100 to between 0 and 30; state the number before, increased from between 0 and 10 to between 1 and 20 - K.NS.1j - Count by tens to 100 included with grouping a collection of up to 100 objects - K.NS.2a-g - Identify, represent, and compare numbers increased from 20 to 30 - K.NS.2d-e - Construct a set that corresponds to a set of 20 objects increased to 30 objects; given a numeral up to 30, construct a set which has more, fewer, or the same number of objects using models; compare and order three sets (of 10 or less) changed to compare two sets (up to 30 objects) - K.CE.1a - Use objects, drawings, words, or numbers to compose and decompose numbers less than or equal to 10, in multiple ways - K.MG.2a - Includes name plane figures

Kindergarten (2016 SOL to 2023 SOL Numbering)	Parameter Changes/Clarifications (2023 SOL)
	• K.MG.3 - Identify a calendar as a tool used to measure time; state the number of days in a week; state the number of months in a year; distinguish between days of the week and months of the year • K.PS.1 - Describe and label attributes (e.g., size, color, shape) of a set of objects (e.g., coins, counters, buttons) that has been sorted • K.PS.1 - Collection of data increased from 16 to 25 data points

Deletions from Kindergarten (2016 SOL)	Additions to Kindergarten (2023 SOL)
• K.2b - Compare and order sets from least to greatest and greatest to least [Included in 1.NS.2] • K.5 - Investigate fractions by representing and solving practical problems [Included in 1.NS.3] • K.7 [EKS] - Identify the numbers of pennies equivalent to a nickel, a dime, and a quarter (i.e., a nickel has the same value as five pennies) [Moved to 1.NS.1] • K.9 [EKS] - Compare and describe temperature of two objects or environment using direct comparison [Included in Science standards] • K.10 [EKS] - Describe the location of one object relative to another, using the terms above, below, and next to [Included in Science standards] • K.13 [EKS] - Compare similarities and differences between patterns • K.13 [KS] - Transfer a repeating pattern from one representation to another [Included in 1.PFA.1]	• K.NS.1i - Compose and decompose numbers 11-19 into a ten and some ones • K.NS.2g - Compare numbers up to 30 to the benchmarks of 5 and 10 using various models • K.CE.1c,e - Model and identify the number that makes 5 or makes 10 when added to a given number less than or equal to 5 or 10 • K.MG.2 - Construct plane figures (circles, triangles, squares, and rectangles) using a variety of tools (e.g., straws, sticks, pipe cleaners) • K.PS.1c,d,g - Additional data analysis knowledge and skills representing the data cycle have been included (e.g., pose questions, determine data needed to answer a posed question, ask and answer questions about the data; draw conclusions)

KEY: NS = Number and Number Sense; CE = Computation and Estimation; MG = Measurement and Geometry; PS = Probability and Statistics; PFA = Patterns, Functions, and Algebra; EKS = Essential Knowledge and Skills (2016); KS = Knowledge and Skills (2023); US = Understanding the Standard

Grade 1 Mathematics *Standards of Learning* - 2023 Overview of Revisions

This overview includes a summary of the content embedded in five content strands.

Number and Number Sense: *Developing a sense of quantity allows us to see relationships between numbers, think flexibly about numbers, and notice patterns that can emerge as we work with numbers to quantify, measure, and make decisions in life.*
- Use flexible counting strategies to determine and describe quantities up to 120
- Represent, compare, and order quantities up to 120
- Use mathematical reasoning and justification to solve contextual problems that involve partitioning models into two and four equal-sized parts

Computation and Estimation: *The operations of addition and subtraction are used to represent and solve many different types of problems.*
- Recall with automaticity addition and subtraction facts within 10
- Represent, solve, and justify solutions to single-step problems using addition and subtraction with whole numbers within 20

Measurement and Geometry: *Analyzing and describing geometric objects, the relationships and structures among them, or the space that they occupy can be used to classify, quantify, measure, or count one or more attributes.*
- Use nonstandard units to measure and compare objects by length, weight, and volume
- Describe, sort, draw, and name plane figures (circles, triangles, squares and rectangles)
- Compose larger plane figures by combining simple plane figures
- Demonstrate an understanding of the concept of passage of time (to the nearest hour and half-hour) and the calendar

Probability and Statistics: *The world can be investigated through posing questions and collecting, representing, analyzing, and interpreting data to describe and predict events and real-world phenomena.*
- Apply the data cycle (pose questions; collect or acquire data; organize and represent data; and analyze data and communicate results) with a focus on object graphs, picture graphs, and tables

Patterns, Functions, and Algebra: *Relationships can be described, and generalizations can be made using patterns and relations.*
- Identify, describe, extend, create, and transfer repeating and growing (increasing) patterns using various representations

Comparison of Grade 1 Mathematics *Standards of Learning* – 2016 to 2023

2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Number and Number Sense**	2023 *Standards of Learning* Knowledge and Skills (KS) **Number and Number Sense (NS)**
1.1 The student will a) **count forward orally by ones to 110, starting at any number between 0 and 110;** c) **count backward orally by ones when given any number between 1 and 30; and** d) **count forward orally by ones, twos, fives, and tens to determine the total number of objects to 110.** • Count forward orally, by ones, from 0 to 110 starting at any number between 0 and 110. (a) • Use the oral counting sequence to tell how many objects are in a set. (a) • Count backward orally by ones when given any number between 1 and 30. (c) • Count forward orally by ones, twos, fives, and tens to determine the total number of objects to 110. (d)	**1.NS.1 The student will utilize flexible counting strategies to determine and describe quantities up to 120.** a) Count forward orally by ones from 0 to 120 starting at any number between 0 and 120. b) Count backward orally by ones when given any number between 1 and 30. c) Represent forward counting patterns when counting by groups of 5 and groups of 10 up to 120 using a variety of tools (e.g., objects, coins, 120 chart). d) Represent forward counting patterns when counting by groups of 2 up to at least 30 using a variety of tools (e.g., beaded number strings, number paths [a prelude to number lines], 120 chart). e) Group a collection of up to 120 objects into tens and ones, and count to determine the total (e.g., 5 groups of ten and 6 ones is equal to 56 total objects). f) Identify a penny, nickel, and dime by their attributes and describe the number of pennies equivalent to a nickel and a dime. g) Count by ones, fives, or tens to determine the value of a collection of like coins (pennies, nickels, or dimes), whose total value is 100 cents or less.

2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Number and Number Sense**	2023 *Standards of Learning* Knowledge and Skills (KS) **Number and Number Sense (NS)**
1.1 The student will **b) write the numerals 0 to 110 in sequence and out-of-sequence;** • Write numerals 0-110 in sequence and out of sequence. (b)	**1.NS.2 The student will represent, compare, and order quantities up to 120.** a) Read and write numerals 0-120 in sequence and out of sequence. b) Estimate the number of objects (up to 120) in a given collection and justify the reasonableness of an answer. c) Create a concrete or pictorial representation of a number using tens and ones and write the corresponding numeral up to 120 (e.g., 47 can be represented as 47 ones or it can be grouped into 4 tens with 7 ones leftover). d) Describe the number of groups of tens and ones when given a two-digit number and justify reasoning. e) Compare two numbers between 0 and 120 represented pictorially and with concrete objects using the terms greater than, less than, or equal to. f) Order three sets, each set containing up to 120 objects, from least to greatest, and greatest to least.
1.2 The student, given up to 110 objects, will **a) group a collection into tens and ones and write the corresponding numeral;** **b) compare two numbers between 0 and 110 represented pictorially or with concrete objects, using the words *greater than, less than* or *equal to*; and** **c) order three or fewer sets from least to greatest and greatest to least.**	**[Included in 1.NS.1 and 1.NS.2]**

2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Number and Number Sense**	2023 *Standards of Learning* Knowledge and Skills (KS) **Number and Number Sense (NS)**
• Group a collection of up to 110 objects into sets of tens and ones. (a) • Write the numeral that corresponds to the total number of objects in a given collection of up to 110 objects that have been grouped into sets of tens and ones. (a) • Identify the place and value of each digit in a two-digit numeral (e.g., in the number 23, the 2 is in the tens place and the value of the 2 is 20). (a) • Identify the number of tens and ones that can be made from any number up to 100 (e.g., 47 is 47 ones or can also be grouped into 4 tens with 7 ones left over). (a) • Compare two numbers between 0 and 110 represented pictorially or with concrete objects, using the words *greater than, less than* or *equal to.* (b) • Order three or fewer sets, each set containing up to 110 objects, from least to greatest and greatest to least. (c)	

1.3 The student, given an ordered set of ten objects and/or pictures, will indicate the ordinal position of each object, first through tenth.
 	
• Identify the ordinal positions first through tenth using ordered sets of 10 objects and/or pictures of such sets presented from:
  o left to right;
  o right to left;
  o top to bottom; and/or
  o bottom to top.
	[Ordinals included in 1.MG.3]

2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Number and Number Sense**	2023 *Standards of Learning* Knowledge and Skills (KS) **Number and Number Sense (NS)**
1.4 The student will a) **represent and solve practical problems involving equal sharing with two or four sharers; and** b) **represent and name fractions for halves and fourths, using models.** ● Share a whole equally with two or four sharers, when given a practical situation. (a) ● Represent fair shares pictorially, when given a practical situation. (a) ● Describe shares as equal pieces or parts of the whole (e.g., halves, fourths), when given a practical situation. (a) ● Represent halves and fourths of a whole, using a region/area model (e.g., pie pieces, pattern blocks, paper folding, and drawings). (b) ● Name fractions represented by drawings or concrete materials for halves and fourths. (b)	**1.NS.3 The student will use mathematical reasoning and justification to solve contextual problems that involve partitioning models into two and four equal-sized parts.** a) Represent equal shares of a whole with two or four sharers, when given a contextual problem. b) Represent and name halves and fourths of a whole, using a region/area model (e.g., pie pieces, pattern blocks, paper folding, drawings) and a set model (e.g., eggs, marbles, counters) limited to two or four items. c) Describe and justify how shares are equal pieces or equal parts of the whole (limited to halves, fourths) when given a contextual problem.
1.5 The student, given a familiar problem situation involving magnitude, will a) **select a reasonable order of magnitude from three given quantities: a one-digit numeral, a two-digit numeral, and a three-digit numeral (e.g., 5, 50, 500); and** b) **explain the reasonableness of the choice.** ● Select a reasonable order of magnitude for a given set from three given quantities: a one-digit numeral, a two-digit numeral, and a	**[Magnitude moved to Grade 2; New estimation content included in 1.NS.2]**

2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Number and Number Sense**	2023 *Standards of Learning* Knowledge and Skills (KS) **Number and Number Sense (NS)**
three-digit numeral (e.g., 5, 50, or 500 jelly beans in jars) in a familiar problem situation. (a) ● Explain why a particular estimate was chosen as the most reasonable from three given quantities (a one-digit numeral, a two-digit numeral, and a three-digit numeral), given a familiar problem situation. (b)	

2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Computation and Estimation**	2023 *Standards of Learning* Knowledge and Skills (KS) **Computation and Estimation (CE)**
1.6 The student will create and solve single-step story and picture problems using addition and subtraction within 20. ● Create and solve single-step oral or written story and picture problems, using addition and subtraction within 20. ● Identify a number sentence to solve an oral or written story and picture problem, selecting from among addition and/or subtraction equations (e.g., number sentences). ● Combine parts contained in larger numbers up to 20 by using related combinations (e.g., 9 + 7 can be thought of as 9 broken up into 2 and 7; using doubles, 7 + 7 = 14; 14 + 2 = 16 or 7 broken up into 1 and 6; making a ten, 1 + 9 = 10; 10 + 6 = 16). ● Explain strategies used to solve addition and subtraction problems within 20 using spoken words, objects, pictorial models, and number sentences.	**1.CE.1 The student will recall with automaticity addition and subtraction facts within 10 and represent, solve, and justify solutions to single-step problems, including those in context, using addition and subtraction with whole numbers within 20.** a) Recognize and describe with fluency part-part-whole relationships for numbers up to 10 in a variety of configurations. b) Demonstrate fluency with addition and subtraction within 10 by applying reasoning strategies (e.g., count on/count back, one more/one less, doubles, make ten). c) Recall with automaticity addition and subtraction facts within 10. d) Investigate, recognize, and describe part-part-whole relationships for numbers up to 20 in a variety of configurations (e.g., beaded racks, double ten frames).

2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Computation and Estimation**	2023 *Standards of Learning* Knowledge and Skills (KS) **Computation and Estimation (CE)**
	e) Solve addition and subtraction problems within 20 using various strategies (e.g., inverse relationships: if $9 + 3 = 12$ then $12 - 3 = 9$; decomposition using known sums/differences: $9 + 7$ can be thought of as 9 decomposed into 2 and 7, then use doubles, $7 + 7 = 14$; $14 + 2 = 16$ or decompose the 7 into 1 and 6; make a ten: $1 + 9 = 10$; $10 + 6 = 16$).
	f) Represent, solve, and justify solutions to single-step addition and subtraction problems (join, separate, and part-part-whole) within 20, including those in context, using words, objects, drawings, or numbers.
	g) Determine the unknown whole number that will result in a sum or difference of 10 or 20 (e.g., $14 - \_\_ = 10$ or $15 + \_\_ = 20$).
	h) Identify and use (+) as a symbol for addition and (-) as a symbol for subtraction.
	i) Describe the equal symbol (=) as a balance representing an equivalent relationship between expressions on either side of the equal symbol (e.g., 6 and 1 is the same as 4 and 3; $6 + 1$ is balanced with $4 + 3$; $6 + 1 = 4 + 3$).
	j) Use concrete materials to model, identify, and justify when two expressions are not equal (e.g., $10 - 3$ is not equal to $3 + 5$).
	k) Use concrete materials to model an equation that represents the relationship of two expressions of equal value.
	l) Write an equation that could be used to represent the solution to an oral, written, or picture problem.

2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Computation and Estimation**	2023 *Standards of Learning* Knowledge and Skills (KS) **Computation and Estimation (CE)**
1.7 The student will a) **recognize and describe with fluency part-whole relationships for numbers up to 10; and** b) **demonstrate fluency with addition and subtraction within 10.** ● Recognize and describe with fluency part-whole relationships for numbers up to 10 in a variety of configurations. (a) ● Identify + as a symbol for addition, - as a symbol for subtraction, and = as a symbol for equality. (b) ● Demonstrate fluency with addition and subtraction within 10. (b)	**[Included in 1.CE.1]**

2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Measurement and Geometry**	2023 *Standards of Learning* Knowledge and Skills (KS) **Measurement and Geometry (MG)**
1.8 The student will determine the value of a collection of like coins (pennies, nickels, or dimes) whose total value is 100 cents or less. ● Count by ones to determine the value of a collection of pennies whose total value is 100 cents or less. ● Group a collection of pennies by fives and tens as a way to determine the value. The total value of the collection is 100 cents or less. ● Count by fives to determine the value of a collection of nickels whose total value is 100 cents or less.	**[Included in 1.NS.1]**

2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Measurement and Geometry**	2023 *Standards of Learning* Knowledge and Skills (KS) **Measurement and Geometry (MG)**
● Count by tens to determine the value of a collection of dimes whose total value is 100 cents or less.	
1.9 The student will investigate the passage of time and a) **tell time to the hour and half-hour, using analog and digital clocks; and** b) **read and interpret a calendar.** ● Identify different types of clocks (analog and digital) as instruments to measure time. (a) ● Tell time shown on an analog clock to the hour and half-hour. (a) ● Tell time shown on a digital clock to the hour and half-hour. (a) ● Match a written time (e.g., 1:00, 3:30, 11:00) to the time shown on a digital and analog clock to the hour and half-hour. (a) ● Read a calendar to locate a given day or date (e.g., What day of the week is the 10th? What date is Saturday?). (b) ● Determine the day/date before and after a given day/date (e.g., Today is the 30th, so yesterday must have been the __?). (b) ● Given a calendar, determine the number of any day of the week (e.g., How many Fridays are in the month of October?) (b)	**1.MG.3 The student will demonstrate an understanding of the concept of passage of time (to the nearest hour and half-hour) and the calendar.** a) Identify different tools to measure time including clocks (analog and digital) and calendar. b) Describe the units of time represented on a clock as minutes and hours. c) Tell time to the hour and half-hour, using analog and digital clocks. d) Describe the location of the hour hand relative to time to the hour and half-hour on an analog clock. e) Describe the location of the minute hand relative to time to the hour and half-hour on an analog clock. f) Match the time shown on a digital clock to an analog clock to the hour and half-hour. g) Identify specific days/dates on a calendar (e.g., What date is Saturday? How many Fridays are in October?).

2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Measurement and Geometry**	2023 *Standards of Learning* Knowledge and Skills (KS) **Measurement and Geometry (MG)**
	h) Use ordinal numbers first through tenth to describe the relative position of specific days/dates (e.g., What is the first Monday in October? What day of the week is May 6th?).
	i) Determine the day/date before and after a given day/date (e.g., Today is the 8th, so yesterday was the ?), and a date that is a specific number of days/weeks in the past or future (e.g., Tim's birthday is in 10 days, what will be the date of his birthday?).

1.10 The student will use nonstandard units to measure and compare length, weight, and volume.
 	
● Measure the length of objects, using various nonstandard units (e.g., connecting cubes, paper clips, erasers).
● Compare the length of two objects, using the terms *longer/shorter, taller/shorter,* or *same as*.
● Measure the weight of objects, using a balance or pan scale with various nonstandard units (e.g., paper clips, bean bags, cubes).
● Identify a balance scale or a pan scale as a tool for measuring weight.
● Compare the weight of two objects, using the terms *lighter, heavier*, or *the same*, using a balance scale.
● Measure the volume of objects, using various nonstandard units (e.g., connecting cubes, blocks, rice, water).
● Compare the volumes of two containers to determine whether the volume of one is *more*, *less*, or *equivalent to* the other, using nonstandard units of measure (e.g., a spoonful or scoopful of rice, sand, jelly beans).
	1.MG.1 The student will reason mathematically using nonstandard units to measure and compare objects by length, weight, and volume.
	
	a) Use nonstandard units to measure the:
	  i) lengths of two objects (units laid end to end with no gaps or overlaps) and compare the measurements using the terms longer/shorter, taller/shorter, or the same as;
	  ii) weights of two objects (using a balance scale or a pan scale) and compare the measurements using the terms lighter, heavier, or the same as; and
	  iii) volumes of two containers and compare the measurements using the terms more, less, or the same as.
	b) Measure the length, weight, or volume of the same object or container with two different units and describe how and why the measurements differ.

2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Measurement and Geometry**	2023 *Standards of Learning* Knowledge and Skills (KS) **Measurement and Geometry (MG)**
• Compare the volumes of two containers to determine whether the volume of one is *more*, *less*, or *equivalent to* the other by pouring the contents of one container into the other.	
1.11 The student will a) **identify, trace, describe, and sort plane figures (triangles, squares, rectangles, and circles) according to number of sides, vertices, and angles; and** b) **identify and describe representations of circles, squares, rectangles, and triangles in different environments, regardless of orientation, and explain reasoning.** • Identify the name of the plane figure when given information about the number of sides, vertices, and angles. (a) • Trace triangles, squares, rectangles, and circles. (a) • Describe a circle using terms such as *round* and *curved*. (a) • Describe triangles, squares, and rectangles by the number of sides, vertices, and angles. (a) • Recognize that rectangles and squares have special types of angles called right angles. (a) • Sort plane figures based on their characteristics (number of sides, vertices, angles, curved, etc.). (a) • Identify and describe representations of circles, squares, rectangles, and triangles, regardless of orientation, in different environments and explain reasoning. (b)	**1.MG.2 The student will describe, sort, draw, and name plane figures (circles, triangles, squares, and rectangles), and compose larger plane figures by combining simple plane figures.** a) Describe triangles, squares, and rectangles using the terms sides, vertices, and angles. Describe a circle using terms such as *round* and *curved*. b) Sort plane figures based on their characteristics (number of sides, vertices, angles, curved, etc.). c) Draw and name the plane figure (circle, square, rectangle, triangle) when given information about the number of sides, vertices, and angles. d) Identify, name, and describe representations of circles, squares, rectangles, and triangles, regardless of orientation, in different environments and explain reasoning. e) Recognize and name the angles found in rectangles and squares as right angles. f) Compose larger plane figures by joining two or three simple plane figures (triangles, squares, and/or rectangles).

2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Probability and Statistics**	2023 *Standards of Learning* Knowledge and Skills (KS) **Probability and Statistics (PS)**
1.12 The student will a) **collect, organize, and represent various forms of data using tables, picture graphs, and object graphs; and** b) **read and interpret data displayed in tables, picture graphs, and object graphs, using the vocabulary** *more, less, fewer, greater than, less than,* **and** *equal to***.** • Collect and organize data using various forms of data collection (e.g., counting and tallying, informal surveys, observations, voting). Data points, collected by students, should be limited to 16 or fewer for no more than four categories. (a) • Represent data in tables, picture graphs, and object graphs. (a) • Analyze information displayed in tables, picture graphs, and object graphs (horizontally or vertically represented): • Read the graph to determine the categories of data and the data as a whole (e.g., the total number of responses) and its parts (e.g., 15 people are wearing sneakers); and • Interpret the data that represents numerical relationships, to include using the words *more, less, fewer, greater than, less than, and equal to*. (b)	**1.PS.1 The student will apply the data cycle (pose questions; collect or acquire data; organize and represent data; and analyze data and communicate results) with a focus on object graphs, picture graphs, and tables.** a) Sort and classify concrete objects into appropriate subsets (categories) based on one or two attributes, such as size, shape, color, and/or thickness (e.g., sort a set of objects that are both red and thick). b) Describe and label attributes of a set of objects that has been sorted. c) Pose questions, given a predetermined context, that require the collection of data (limited to 25 or fewer data points for no more than four categories). d) Determine the data needed to answer a posed question and collect the data using various methods (e.g., counting objects, drawing pictures, tallying). e) Organize and represent a data set by sorting the collected data using various methods (e.g., tallying, T-charts). f) Represent a data set (vertically or horizontally) using object graphs, picture graphs, and tables. g) Analyze data represented in object graphs, picture graphs, and tables and communicate results: i) ask and answer questions about the data represented in object graphs, picture graphs, and tables (e.g., total number of data points represented, how many in each category, how many more or less are in one category than another); and

2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Probability and Statistics**	2023 *Standards of Learning* Knowledge and Skills (KS) **Probability and Statistics (PS)**
	ii) draw conclusions about the data and make predictions based on the data.

2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Patterns, Functions, and Algebra**	2023 *Standards of Learning* Knowledge and Skills (KS) **Patterns, Functions, and Algebra (PFA)**
1.13 The student will sort and classify concrete objects according to one or two attributes. ● Sort and classify concrete objects into appropriate subsets (categories) based on one or two attributes, such as size, shape, color, and/or thickness (e.g., sort a set of objects that are both red and thick). ● Label attributes of a set of objects that has been sorted. ● Name multiple ways to sort a set of objects.	**[Included in 1.PS.1]**
1.14 The student will identify, describe, extend, create, and transfer growing and repeating patterns. ● Identify the pattern in a given rhythmic, color, geometric figure, or numerical sequence. ● Describe the pattern in a given rhythmic, color, geometric figure, or numerical sequence in terms of the core (the part of the sequence that repeats).	**1.PFA.1 The student will identify, describe, extend, create, and transfer repeating patterns and increasing patterns using various representations.** a) Identify and describe repeating and increasing patterns. b) Analyze a repeating or increasing pattern and generalize the change to extend the pattern using objects, pictures, movements, colors, or geometric figures.

2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Patterns, Functions, and Algebra**	2023 *Standards of Learning* Knowledge and Skills (KS) **Patterns, Functions, and Algebra (PFA)**
● Extend a repeating or growing pattern, using manipulatives, geometric figures, numbers, or calculators. ● Create a repeating or growing pattern, using manipulatives, geometric figures, numbers, or calculators (e.g., the growing patterns 2, 3, 2, 4, 2, 5, 2, 6, 2, 7). ● Transfer a pattern from one form to another.	c) Create repeating and increasing patterns using objects, pictures, movements, colors, or geometric figures. d) Transfer a repeating or increasing pattern from one form to another.
1.15 The student will demonstrate an understanding of equality through the use of the equal symbol. ● Describe the concept of equality. ● Identify equivalent values and represent equalities through the use of objects, words, and the equal (=) symbol. ● Identify and describe expressions that are not equal (e.g., 4 + 3 is not equal to 3 + 5). ● Recognize that equations can be used to represent the relationship between two expressions of equal value (e.g., 4 + 2 = 2 + 4 and 6 + 1= 4 + 3). ● Model an equation that represents the relationship of two expressions of equal value.	**[Included in 1.CE.1]**

2023 Grade 1 Mathematics SOL – Summary of Changes

Grade 1 (2016 SOL to 2023 SOL Numbering)	Parameter Changes/Clarifications (2023 SOL)
1.1a,c,d → 1.NS.1 1.1b → 1.NS.2 1.2a-c → 1.NS.2 1.3 → 1.MG.3 1.4a-b → 1.NS.3 1.5a-b → [Moved to Grade 2; new estimation content in 1.NS.2] 1.6 → 1.CE.1 1.7a-b → 1.CE.1 1.8 → 1.NS.1 1.9a-b → 1.MG.3 1.10 → 1.MG.1 1.11a-b → 1.MG.2 1.12a-b → 1.PS.1 1.13 → 1.PS.1 1.14 → 1.PFA.1 1.15 → 1.CE.1	1.NS.1a - Count forward by ones increased from 110 to 120 1.NS.1c - Represent forward counting patterns increased from 110 to 120 when counting by groups of 5 or groups of 10 1.NS.1d - Represent forward counting patterns decreased from 110 to 30 when counting by groups of 2 1.NS.1e - Group a collection of objects into tens and ones increased from 110 to 120 1.NS.2a - Read and write numerals, in sequence and out of sequence, increased from 110 to 120 1.NS. 2b - Estimate the number of objects in a collection (up to 120) and justify reasonableness replaces magnitude to 500 1.NS.2c - Create a concrete or pictorial representation of a number using tens and ones increased from 110 to 120 1.NS.2e - Compare two numbers and order three sets increased from 110 to 120 objects 1.NS.3b - Represent and name halves and fourths of a whole, using a set model (limited to two or four items) 1.CE.1c - 'Demonstrate fluency within 10' expanded to include 'Recall with automaticity' 1.CE.1d - Investigate, recognize, and describe part-part-whole relationships to 20 in a variety of configurations 1.CE.1(l) - Write an equation that could be used to represent the solution to an oral, written, or picture problem 1.PS.1c - Collect data points increased from 16 to 25

Deletions from Grade 1 (2016 SOL)	Additions to Grade 1 (2023 SOL)
1.2a [EKS] - Identify the place and value of each digit in a two-digit numeral [Included in Grade 2] 1.5 - Magnitude to 500 [Moved to Grade 2] 1.6 [EKS] - Create single-step oral or written story and picture problems, using addition and subtraction within 20 1.11 [EKS] - Trace triangles, squares, rectangles, and circles	1.NS.1f - Identify penny, nickel, dime and describe the number of pennies equivalent to a nickel or dime [Moved from Kindergarten] 1.NS.2d - Describe the number of groups of tens and ones when given a two-digit number and justify reasoning 1.CE.1g - Determine the unknown whole number that will result in a sum or difference of 10 or 20 (e.g., $14 - \underline{} = 10$ or $15 + \underline{} = 20$) 1.MG.1b - Measure the length, weight, or volume of the same object or container with two different units and describe how and why the measurements differ 1.MG.2c - Draw and name the plane figure (circle, square, rectangle, triangle) when given information about the number of sides, vertices, and angles 1.MG.2f - Compose larger plane figures by combining two or three simple plane figures (triangles, squares, and/or rectangles) 1.MG.3b - Describe the units of time represented on a clock as minutes and hours 1.MG.3e - Describe the location of the minute hand and the hour hand relative to time to the hour and half-hour on analog clock 1. PS.1a-g - Additional data analysis knowledge and skills representing the data cycle have been included (e.g., pose questions, determine data needed to answer a posed question, ask and answer questions about the data; draw conclusions)

KEY: NS = Number and Number Sense; CE = Computation and Estimation; MG = Measurement and Geometry; PS = Probability and Statistics; PFA = Patterns, Functions, and Algebra; EKS = Essential Knowledge and Skills (2016); KS = Knowledge and Skills (2023); US = Understanding the Standard

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Grade 2 Mathematics *Standards of Learning* - 2023 Overview of Revisions

This overview includes a summary of the content embedded in five content strands.

Number and Number Sense: *Flexibility with composing and decomposing base 10 numbers and understanding the structure to build relationships among numbers allows us to quantify, measure and make decisions in life.*
- Use flexible counting strategies to determine and describe quantities up to 200
- Demonstrate an understanding of the ten-to-one relationships of the base 10 number system to represent, compare, and order whole numbers up to 999
- Use mathematical reasoning and justification to solve contextual problems that involve partitioning models into equal-sized parts (halves, fourths, eighths, thirds, and sixths)
- Solve problems that involve counting and representing money amounts up to $2.00

Computation and Estimation: *The operations of addition and subtraction are used to represent and solve many different types of problems.*
- Recall with automaticity addition and subtraction facts within 20
- Estimate, represent, solve, and justify solutions to single-step and multistep addition and subtraction problems where addends or minuends do not exceed 100

Measurement and Geometry: *Analyzing and describing geometric objects, the relationships and structures among them, or the space that they occupy can be used to classify, quantify, measure, or count one or more attributes.*
- Reason mathematically using standard units (U.S. Customary) with appropriate tools to estimate, measure, and compare objects by length, weight, and liquid volume to the nearest whole unit
- Demonstrate an understanding of the concept of time to the nearest five minutes, using analog and digital clocks
- Identify, describe, and create plane figures (including circles, triangles, squares, and rectangles) that have at least one line of symmetry and explain its relationship with congruency
- Describe, name, compare, and contrast plane and solid figures (circles/spheres, squares/cubes, and rectangles/rectangular prisms).

Probability and Statistics: *The world can be investigated through posing questions and collecting, representing, analyzing, and interpreting data to describe and predict events and real-world phenomena.*
- Apply the data cycle (pose questions; collect or acquire data; organize and represent data; and analyze data and communicate results) with a focus on pictographs and bar graphs

Patterns, Functions, and Algebra: *Relationships can be described, and generalizations can be made using patterns and relations.*
- Describe, extend, create, and transfer growing (increasing) patterns using various representations

Comparison of Grade 2 Mathematics *Standards of Learning* – 2016 to 2023

2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Number and Number Sense**	2023 *Standards of Learning* Knowledge and Skills (KS) **Number and Number Sense (NS)**
2.1 The student will **a) read, write, and identify the place and value of each digit in a three-digit numeral, with and without models;** **c) compare and order whole numbers between 0 and 999; and** • Demonstrate understanding of the ten-to-one relationships among ones, tens, and hundreds, using manipulatives. (a) • Write numerals, using a model or pictorial representation (i.e., a picture of base-10 blocks). (a) • Read three-digit numbers when shown a numeral, a model of the number, or a pictorial representation of the number. (a) • Identify and write the place (ones, tens, hundreds) of each digit in a three-digit numeral. (a) • Determine the value of each digit in a three-digit numeral (e.g., in 352, the 5 represents 5 tens and its value is 50). (a) • Use models to represent numbers in multiple ways, according to place value (e.g., 256 can be 1 hundred, 14 tens, and 16 ones, 25 tens and 6 ones, etc.). (a) • Compare two numbers between 0 and 999 represented with concrete objects, pictorially or symbolically, using the symbols (>, <, or =) and the words greater than, less than or equal to. (c) • Order three whole numbers between 0 and 999 represented with concrete objects, pictorially, or symbolically from least to greatest and greatest to least. (c)	**2.NS.2 The student will demonstrate an understanding of the ten-to-one relationships of the base 10 number system to represent, compare, and order whole numbers up to 999.** a) Write the three-digit whole number represented by a given model (e.g., concrete objects, pictures of base 10 blocks). b) Read, write, and represent three-digit numbers in standard form, expanded form, and word form, using concrete or pictorial representations. c) Apply patterns within the base 10 system to determine and communicate, orally and in written form, the place (ones, tens, hundreds) and value of each digit in a three-digit whole number (e.g., in 352, the 5 represents 5 tens and its value is 50). d) Investigate and explain the ten-to-one relationships among ones, tens, and hundreds, using models. e) Compose and decompose whole numbers up to 200 by making connections between a variety of models (e.g., base 10 blocks, place value cards, presented orally, in expanded or standard form) and counting strategies (e.g., 156 can be 1 hundred, 5 tens, 6 ones; 1 hundred, 4 tens, 16 ones; 15 tens, 6 ones). f) Plot and justify the position of a given number up to 100 on a number line with pre-marked benchmarks of 1s, 2s, 5s, 10s, or 25s. g) Compare two whole numbers, each 999 or less, represented concretely, pictorially, or symbolically, using words (greater

2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Number and Number Sense**	2023 *Standards of Learning* Knowledge and Skills (KS) **Number and Number Sense (NS)**
	than, less than, or equal to) and symbols (>, <, or =). Justify reasoning orally, in writing, or with a model. h) Order up to three whole numbers, each 999 or less, represented concretely, pictorially, or symbolically from least to greatest and greatest to least.
2.1 The student will **b) identify the number that is 10 more, 10 less, 100 more, and 100 less than a given number up to 999;** ● Use place value understanding to identify the number that is 10 more, 10 less, 100 more, or 100 less than a given number, up to 999. (b)	**[Deleted]**
2.1 The student will **d) round two-digit numbers to the nearest ten.** ● Round two-digit numbers to the nearest ten. (d)	**[Included in 2.CE.1]**
2.2 The student will **a) count forward by twos, fives, and tens to 120, starting at various multiples of 2, 5, or 10;** **b) count backward by tens from 120; and** **c) use objects to determine whether a number is even or odd.** ● Determine patterns created by counting by twos, fives, and tens to 120 on number charts. (a)	**2.NS.1 The student will utilize flexible counting strategies to determine and describe quantities up to 200.** a) Represent forward counting patterns when counting by groups of 2 up to at least 50, starting at various multiples of 2 and using a variety of tools (e.g., objects, number lines, hundreds charts). b) Represent forward counting patterns created when counting by groups of 5s, 10s, and 25s starting at various multiples up to at least 200 using a variety of tools (e.g., objects, number lines, hundreds charts).

2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Number and Number Sense**	2023 *Standards of Learning* Knowledge and Skills (KS) **Number and Number Sense (NS)**
- Describe patterns in skip counting and use those patterns to predict the next number in the counting sequence. (a) - Skip count by twos, fives, and tens to 120 from various multiples of 2, 5 or 10, using manipulatives, a hundred chart, mental mathematics, a calculator, and/or paper and pencil. (a) - Skip count by two to 120 starting from any multiple of 2. (a) - Skip count by five to 120 starting at any multiple of 5. (a) - Skip count by 10 to 120 starting at any multiple of 10. (a) - Count backward by 10 from 120. (b) - Use objects to determine whether a number is even or odd (e.g., dividing collections of objects into two equal groups or pairing objects). (c)	c) Describe and use patterns in skip counting by multiples of 2 (to at least 50), and multiples of 5, 10, and 25 (to at least 200) to justify the next number in the counting sequence. d) Represent forward counting patterns when counting by groups of 100 up to at least 1,000 starting at 0 using a variety of tools (e.g., objects, number lines, calculators, one thousand charts). e) Represent backward counting patterns when counting by groups of 10 from 200 or less using a variety of tools including objects, number lines, calculators, and hundreds charts. f) Describe and use patterns in skip counting backwards by 10s (from at least 200) to justify the next number in the counting sequence. g) Choose a reasonable estimate up to 1,000 when given a contextual problem (e.g., What would be the best estimate for the number of students in our school – 5, 50, or 500?). h) Represent even numbers (up to 50) with concrete objects, using two equal groups or two equal addends. i) Represent odd numbers (up to 50) with concrete objects, using two equal groups with one leftover or two equal addends plus 1. j) Determine whether a number (up to 50) is even or odd using concrete objects and justify reasoning (e.g., dividing collections of objects into two equal groups, pairing objects).
2.3 The student will **a) count and identify the ordinal positions first through twentieth, using an ordered set of objects; and** **b) write the ordinal numbers 1st through 20th.**	**[Deleted; Ordinal numbers to 10th remain in Grade 1]**

2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Number and Number Sense**	2023 *Standards of Learning* Knowledge and Skills (KS) **Number and Number Sense (NS)**

• Count an ordered set of objects, using the ordinal number words *first* through *twentieth*. (a)
• Identify the ordinal positions first through twentieth, using an ordered set of objects presented in lines or rows from
    o left to right;
    o right to left;
    o top to bottom; and
    o bottom to top**.** (a)
• Write 1st, 2nd, 3rd, through 20th in numerals. (b)	

2.4 The student will **a) name and write fractions represented by a set, region, or length model for halves, fourths, eighths, thirds, and sixths;** **b) represent fractional parts with models and with symbols; and** **c) compare the unit fractions for halves, fourths, eighths, thirds, and sixths, with models.** • Recognize fractions as representing equal-size parts of a whole. (a) • Name and write fractions represented by a set model showing halves, fourths, eighths, thirds, and sixths. (a, b) • Name and write fractions represented by a region/area model showing halves, fourths, eighths, thirds, and sixths. (a, b) • Name and write fractions represented by a length model showing halves, fourths, eighths, thirds, and sixths. (a, b) • Represent, with models and with symbols, fractional parts of a whole for halves, fourths, eighths, thirds, and sixths, using: o region/area models (e.g., pie pieces, pattern blocks, geoboards);	**2.NS.3 The student will use mathematical reasoning and justification to solve contextual problems that involve partitioning models into equal-sized parts (halves, fourths, eighths, thirds, and sixths).** a) Model and describe fractions as representing equal-size parts of a whole. b) Describe the relationship between the number of fractional parts needed to make a whole and the size of the parts (i.e., as the whole is divided into more parts, each part becomes smaller). c) Compose the whole for a given fractional part and its value (in context) for halves, fourths, eighths, thirds, and sixths (e.g., when given $\frac{1}{4}$, determine how many pieces would be needed to make $\frac{4}{4}$). d) Using same-size fraction pieces, from a region/area model, count by unit fractions up to two wholes (e.g., zero one-fourths, one one-fourth, two one-fourths, three one-fourths, four one-fourths,

2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Number and Number Sense**	2023 *Standards of Learning* Knowledge and Skills (KS) **Number and Number Sense (NS)**

o   sets (e.g., chips, counters, cubes); and
   o   length/measurement models (e.g., fraction strips or bars,
     rods, connecting cube trains). (b)
● Compare unit fractions for halves, fourths, eighths, thirds, and
sixths), using words (greater than, less than or equal to) and
symbols (>, <, =), with models. (c)
● Using same-size fraction pieces, from region/area models or
length/measurement models, count the pieces (e.g., *one-fourth,
two-fourths, three-fourths*, etc.) and compare those pieces to one
whole (e.g., *four-fourths* will make one whole*; one-fourth* is less
than a whole). (c)
	five one-fourths; or zero-fourths, one-fourth, two-fourths, three-fourths, four-fourths, five-fourths).
	e) Given a context, represent, name, and write fractional parts of a
	whole for halves, fourths, eighths, thirds, and sixths using:
	   i) region/area models (e.g., pie pieces, pattern blocks,
	     geoboards);
	   ii) length models (e.g., paper fraction strips, fraction bars, rods,
	     number lines); and
	   iii) set models (e.g., chips, counters, cubes).
	f) Compare unit fractions for halves, fourths, eighths, thirds, and
	sixths using words (greater than, less than, or equal to) and
	symbols (>, <, =), with region/area and length models.

[Moved from 2.7]	**2.NS.4 The student will solve problems that involve counting and representing money amounts up to \$2.00.** a) Identify a quarter and its value and determine multiple ways to represent the value of a quarter using pennies, nickels, and/or dimes. b) Count by ones, fives, tens, and twenty-fives to determine the value of a collection of mixed coins and one-dollar bills whose total value is \$2.00 or less. c) Construct a set of coins and/or bills to total a given amount of money whose value is \$2.00 or less. d) Represent the value of a collection of coins and one-dollar bills (limited to \$2.00 or less) using the cent (¢) and dollar (\$) symbols and decimal point (.).

2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Computation and Estimation**	2023 *Standards of Learning* Knowledge and Skills (KS) **Computation and Estimation (CE)**
2.5 The student will **a) recognize and use the relationships between addition and subtraction to solve single-step practical problems, with whole numbers to 20; and** **b) demonstrate fluency with addition and subtraction within 20.** ● Recognize and use the relationship between addition and subtraction to solve single-step practical problems, with whole numbers to 20. (a) ● Determine the missing number in an equation (number sentence) (e.g., 3 + __ = 5 or __ + 2 = 5; 5 − __ = 3 or 5 − 2 = __). (a) ● Write the related facts for a given addition or subtraction fact (e.g., given 3 + 4 = 7, write 7 − 4 = 3 and 7 − 3 = 4). (a) ● Demonstrate fluency with addition and subtraction within 20. (b)	**2.CE.1 The student will recall with automaticity addition and subtraction facts within 20 and estimate, represent, solve, and justify solutions to single-step and multistep problems, including those in context, using addition and subtraction with whole numbers where addends or minuends do not exceed 100.** a) Apply strategies, (e.g., rounding to the nearest 10, compatible numbers, other number relationships), to estimate a solution for single-step addition or subtraction problems, including those in context, where addends and minuends do not exceed 100. b) Apply strategies, (e.g., the use of concrete and pictorial models, place value, properties of addition, the relationship between addition and subtraction) to determine the sum or difference of two whole numbers where addends or minuends do not exceed 100. c) Represent, solve, and justify solutions to single-step and multistep contextual problems (e.g., join, separate, part-part-whole, comparison) involving addition or subtraction of whole numbers where addends or minuends do not exceed 100. d) Demonstrate fluency with addition and subtraction within 20 by applying reasoning strategies (e.g., doubles, near doubles, make-a-ten, compensations, inverse relationships). e) Recall with automaticity addition and subtraction facts within 20.

2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Computation and Estimation**	2023 *Standards of Learning* Knowledge and Skills (KS) **Computation and Estimation (CE)**
	f) Use patterns, models, and strategies to make generalizations about the algebraic properties for fluency (e.g., $4 + 3$ is equal to $3 + 4$; $0 + 8 = 8$).
	g) Determine the missing number in an equation (number sentence) through modeling and justification with addition and subtraction within 20 (e.g., $3 + \_ = 5$ or $\_ + 2 = 5$; $5 - \_ = 3$ or $5 - 2 = \_$).
	h) Use inverse relationships to write all related facts connected to a given addition or subtraction fact model within 20 (e.g., given a model for $3 + 4 = 7$, write $4 + 3 = 7$, $7 - 4 = 3$, and $7 - 3 = 4$).
	i) Describe the not equal symbol ($\neq$) as representing a relationship where expressions on either side of the not equal symbol represent different values and justify reasoning.
	j) Represent and justify the relationship between values and expressions as equal or not equal using appropriate models and/or symbols (e.g., $9 + 24 = 10 + 23$; $45 - 9 = 46 - 10$; $15 + 16 \neq 31 + 15$).
2.6 The student will **a) estimate sums and differences;** **b) determine sums and differences, using various methods; and** **c) create and solve single-step and two-step practical problems involving addition and subtraction.** ● Estimate the sum of two whole numbers whose sum is 99 or less and recognize whether the estimation is reasonable (e.g., $27 + 41$ is about 70, because 27 is about 30 and 41 is about 40, and $30 + 40$ is 70). (a)	**[Included in 2.CE.1]**

2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Computation and Estimation**	2023 *Standards of Learning* Knowledge and Skills (KS) **Computation and Estimation (CE)**
• Estimate the difference between two whole numbers each 99 or less and recognize whether the estimate is reasonable. (a) • Determine the sum of two whole numbers whose sum is 99 or less, using various methods. (b) • Determine the difference of two whole numbers each 99 or less, using various methods. (b) • Create and solve single-step practical problems involving addition or subtraction. (c) • Create and solve two-step practical problems involving addition, subtraction, or both addition and subtraction. (c)	

2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Measurement and Geometry**	2023 *Standards of Learning* Knowledge and Skills (KS) **Measurement and Geometry (MG)**
2.7 The student will **a) count and compare a collection of pennies, nickels, dimes, and quarters whose total value is $2.00 or less; and** **b) use the cent symbol, dollar symbol, and decimal point to write a value of money.** • Determine the value of a collection of coins and one-dollar bills whose total value is $2.00 or less. (a) • Count by ones, fives, tens, and twenty-fives to determine the value of a collection of coins whose total value is $2.00 or less. (a)	**[Included in 2.NS.4]**

2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Measurement and Geometry**	2023 *Standards of Learning* Knowledge and Skills (KS) **Measurement and Geometry (MG)**
● Compare the values of two sets of coins and one-dollar bills (each set having a total value of $2.00 or less), using the terms *greater than, less than*, or *equal to*. (a) ● Use the cent (¢) and dollar ($) symbols and decimal point (.) to write a value of money which is $2.00 or less. (b)	
2.8 The student will estimate and measure **a) length to the nearest inch; and** **b) weight to the nearest pound.** ● Identify a ruler as an instrument to measure length. (a) ● Estimate and then measure the length of various line segments and objects to the nearest inch using a ruler. (a) ● Identify different types of scales as instruments to measure weight. (b) ● Estimate and then measure the weight of objects to the nearest pound using a scale. (b)	**2.MG.1 The student will reason mathematically using standard units (U.S. Customary) with appropriate tools to estimate, measure, and compare objects by length, weight, and liquid volume to the nearest whole unit.** a) Explain the purpose of various measurement tools and how to use them appropriately by: i) identifying a ruler as an instrument to measure length; ii) identifying different types of scales as instruments to measure weight; and iii) identifying different types of measuring cups as instruments to measure liquid volume. b) Use U.S. Customary units to estimate, measure, and compare the two for reasonableness: i) the length of an object to the nearest inch, using a ruler; ii) the weight of an object to the nearest pound, using a scale; and iii) the liquid volume of a container to the nearest cup, using a measuring cup.

2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Measurement and Geometry**	2023 *Standards of Learning* Knowledge and Skills (KS) **Measurement and Geometry (MG)**
2.9 The student will tell time and write time to the nearest five minutes, using analog and digital clocks. ● Show, tell, and write time to the nearest five minutes, using an analog and digital clock. ● Match a written time (e.g., 4:20, 10:05, 1:50) to a time shown on a clock face to the nearest five minutes. ● Match the time (to the nearest minutes) shown on a clock face to a written time.	**2.MG.2 The student will demonstrate an understanding of the concept of time to the nearest five minutes, using analog and digital clocks.** a) Identify the number of minutes in an hour (60 minutes) and the number of hours in a day (24 hours). b) Determine the unit of time (minutes, hours, days, or weeks) that is most appropriate when measuring a given activity or context and explain reasoning (e.g., Would you measure the time it takes to brush your teeth in minutes or hours?). c) Show, tell, and write time to the nearest five minutes, using analog and digital clocks. d) Match a written time (e.g., 1:35, 6:20, 9:05) to the time shown on an analog clock to the nearest five minutes.
2.10 The student will **a) determine past and future days of the week; and** **b) identify specific days and dates on a given calendar.** ● Determine the day that is a specific number of days or weeks in the past or in the future from a given date, using a calendar. (a) ● Identify specific days and dates (e.g., What is the third Monday in a given month? What day of the week is May 11?). (b)	**[Included in Kindergarten and Grade 1]**

2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Measurement and Geometry**	2023 *Standards of Learning* Knowledge and Skills (KS) **Measurement and Geometry (MG)**
2.11 The student will read temperature to the nearest 10 degrees. ● Identify different types of thermometers as instruments used to measure temperature. ● Read temperature in Fahrenheit to the nearest ten degrees on thermometers (real world, physical model, and pictorial representations).	**[Deleted; included in Grade 2 Science]**
2.12 The student will **a) Draw a line of symmetry in a figure; and** **b) Identify and create figures with at least one line of symmetry.** ● Draw a line of symmetry in a figure. (a) ● Determine a line of symmetry that results in two figures that have the same size and shape and explain reasoning. (a, b) ● Identify figures with at least one line of symmetry, using various concrete materials (e.g., mirrors, paper folding, pattern blocks). (b) ● Determine a line of symmetry that results in two figures that have the same size and shape and explain reasoning. (a, b) ● Create figures with at least one line of symmetry using various concrete materials. (b)	**2.MG.3 The student will identify, describe, and create plane figures (including circles, triangles, squares, and rectangles) that have at least one line of symmetry and explain its relationship with congruency.** a) Explore a figure using a variety of tools (e.g., paper folding, geoboards, drawings) to show and justify a line of symmetry, if one exists. b) Create figures with at least one line of symmetry using various concrete and pictorial representations. c) Describe the two resulting figures formed by a line of symmetry as being congruent (having the same shape and size).

2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Measurement and Geometry**	2023 *Standards of Learning* Knowledge and Skills (KS) **Measurement and Geometry (MG)**
2.13 The student will identify, describe, compare, and contrast plane and solid figures (circles/spheres, squares/cubes, and rectangles/rectangular prisms). ● Determine similarities and differences between related plane and solid figures (circles/spheres, squares/cubes, rectangles/rectangular prisms), using models and cutouts. ● Trace faces of solid figures (cubes and rectangular prisms) to create the set of plane figures related to the solid figure. ● Identify and describe plane figures (circles, squares, and rectangles), according to their characteristics (number of sides, vertices, and angles). Squares and rectangles have four right angles. ● Identify and describe solid figures (spheres, cubes, and rectangular prisms), according to the shape of their faces, number of edges, and number of vertices, using models. ● Compare and contrast plane and solid figures (circles/spheres, squares/cubes, and rectangles/rectangular prisms) according to their characteristics (number and shape of their faces, edges, vertices, and angles).	**2.MG.4 The student will describe, name, compare, and contrast plane and solid figures (circles/spheres, squares/cubes, and rectangles/rectangular prisms).** a) Trace faces of solid figures (cubes and rectangular prisms) to create the set of plane figures related to the solid figure. b) Compare and contrast models and nets (cutouts) of cubes and rectangular prisms (e.g., number and shapes of faces, edges, vertices). c) Given a concrete or pictorial model, name and describe the solid figure (sphere, cube, and rectangular prism) by its characteristics (e.g., number of edges, number of vertices, shapes of faces). d) Compare and contrast plane and solid figures (circles/spheres, squares/cubes, and rectangles/rectangular prisms) according to their characteristics (e.g., number and shapes of their faces, edges, vertices).

2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Probability and Statistics**	2023 *Standards of Learning* Knowledge and Skills (KS) **Probability and Statistics (PS)**
2.14 The student will use data from probability experiments to predict outcomes when the experiment is repeated. • Conduct probability experiments using multicolored spinners, colored tiles, or number cubes and use the data from the experiments to predict outcomes if the experiment is repeated. • Record the results of probability experiments, using tables, charts, and tally marks. • Interpret the results of probability experiments. • Predict which of two events is more or less likely to occur if an experiment is repeated.	**[Deleted]**
2.15 The student will **a) collect, organize, and represent data in pictographs and bar graphs; and** **b) read and interpret data represented in pictographs and bar graphs** • Collect and organize data using various forms of data collection (e.g., lists, tables, objects, pictures, symbols, tally marks, charts). Data points, collected by students, should be limited to 16 or fewer for no more than four categories. (a) • Represent data in pictographs and bar graphs (limited to 16 or fewer data points for no more than four categories). (a) • Read and interpret data represented in pictographs and bar graphs with up to 25 data points for no more than six categories (represented horizontally or vertically). State orally and in writing (at least one statement) that includes one or more of the following:	**2.PS.1 The student will apply the data cycle (pose questions; collect or acquire data; organize and represent data; and analyze data and communicate results) with a focus on pictographs and bar graphs.** a) Pose questions, given a predetermined context, that require the collection of data (limited to 25 or fewer data points for no more than six categories). b) Determine the data needed to answer a posed question and collect the data using various methods (e.g., voting; creating lists, tables, or charts; tallying). c) Organize and represent a data set using a pictograph where each symbol represents up to 2 data points. Determine and use a key to assist in the analysis of the data.

2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Probability and Statistics**	2023 *Standards of Learning* Knowledge and Skills (KS) **Probability and Statistics (PS)**

o describes the categories of data and the data as a whole (e.g., adding together all data points will equal the total number of responses);
o identifies parts of the data that have special characteristics; including categories with the greatest, the least, or the same;
o uses the data to make comparisons; and
o makes predictions and generalizations. (b)
	d) Organize and represent a data set using a bar graph with a title and labeled axes (limited to 25 or fewer data points for up to six categories, and limit increments of scale to multiples of 1 or 2).
	e) Analyze data represented in pictographs and bar graphs and communicate results:
	  i) ask and answer questions about the data represented in pictographs and bar graphs (e.g., total number of data points represented, how many in each category, how many more or less are in one category than another). Pictograph keys will be limited to symbols representing 1, 2, 5, or 10 pieces of data and bar graphs will be limited to scales with increments in multiples of 1, 2, 5, or 10; and
	  ii) draw conclusions about the data and make predictions based on the data.

2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Patterns, Functions, and Algebra**	2023 *Standards of Learning* Knowledge and Skills (KS) **Patterns, Functions, and Algebra (PFA)**
2.16 The student will identify, describe, create, extend, and transfer patterns found in objects, pictures, and numbers. ● Identify a pattern as growing or repeating. ● Describe the core (the part of the sequence that repeats) of a given repeating pattern. ● Describe how a given growing pattern is changing.	**2.PFA.1 The student will describe, extend, create, and transfer repeating and increasing patterns (limited to addition of whole numbers) using various representations.** a) Identify and describe repeating and increasing patterns.

2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Patterns, Functions, and Algebra**	2023 *Standards of Learning* Knowledge and Skills (KS) **Patterns, Functions, and Algebra (PFA)**
• Create a growing or repeating pattern, using objects, pictures, or numbers. • Extend a given pattern, using objects, pictures, or numbers. • Transfer a given growing or repeating pattern from one form to another using objects, pictures, or numbers.	b) Analyze a repeating or increasing pattern and generalize the change to extend the pattern using objects, pictures, and numbers. c) Create repeating and increasing patterns using various representations (e.g., objects, pictures, numbers). d) Transfer a given repeating or increasing pattern from one form to another (e.g., objects, pictures, numbers), and explain the connection between the two patterns.
2.17 The student will demonstrate an understanding of equality through the use of the equal symbol and the use of the not equal symbol. • Identify the equal symbol (=) as the symbol used to indicate that the values on either side are equal. • Identify the not equal symbol ($\neq$) as the symbol used to indicate that two values on either side are not equal. • Identify values and expressions that are equal (e.g., $8 = 8$, $8 = 4 + 4$). • Identify values and expressions that are not equal (e.g., $8 \neq 9$, $4 + 3 \neq 8$). • Identify and use the appropriate symbol to distinguish between equal and not equal quantities (e.g., $9 + 24 = 10 + 23$; $45 - 9 = 46 - 10$; $15 + 16 \neq 31 + 15$). • Use a model to represent the relationship of two expressions of equal value and two expressions that are not equivalent.	[Included in 2.CE.1]

2023 Grade 2 Mathematics SOL – Summary of Changes

Grade 2 (2016 SOL to 2023 SOL Numbering)	Parameter Changes/Clarifications (2023 SOL)
2.1a,c → 2.NS.2 2.1b → [Deleted] 2.1d → 2.CE.1 2.2a-c → 2.NS.1 2.3a-b → [Deleted] 2.4a-c → 2.NS.3 2.5a-b → 2.CE.1 2.6a-c → 2.CE.1 2.7a-b → 2.NS.4 2.8a-b → 2.MG.1 2.9 → 2.MG.2 2.10a-b → [Deleted; included in Grade 1] 2.11 → [Deleted; included in Science standards] 2.12a-b → 2.MG.3 2.13 → 2.MG.4 2.14 → [Deleted] 2.15a-b → 2.PS.1 2.16 → 2.PFA.1 2.17 → 2.CE.1	2.NS.1a - Represent forward counting patterns with groups of 2s to 50, and groups of 5s, 10s, and 25s to 200 2.NS.1c - Describe and use patterns in skip counting by multiples of 2 (to at least 50), and multiples of 5, 10, and 25 (to at least 200) to justify the next number in the counting sequence 2.NS.1e-f - Represent, describe, and use patterns in skip counting backwards by 10's to justify the number in the counting sequence 2.NS.1h - Represent/determine even/odd numbers up to 50 using concrete objects and justify reasoning 2.NS.3b - Describe the relationship between the number of fractional parts needed to make a whole and the size of the parts 2.NS.3d - Use same-size fraction pieces, count unit fractions increased from one whole to two wholes 2.NS.3 - Given a context, represent, name, and write fractional parts of a whole for halves, fourths, eighths, thirds, and sixths 2.CE.1 - *Create and solve* problems has been replaced with *estimate, represent, solve, and justify* solutions; solve addition and subtraction problems where addends or minuends do not exceed 100 (*previously sums to 99 or less; difference of two whole numbers each 99 or less*) 2.CE.1a - Rounding to nearest ten included as strategy to estimate a solution for addition or subtraction problems 2.CE.1e - 'Demonstrate fluency within 20' expanded to include 'Recall with automaticity' 2.CE.1f - Use patterns, models, and strategies to make generalizations about the algebraic properties for fluency

Grade 2 (2016 SOL to 2023 SOL Numbering)	Parameter Changes/Clarifications (2023 SOL)
	● 2.CE.1i-j - 'Identify and use' replaced with 'describe' the not equal symbol (≠) as representing a relationship where expressions on either side of the not equal symbol represent different values and 'justify' reasoning ● 2.MG.3c - Determine a line of symmetry that results in two figures that are *congruent* [Congruent moved from Grade 3] ● 2.PS.1 - Number of data points when creating pictographs and bar graphs increased from 16 to 25 ● 2.PFA.1a - Growing patterns (limited to increasing patterns) ● 2.PFA.1d - Transfer a given increasing pattern from one form to another and explain the connection between the two patterns

Deletions from Grade 2 (2016 SOL)	Additions to Grade 2 (2023 SOL)
● 2.1b - Identify the number that is 10 more, 10 less, 100 more, and 100 less than a given number up to 999 ● 2.3 - Ordinal numbers [Ordinals to tenth remains in Grade 1] ● 2.6c - Create single-step and two-step practical problems involving addition and subtraction ● 2.7a [EKS] - Compare the value of two sets of coins/dollars to $2.00 [Included in Grade 3] ● 2.10 - Calendar standard [Included in Grade 1] ● 2.11 - Read temperature [Included in Science standards] ● 2.13 - Identify and describe plane figures (circles, squares, and rectangles), according to their characteristics [Included in Grades K and 1] ● 2.14 - Probability	● 2.NS.1d - Represent forward counting patterns when counting by groups of 100 up to at least 1,000 ● 2.NS.1g - Choose a reasonable estimate up to 1,000 when given a contextual problem [Magnitude moved from Grade 1] ● 2.NS.2e - Compose and decompose whole numbers up to 200 by making connections between a variety of models and strategies ● 2.NS.2f - Plot and justify the position of a given number up to 100 on a number line with pre-marked benchmarks ● 2.NS.3c - Compose the whole for a given fractional part and its value for halves, fourths, eighths, thirds, and sixths ● 2.NS.4a - Identify a quarter and its value and determine multiple ways to represent the value of a quarter using pennies, nickels, and/or dimes [Identify quarter moved from Grade 1] ● 2.NS.4c - Construct a set of coins and/or bills to total a given amount of money whose value is $2.00 or less

Deletions from Grade 2 (2016 SOL)	Additions to Grade 2 (2023 SOL)
	• 2.MG.1a-b - Estimate/measure liquid volume to the nearest cup and identify different types of measuring cups as instruments to measure liquid volume • 2.MG.2a - Identify the number of minutes in an hour (60 minutes) and the number of hours in a day (24 hours) • 2.MG.2b - Determine the unit of time (minutes, hours, days, or weeks) that is most appropriate when measuring a given activity or context and explain reasoning • 2.MG.4b – Compare and contrast models and nets (cutouts) of cubes and rectangular prisms (e.g., number and shapes of faces, edges, vertices) • 2.PS.1 - Additional data analysis knowledge and skills representing the data cycle have been included (e.g., pose questions, determine data needed to answer a posed question, ask and answer questions about the data; draw conclusions)

KEY: NS = Number and Number Sense; CE = Computation and Estimation; MG = Measurement and Geometry; PS = Probability and Statistics; PFA = Patterns, Functions, and Algebra; EKS = Essential Knowledge and Skills (2016); KS = Knowledge and Skills (2023); US = Understanding the Standard

Grade 3 Mathematics *Standards of Learning* - 2023 Overview of Revisions

This overview includes a summary of the content embedded in five content strands.

Number and Number Sense: *Flexibility with composing and decomposing base 10 numbers and understanding the structure to build relationships among numbers allows us to quantify, measure and make decisions in life.*
- Read, write, and determine the place and value of each digit in a whole number, up to six digits
- Compare and order whole numbers up to 9,999
- Represent and compare fractions (proper, improper, or mixed numbers with denominators of 2, 3, 4, 5, 6, 8, and 10)
- Count, compare, represent, and make change for money amounts up to $5.00

Computation and Estimation: *The operations of addition, subtraction, multiplication, and division are used to represent and solve many different types of problems.*
- Represent, solve, and justify solutions to single-step and multistep problems, including those in context, using addition and subtraction with whole numbers where addends and minuends do not exceed 1,000
- Represent, solve, and justify solutions to single-step problems using multiplication and division
- Recall with automaticity multiplication facts through 10×10 and the corresponding division facts

Measurement and Geometry: *Analyzing and describing geometric objects, the relationships and structures among them, or the space that they occupy can be used to classify, quantify, measure, or count one or more attributes.*
- Estimate and measure objects by length, weight, and liquid volume to the nearest half or whole unit (U.S. Customary and metric)
- Solve problems involving area and perimeter (in both U.S. Customary and metric units)
- Demonstrate an understanding of the concept of time to the nearest minute and solve elapsed time problems in one-hour increments
- Identify, describe, classify, and compare polygons (triangles, quadrilaterals, pentagons, hexagons, octagons)
- Combine and subdivide triangles and quadrilaterals to create new polygons

Probability and Statistics: *The world can be investigated through posing questions and collecting, representing, analyzing, and interpreting data to describe and predict events and real-world phenomena.*
- Apply the data cycle (formulate questions; collect or acquire data; organize and represent data; and analyze data and communicate results) with a focus on pictographs and bar graphs

Patterns, Functions, and Algebra: *Relationships can be described, and generalizations can be made using patterns and relations.*
- Identify, describe, extend, and create increasing and decreasing patterns using various representations

Comparison of Grade 3 Mathematics *Standards of Learning* – 2016 to 2023

2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Number and Number Sense**	2023 *Standards of Learning* Knowledge and Skills (KS) **Number and Number Sense (NS)**
3.1 The student will **a) read, write, and identify the place and value of each digit in a six-digit whole number, with and without models;** ● Read six-digit numerals orally. (a) ● Write six-digit numerals in standard form that are stated verbally or written in words. (a) ● Determine the value of each digit in a six-digit whole number (e.g., in 165,724, the 7 represents 7 hundreds and its value is 700). (a) ● Represent numbers up to 9,999 in multiple ways, according to place value (e.g., 256 can be 1 hundred, 14 tens, and 16 ones, but also 25 tens and 6 ones), with and without models. (a)	**3.NS.1 The student will use place value understanding to read, write, and determine the place and value of each digit in a whole number, up to six digits, with and without models.** a) Read and write six-digit whole numbers in standard form, expanded form, and word form. b) Apply patterns within the base 10 system to determine and communicate, orally and in written form, the place and value of each digit in a six-digit whole number (e.g., in 165,724, the 5 represents 5 thousands and its value is 5,000). c) Compose, decompose, and represent numbers up to 9,999 in multiple ways, according to place value (e.g., 256 can be 1 hundred, 14 tens, 16 ones, but also 25 tens, 6 ones), with and without models.
3.1 The student will **b) round whole numbers, 9,999 or less, to the nearest ten, hundred, and thousand;** ● Round a given whole number, 9,999 or less, to the nearest ten, hundred, and thousand. (b) ● Solve problems, using rounding of numbers, 9,999 or less, to the nearest ten, hundred, and thousand. (b)	**[Included in 3.CE.1]**

2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Number and Number Sense**	2023 *Standards of Learning* Knowledge and Skills (KS) **Number and Number Sense (NS)**
3.1 The student will **c) compare and order whole numbers, each 9,999 or less.** ● Compare two whole numbers, each 9,999 or less, using symbols (>, <, =, or ≠) and/or words (*greater than, less than*, *equal to,* and *not equal to)*. (c) ● Order up to three whole numbers, each 9,999 or less, represented with concrete objects, pictorially, or symbolically from least to greatest and greatest to least. (c)	**3.NS.2 The student will demonstrate an understanding of the base 10 system to compare and order whole numbers up to 9,999.** a) Compare two whole numbers, each 9,999 or less, using symbols (>, <, =, ≠) and/or words (*greater than*, *less than*, *equal to*, *not equal to*), with and without models. b) Order up to three whole numbers, each 9,999 or less, represented with and without models, from least to greatest and greatest to least.
3.2 The student will **a) name and write fractions and mixed numbers represented by a model;** **b) represent fractions and mixed numbers, with models and symbols; and** **c) compare fractions having like and unlike denominators, using words and symbols (>, <, =, or ≠), with models.** ● Name and write fractions (proper and improper) and mixed numbers with denominators of 12 or less in symbols represented by concrete and/or pictorial models. (a) ● Represent a given fraction (proper or improper) and mixed numbers, using concrete or pictorial set, area/region, length/measurement models and symbols. (b) ● Identify a fraction represented by a model as the sum of unit fractions. (b)	**3.NS.3 The student will use mathematical reasoning and justification to represent and compare fractions (proper and improper) and mixed numbers with denominators of 2, 3, 4, 5, 6, 8, and 10), including those in context.** a) Represent, name, and write a given fraction (proper or improper) or mixed number with denominators of 2, 3, 4, 5, 6, 8, and 10 using: 　i) region/area models (e.g., pie pieces, pattern blocks, geoboards); 　ii) length models (e.g., paper fraction strips, fraction bars, rods, number lines); and 　iii) set models (e.g., chips, counters, cubes). b) Identify a fraction represented by a model as the sum of unit fractions. c) Using a model of a fraction greater than one, count the fractional parts to name and write it as an improper fraction and as a mixed number (e.g., $\frac{1}{4}, \frac{2}{4}, \frac{3}{4}, \frac{4}{4}, \frac{5}{4} = 1\frac{1}{4}$).

2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Number and Number Sense**	2023 *Standards of Learning* Knowledge and Skills (KS) **Number and Number Sense (NS)**
• Using a model of a fraction greater than one, count the fractional parts to name and write it as an improper fraction and as a mixed number (e.g., $\frac{1}{4}, \frac{2}{4}, \frac{3}{4}, \frac{4}{4}, \frac{5}{4} = 1\frac{1}{4}$, or $2\frac{1}{3} = \frac{7}{3}$). (b) • Compare a model of a fraction, less than or equal to one, to the benchmarks of 0, $\frac{1}{2}$, and 1. (c) • Compare proper fractions using the terms *greater than, less than, equal to, or not equal to* and the symbols ($<, >, =,$ and $\neq$). Comparisons are made between fractions with both like and unlike denominators, with concrete or pictorial models. (c)	d) Compose and decompose fractions (proper and improper) with denominators of 2, 3, 4, 5, 6, 8, and 10 in multiple ways (e.g., $\frac{7}{4} = \frac{4}{4} + \frac{3}{4}$ or $\frac{4}{6} = \frac{3}{6} + \frac{1}{6} = \frac{2}{6} + \frac{2}{6}$) with models. e) Compare a fraction, less than or equal to one, to the benchmarks of 0, $\frac{1}{2}$, and 1 using area/region models, length models, and without models. f) Compare two fractions (proper or improper) and/or mixed numbers with like numerators of 2, 3, 4, 5, 6, 8, and 10 (e.g., $\frac{2}{3} > \frac{2}{8}$) using words (*greater than, less than, equal to*) and/or symbols ($>, <, =$) using area/region models, length models, and without models. g) Compare two fractions (proper or improper) and/or mixed numbers with like denominators of 2, 3, 4, 5, 6, 8, and 10 (e.g., $\frac{3}{6} < \frac{4}{6}$) using words (*greater than, less than, equal to*) and/or symbols ($>, <, =$), using area/region models, length models, and without models. h) Represent equivalent fractions with denominators of 2, 3, 4, 5, 6, 8, or 10, using region/area models and length models.
[Previously 3.6]	**3.NS.4 The student will solve problems, including those in context, that involve counting, comparing, representing, and making change for money amounts up to $5.00.** a) Determine the value of a collection of bills and coins whose total is $5.00 or less.

2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Number and Number Sense**	2023 *Standards of Learning* Knowledge and Skills (KS) **Number and Number Sense (NS)**
	b) Construct a set of bills and coins to total a given amount of money whose value is $5.00 or less. c) Compare the values of two sets of coins or two sets of bills and coins, up to $5.00, with words (*greater than, less than, equal to)* and/or symbols ($>$, $<$, $=$), using concrete or pictorial models. d) Solve contextual problems to make change from $5.00 or less by using counting on or counting back strategies with concrete or pictorial models.

2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Computation and Estimation**	2023 *Standards of Learning* Knowledge and Skills (KS) **Computation and Estimation (CE)**
3.3 The student will **a) estimate and determine the sum or difference of two whole numbers; and** **b) create and solve single-step and multistep practical problems involving sums or differences of two whole numbers, each 9,999 or less.** ● Determine whether an estimate or an exact answer is an appropriate solution for practical addition and subtraction problems involving single-step and multistep problems. (a, b) ● Estimate the sum of two whole numbers with sums to 9,999. (a)	**3.CE.1 The student will estimate, represent, solve, and justify solutions to single-step and multistep problems, including those in context, using addition and subtraction with whole numbers where addends and minuends do not exceed 1,000.** a) Determine and justify whether an estimate or an exact answer is appropriate when solving single-step and multistep contextual problems involving addition and subtraction, where addends and minuends do not exceed 1,000. b) Apply strategies (e.g., rounding to the nearest 10 or 100, using compatible numbers, using other number relationships) to estimate a solution for single-step or multistep addition or

2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Computation and Estimation**	2023 *Standards of Learning* Knowledge and Skills (KS) **Computation and Estimation (CE)**
• Estimate the difference of two whole numbers, each 9,999 or less. (a) • Apply strategies, including place value and the properties of addition, to add two whole numbers with sums to 9,999. (a, b) • Apply strategies, including place value and the properties of addition, to subtract two whole numbers, each 9,999 or less. (a, b) • Use inverse relationships between addition and subtraction facts to solve practical problems. (b) • Create and solve single-step and multistep practical problems involving the sum or difference of two whole numbers, each 9,999 or less. (b)	subtraction problems, including those in context, where addends or minuends do not exceed 1,000. c) Apply strategies (e.g., place value, properties of addition, other number relationships) and algorithms, including the standard algorithm, to determine the sum or difference of two whole numbers where addends and minuends do not exceed 1,000. d) Identify and use the appropriate symbol to distinguish between expressions that are equal and expressions that are not equal (e.g., 256 - 13 = 220 + 23; 457 + 100 ≠ 557 + 100). e) Represent, solve, and justify solutions to single-step and multistep contextual problems involving addition and subtraction with whole numbers where addends and minuends do not exceed 1,000.
3.4 The student will **a) represent multiplication and division through 10×10, using a variety of approaches and models;** **b) create and solve single-step practical problems that involve multiplication and division through 10×10;** **c) demonstrate fluency with multiplication facts of 0, 1, 2, 5, and 10; and** • Represent multiplication using a variety of approaches and models (e.g., repeated addition, equal-sized groups, arrays, equal jumps on a number line, skip counting). (a) • Represent division using a variety of approaches and models (e.g., repeated subtraction, equal sharing, equal groups). (a)	**3.CE.2 The student will recall with automaticity multiplication and division facts, through 10×10; and represent, solve, and justify solutions to single-step contextual problems using multiplication and division with whole numbers.** a) Represent multiplication and division of whole numbers through 10×10, including in a contextual situation, using a variety of approaches and models (e.g., repeated addition/subtraction, equal-sized groups/sharing, arrays, equal jumps on a number line, using multiples to skip count). b) Use inverse relationships to write the related facts connected to a given model for multiplication and division of whole numbers through 10×10.

2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Computation and Estimation**	2023 *Standards of Learning* Knowledge and Skills (KS) **Computation and Estimation (CE)**
- Write three related equations (fact sentences) when given one equation (fact sentence) for multiplication or division (e.g., given $6 \times 7 = 42$, write $7 \times 6 = 42$, $42 \div 7 = 6$, and $42 \div 6 = 7$. (a) - Create practical problems to represent a multiplication or division fact. (b) - Use multiplication and division basic facts to represent a given situation, using a number sentence. (b) - Recognize and use the inverse relationship between multiplication and division to solve practical problems. (b) - Solve single-step practical problems that involve multiplication and division of whole numbers through 10×10. (b) - Demonstrate fluency with multiplication facts of 0, 1, 2, 5, and 10. (c) - Apply strategies, including place value and the properties of multiplication and/or addition when multiplying and dividing whole numbers. (a, b, c, d)	c) Apply strategies (e.g., place value, the properties of multiplication and/or addition) when multiplying and dividing whole numbers. d) Demonstrate fluency with multiplication facts through 10×10 by applying reasoning strategies (e.g., doubling, add-a-group, subtract-a-group, near squares, and inverse relationships). e) Represent, solve, and justify solutions to single-step contextual problems that involve multiplication and division of whole numbers through 10×10. f) Recall with automaticity the multiplication facts through 10×10 and the corresponding division facts. g) Create an equation to represent the mathematical relationship between equivalent expressions using multiplication and/or division facts through 10×10 (e.g., $4 \times 3 = 14 - 2$, $35 \div 5 = 1 \times 7$).
3.4 The student will **d) solve single-step practical problems involving multiplication of whole numbers, where one factor is 99 or less and the second factor is 5 or less.** Solve single-step practical problems involving multiplication of whole numbers, where one factor is 99 or less and the second factor is 5 or less. (d)	**[Included in Grade 4]**

2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Computation and Estimation**	2023 *Standards of Learning* Knowledge and Skills (KS) **Computation and Estimation (CE)**
3.5 The student will solve practical problems that involve addition and subtraction with proper fractions having like denominators of 12 or less. ● Solve practical problems that involve addition and subtraction with proper fractions having like denominators of 12 or less, using concrete and pictorial models representing area/regions (e.g., circles, squares, and rectangles), length/measurements (e.g., fraction bars and strips), and sets (e.g., counters).	**[Included in Grade 4]**

2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Measurement and Geometry**	2023 *Standards of Learning* Knowledge and Skills (KS) **Measurement and Geometry (MG)**
3.6 The student will **a) determine the value of a collection of bills and coins whose total value is $5.00 or less;** **b) compare the value of two sets of coins or two sets of coins and bills; and** **c) make change from $5.00 or less.** ● Determine the value of a collection of coins and bills whose total value is $5.00 or less. (a) ● Compare the values of two sets of coins or two sets of coins and bills, up to $5.00, using the terms *greater than, less than*, and *equal to*. (b)	**[Moved to 3.NS.4]**

2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Measurement and Geometry**	2023 *Standards of Learning* Knowledge and Skills (KS) **Measurement and Geometry (MG)**
● Make change from $5.00 or less. (c)	

3.7 The student will estimate and use U.S. Customary and metric units to measure
a) length to the nearest $\frac{1}{2}$ inch, inch, foot, yard, centimeter, and meter; and
b) liquid volume in cups, pints, quarts, gallons, and liters.
 	
● Estimate and use U.S. Customary and metric units to measure lengths of objects to the nearest $\frac{1}{2}$ inch, inch, foot, yard, centimeter, and meter. (a)
● Determine the actual measure of length using U.S. Customary and metric units to measure objects to the nearest $\frac{1}{2}$ inch, foot, yard, centimeter, and meter. (a)
● Estimate and use U.S. Customary and metric units to measure liquid volume to the nearest cup, pint, quart, gallon, and liter. (b)
● Determine the actual measure of liquid volume using U.S. Customary and metric units to measure to the nearest cup, pint, quart, gallon, and liter. (b)
	3.MG.1 The student will reason mathematically using standard units (U.S. Customary and metric) with appropriate tools to estimate and measure objects by length, weight/mass, and liquid volume to the nearest half or whole unit.
	
	a) Justify whether an estimate or an exact measurement is needed for a contextual situation and choose an appropriate unit.
	b) Estimate and measure:
	  i) length of an object to the nearest U.S. Customary unit ($\frac{1}{2}$ inch, inch, foot, yard) and metric unit (centimeter, meter);
	  ii) weight/mass of an object to the nearest U.S. Customary unit (pound) and metric unit (kilogram); and
	  iii) liquid volume to the nearest U.S. Customary unit (cup, pint, quart, gallon) and metric unit (liter).
	c) Compare estimates of length, weight/mass, or liquid volume with the actual measurements.

3.8 The student will estimate and **a) measure the distance around a polygon in order to determine its perimeter using U.S. Customary and metric units; and** **b) count the number of square units needed to cover a given surface in order to determine its area.**	**3.MG.2 The student will use multiple representations to estimate and solve problems, including those in context, involving area and perimeter (in both U.S. Customary and metric units).** a) Solve problems, including those in context, involving area: i) describe and give examples of area as a measurement in contextual situations; and

2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Measurement and Geometry**	2023 *Standards of Learning* Knowledge and Skills (KS) **Measurement and Geometry (MG)**

• Estimate and use U.S. Customary and metric units to measure the distance around a polygon with no more than six sides to determine the perimeter. (a)
• Determine the area of a given surface by estimating and then counting the number of square units needed to cover the surface. (b)
	ii) estimate and determine the area of a given surface by counting the number of square units, describe the measurement (using the number and unit) and justify the measurement.
	b) Solve problems, including those in context, involving perimeter:
	   i) describe and give examples of perimeter as a measurement in contextual situations;
	   ii) estimate and measure the distance around a polygon (with no more than six sides) to determine the perimeter and justify the measurement; and
	   iii) given the lengths of all sides of a polygon (with no more than six sides), determine its perimeter and justify the measurement.

3.9 The student will
a) tell time to the nearest minute, using analog and digital clocks;
b) solve practical problems related to elapsed time in one-hour increments within a 12-hour period; and
 	
• Tell time to the nearest minute, using analog and digital clocks. (a)
• Match a written time (e.g., 4:38, 7:09, 12:51) to the time shown on analog and digital clocks to the nearest minute. (a)
• Solve practical problems related to elapsed time in one-hour increments, within a 12-hour period (within a.m. or within p.m.):
   o when given the beginning time and the ending time, determine the time that has elapsed; (b)
   o when given the beginning time and amount of elapsed time in one-hour increments, determine the ending time; or (b)
	3.MG.3 The student will demonstrate an understanding of the concept of time to the nearest minute and solve single-step contextual problems involving elapsed time in one-hour increments within a 12-hour period.
	
	a) Tell and write time to the nearest minute, using analog and digital clocks.
	b) Match a written time (e.g., 4:38, 7:09, 12:51) to the time shown on analog and digital clocks to the nearest minute.
	c) Solve single-step contextual problems involving elapsed time in one-hour increments, within a 12-hour period (within a.m. or within p.m.) when given:
	   i) the starting time and the ending time, determine the amount of time that has elapsed;

2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Measurement and Geometry**	2023 *Standards of Learning* Knowledge and Skills (KS) **Measurement and Geometry (MG)**
o when given the ending time and the elapsed time in one-hour increments, determine the beginning time. (b)	ii) the starting time and amount of elapsed time in one-hour increments, determine the ending time; or iii) the ending time and the amount of elapsed time in one-hour increments, determine the starting time.

3.9 The student will
c) identify equivalent periods of time and solve practical problems related to equivalent periods of time.
 	
● Identify the number of minutes in an hour and the number of hours in a day. (c)
● Identify equivalent relationships observed in a calendar, including the approximate number of days in a given month (about 30), the number of days in a week, the number of days in a year (about 365 $\frac{1}{4}$), and the number of months in a year. (c)
● Solve practical problems related to equivalent periods of time to include:
  o approximate days in five or fewer months;
  o days in five or fewer weeks;
  o months in five or fewer years;
  o minutes in five or fewer hours; and
  o hours in five or fewer days. (c)
	[Minutes in an hour and hours in a day moved to Grade 2]
	[Equivalent relationships and practical problems related to equivalent periods of time deleted]

3.10 The student will read temperature to the nearest degree. ● Read Celsius and Fahrenheit temperatures to the nearest degree using real thermometers, physical models, or pictorial representations.	**[Included in Grade 3 Science standards]**

2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Measurement and Geometry**	2023 *Standards of Learning* Knowledge and Skills (KS) **Measurement and Geometry (MG)**
3.11 The student will identify and draw representations of points, lines, line segments, rays, and angles. ● Identify examples of points, lines, line segments, rays, and angles. ● Describe endpoints and vertices as they relate to lines, line segments, rays, and angles. ● Draw representations of points, line segments, rays, angles, and lines, using a ruler or straightedge.	**[Included in Grade 4]**
3.12 The student will **a) define polygon;** **b) identify and name polygons with 10 or fewer sides; and** **c) combine and subdivide polygons with three or four sides and name the resulting polygon(s).** ● Define polygon. (a) ● Classify figures as polygons or not polygons. (a) ● Identify and name polygons with 10 or fewer sides in various orientations: triangle is a three-sided polygon; quadrilateral is a four-sided polygon; pentagon is a five-sided polygon; hexagon is a six-sided polygon; heptagon is a seven-sided polygon; octagon is an eight-sided polygon; nonagon is a nine-sided polygon; and decagon is a ten-sided polygon. (b) ● Combine no more than three polygons, where each has three or four sides, and name the resulting polygon. (c) ● Subdivide a three-sided or four-sided polygon into no more than three parts and name the resulting polygon(s). (c)	**3.MG.4 The student will identify, describe, classify, compare, combine, and subdivide polygons.** a) Describe a polygon as a closed plane figure composed of at least three line segments that do not cross. b) Classify figures as polygons or not polygons and justify reasoning. c) Identify and describe triangles, quadrilaterals, pentagons, hexagons, and octagons in various orientations, with and without contexts. d) Identify and name examples of polygons (triangles, quadrilaterals, pentagons, hexagons, octagons) in the environment. e) Classify and compare polygons (triangles, quadrilaterals, pentagons, hexagons, octagons). f) Combine no more than three polygons, where each has three or four sides, and name the resulting polygon (triangles, quadrilaterals, pentagons, hexagons, octagons).

2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Measurement and Geometry**	2023 *Standards of Learning* Knowledge and Skills (KS) **Measurement and Geometry (MG)**
	g) Subdivide a three-sided or four-sided polygon into no more than three parts and name the resulting polygon(s).
3.13 The student will identify and describe congruent and noncongruent figures. ● Identify examples of congruent and noncongruent figures. ● Determine and explain why plane figures are congruent or noncongruent.	**[Moved to Grade 2]**

2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Probability and Statistics**	2023 *Standards of Learning* Knowledge and Skills (KS) **Probability and Statistics (PS)**
3.14 The student will investigate and describe the concept of probability as a measurement of chance and list possible outcomes for a single event. ● Define probability as the measurement of chance that an event will happen. ● List all possible outcomes for a single event (e.g., heads and tails are the two possible outcomes of flipping a coin). Limit the number of outcomes to 12 or fewer. ● Describe the degree of likelihood of an outcome occurring using terms such as *impossible, unlikely, equally likely, likely,* and *certain*.	**[Included in Grade 4]**

2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Probability and Statistics**	2023 *Standards of Learning* Knowledge and Skills (KS) **Probability and Statistics (PS)**

3.15 The student will
a) **collect, organize, and represent data in pictographs or bar graphs; and**
b) **read and interpret data represented in pictographs and bar graphs.**
 	
● Formulate questions to investigate. (a)
● Design data investigations to answer formulated questions, limiting the number of categories for data collection to four. (a)
● Collect and organize data, using various forms of data collections (e.g., surveys, polls, questionnaires, scientific experiments, observations). (a)
● Represent data in a pictograph (limited to 16 or fewer data points for no more than four categories). (a)
● Represent data in a bar graph (limited to 16 or fewer data points for no more than four categories). (a)
  o label each axis on a bar graph and give the bar graph a title. Limit increments on the numerical axis to whole numbers representing multiples of 1, 2, 5, or 10. (a)
● Analyze data represented in pictographs and bar graphs, orally and in writing. (b)
  o read the information presented on a bar or pictograph (e.g., the title, the categories, the description of the two axes). (b)
● Interpret information from pictographs and bar graphs, with up to 30 data points and up to eight categories, describe interpretation orally and by writing at least one sentence. (b)
	3.PS.1 The student will apply the data cycle (formulate questions; collect or acquire data; organize and represent data; and analyze data and communicate results) with a focus on pictographs and bar graphs.
	
	a) Formulate questions that require the collection or acquisition of data.
	b) Determine the data needed to answer a formulated question and collect or acquire existing data (limited to 30 or fewer data points for no more than eight categories) using various methods (e.g., polls, observations, tallies).
	c) Organize and represent a data set using pictographs that include an appropriate title, labeled axes, and key. Each pictograph symbol should represent 1, 2, 5 or 10 data points.
	d) Organize and represent a data set using bar graphs with a title and labeled axes, with and without the use of technology tools. Determine and use an appropriate scale (increments limited to multiples of 1, 2, 5 or 10).
	e) Analyze data represented in pictographs and bar graphs, and communicate results orally and in writing:
	  i) describe the categories of data and the data as a whole (e.g., data were collected on preferred ways to cook or prepare eggs - scrambled, fried, hard boiled, and egg salad);
	  ii) identify parts of the data that have special characteristics, including categories with the greatest, the least, or the same (e.g., most students prefer scrambled eggs);
	  iii) make inferences about data represented in pictographs and bar graphs;

2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Probability and Statistics**	2023 *Standards of Learning* Knowledge and Skills (KS) **Probability and Statistics (PS)**
o describe the categories of data and the data as a whole (e.g., data were collected on preferred ways to cook or prepare eggs — scrambled, fried, hard boiled, and egg salad); (b) o identify parts of the data that have special characteristics, including categories with the greatest, the least, or the same (e.g., most students prefer scrambled eggs); and (b) o select a correct interpretation of a graph from a set of interpretations, where one is correct and the remaining are incorrect. (b)	iv) use characteristics of the data to draw conclusions about the data and make predictions based on the data (e.g., it is unlikely that a third grader would like hard boiled eggs); and v) solve one- and two-step addition and subtraction problems using data from pictographs and bar graphs.

2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Patterns, Functions, and Algebra**	2023 *Standards of Learning* Knowledge and Skills (KS) **Patterns, Functions, and Algebra (PFA)**
3.16 The student will identify, describe, create, and extend patterns found in objects, pictures, numbers, and tables. ● Identify and describe repeating and growing patterns using words, objects, pictures, numbers, and tables. ● Identify a missing term in a pattern (e.g., 4, 6, _, 10, 12, 14). ● Create repeating and growing patterns using objects, pictures, numbers, and tables. ● Extend or identify missing parts in repeating and growing patterns using objects, pictures, numbers, and tables. ● Solve problems that involve the application of input and output rules limited to addition and subtraction of whole numbers.	**3.PFA.1 The student will identify, describe, extend, and create increasing and decreasing patterns (limited to addition and subtraction of whole numbers), including those in context, using various representations.** a) Identify and describe increasing and decreasing patterns using various representations (e.g., objects, pictures, numbers, number lines). b) Analyze an increasing or decreasing pattern and generalize the change to extend the pattern or identify missing terms using various representations.

2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Patterns, Functions, and Algebra**	2023 *Standards of Learning* Knowledge and Skills (KS) **Patterns, Functions, and Algebra (PFA)**
● When given the rule, determine the missing values in a list or table. (Rules will be limited to addition and subtraction of whole numbers.)	c) Solve contextual problems that involve identifying, describing, and extending patterns. d) Create increasing and decreasing patterns using objects, pictures, numbers, and number lines. e) Investigate and explain the connection between two different representations of the same increasing or decreasing pattern.
3.17 The student will create equations to represent equivalent mathematical relationships. ● Identify and use the appropriate symbol to distinguish between expressions that are equal and expressions that are not equal (e.g., $256 - 13 = 220 + 23$; $143 + 17 = 140 + 20$; $457 + 100 \neq 557 +100$). ● Create equations to represent equivalent mathematical relationships (e.g., $4 \times 3 = 14 - 2$).	**[Included in 3.CE.1 and 3.CE.2]**

2023 Grade 3 Mathematics SOL – Summary of Changes

Grade 3 (2016 SOL to 2023 SOL Numbering)	Parameter Changes/Clarifications (2023 SOL)
3.1a → 3.NS.1 3.1b → [Included in 3.CE.1] 3.1c → 3.NS.2 3.2a-c → 3.NS.3 3.3a-b → 3.CE.1 3.4a-c → 3.CE.2 3.4d → [Included in Grade 4] 3.5 → [Included in Grade 4] 3.6a-c → 3.NS.4 3.7a-b → 3.MG.1 3.8a-b → 3.MG.2 3.9a-b → 3.MG.3 3.9c → [Minutes in an hour/hours in a day -Moved to Grade 2; Remaining content deleted] 3.10 → [Included in Science standards] 3.11 → [Included in Grade 4] 3.12a-c → 3.MG.4 3.13 → [Moved to Grade 2] 3.14 → [Included in Grade 4] 3.15a-b → 3.PS.1 3.16 → 3.PFA.1 3.17 → [Included in 3.CE.1 and 3.CE.2]	3.NS.1a – Read and write six-digit whole numbers includes expanded form (previously only standard and word form) 3.NS.1c – Compose, decompose, and represent numbers up to 9,999 in multiple ways (previously only represent) 3.NS.3a-h - Denominators limited to 2, 3, 4, 5, 6, 8, 10 (previously 12 or less); comparison of two fractions includes proper fractions, improper fractions, and mixed numbers (previously limited to proper fractions) 3.NS.3e,f,g - Fraction comparison strategies clarified to include comparing fractions with like numerators or like denominators 3.CE.1 - "Create and solve…" reworded to "Estimate, represent, solve, and justify solutions to…" problems 3.CE.1a-e - Addends and minuends do not exceed 1,000 (previously each 9,999 or less) 3.CE.1b - Rounding included as an estimation strategy 3.CE.2d,f - "Recall with automaticity the multiplication facts through 10×10 and the corresponding division facts" included along with "Demonstrate fluency with multiplication facts through 10×10 by applying reasoning strategies" 3.MG.2b - Determine the perimeter of a polygon, given the lengths of all sides (with no more than six sides) and justify 3.MG.4c-f - Polygons limited to triangles, quadrilaterals, pentagons, hexagons, octagons (heptagons, nonagons, decagons deleted) 3.PS.1b - Data collection increased to 30 or fewer data points for no more than eight categories (previously 16 or fewer data points for no more than four categories)

Deletions from Grade 3 (2016 SOL)	Additions to Grade 3 (2023 SOL)
3.3 - Create single-step and multistep practical problems involving the sum or difference of two whole numbers, each 9,999 or less 3.4 - Create practical problems to represent a multiplication or division fact 3.4 - Solve single-step practical problems involving multiplication of whole numbers, where one factor is 99 or less and the second factor is 5 or less [Moved to Grade 4] 3.5 - Solve practical problems that involve addition and subtraction with proper fractions having like denominators of 12 or less, using concrete and pictorial models [Moved to Grade 4] 3.9c - Identify the number of minutes in an hour and the number of hours in a day [Moved to Grade 2] 3.9c - Identify equivalent relationships observed in a calendar, including the approximate number of days in a given month, the number of days in a week, the number of days in a year, and the number of months in a year 3.9c - Solve practical problems related to equivalent periods of time to include approximate days in five or fewer months; days in five or fewer weeks; months in five or fewer years; minutes in five or fewer hours; and hours in five or fewer days 3.10 - Read Celsius and Fahrenheit temperatures to the nearest degree [Included in Science standards] 3.11 - Identify examples of points, lines, line segments, rays, and angles; describe endpoints and vertices as they relate to lines, line segments, rays, and angles; draw representations of points, line segments, rays, angles, and lines, using a ruler or straightedge [Included in Grade 4] 3.13 - Identify examples of congruent and noncongruent figures; determine and explain why plane figures are congruent or noncongruent [Moved to Grade 2]	3.NS.3d - Compose and decompose fractions (proper and improper) with denominators of 2, 3, 4, 5, 6, 8, or 10 in multiple ways with models 3.NS.3h – Represent equivalent fractions with denominators of 2, 3, 4, 5, 6, 8, and 10 using region/area and length models 3.NS.4b - Construct a set of bills and coins to total a given amount of money whose value is $5.00 or less 3.MG.1a - Justify whether an estimate or an exact measurement is needed for a contextual situation and choose an appropriate unit 3.MG.1b - Estimate and measure the weight/mass of an object to the nearest U.S. Customary unit (pound) and metric unit (kilogram) 3.MG.1c - Compare estimates of length, weight/mass, or liquid volume with the actual measurements 3.MG.2a-b - Describe and give examples of area and perimeter as measurements in contextual situations 3.MG.4e - Classify and compare polygons (triangles, quadrilaterals, pentagons, hexagons, octagons) 3.PS.1a-e - Additional data analysis knowledge and skills representing the data cycle have been included 3.PFA.1e – Investigate and explain connection between two different representations of the same increasing or decreasing pattern

Deletions from Grade 3 (2016 SOL)	Additions to Grade 3 (2023 SOL)
3.14 - Investigate and describe the concept of probability as a measurement of chance and list possible outcomes [Included in Grade 4] 3.16 - Identify, describe, create, extend repeating patterns. [Included in Grades 1 and 2] 3.16 - Patterns in tables, including solving problems that involve the application of input and output rules limited to addition and subtraction of whole numbers and determining the missing values in a table [Included in Grade 4]	

KEY: NS = Number and Number Sense; CE = Computation and Estimation; MG = Measurement and Geometry; PS = Probability and Statistics; PFA = Patterns, Functions, and Algebra; EKS = Essential Knowledge and Skills (2016); KS = Knowledge and Skills (2023); US = Understanding the Standard

Grade 4 Mathematics *Standards of Learning* - 2023 Overview of Revisions

This overview includes a summary of the content embedded in five content strands.

Number and Number Sense: *Exploring relationships between whole numbers, fractions, and decimals and their representations provides meaning and structure and allows us to quantify, measure and make decisions in life.*
- Read, write, and identify the place and value of each digit in a nine-digit whole number
- Compare and order numbers up to seven digits
- Represent, compare, and order fractions (proper, improper, or mixed numbers with denominators 12 or less)
- Represent, compare, and order decimals through thousandths
- Identify and represent fraction and decimal equivalencies (limited to halves, fourths, fifths, tenths, and hundredths)

Computation and Estimation: *The operations of addition, subtraction, multiplication, and division, and estimation, allow us to model, represent, and solve different types of problems with whole numbers and rational numbers (not including integers).*
- Represent, solve, and justify solutions to single-step and multistep problems, using addition, subtraction, and multiplication with whole numbers, and single-step contextual problems using division
- Recall with automaticity multiplication facts through 12×12 and the corresponding division facts
- Solve addition and subtraction problems involving fractions with like denominators
- Solve problems involving multiplication of a whole number and a unit fraction with models
- Solve addition and subtraction problems involving decimals through the thousandths

Measurement and Geometry: *Analyzing and describing geometric objects, the relationships and structures among them, or the space that they occupy can be used to classify, quantify, measure, or count one or more attributes.*
- Solve problems that involve length, weight, and liquid volume using U.S. Customary and metric units
- Solve problems to determine equivalent measures of length, weight, and liquid volume within the U.S. Customary system
- Solve single-step contextual problems involving elapsed time (limited to hours and minutes within a 12-hour period)
- Solve problems involving area and perimeter of rectangles and squares
- Identify, describe, and draw points, rays, line segments, angles, and lines, including intersecting, parallel, and perpendicular lines
- Classify and describe quadrilaterals (parallelograms, rectangles, squares, rhombi, and/or trapezoids)
- Identify, describe, compare, and contrast plane and solid figures

Probability and Statistics: *The world can be investigated through posing questions and collecting, representing, analyzing, and interpreting data to describe and predict events and real-world phenomena.*

- Apply the data cycle (formulate questions; collect or acquire data; organize and represent data; and analyze data and communicate results) with a focus on line graphs
- Determine the probability of an outcome of a simple event and model, predict, and justify what might occur in the future.

Patterns, Functions, and Algebra: *Relationships can be described and generalizations can be made using patterns and relations.*

- Identify, describe, extend, and create increasing and decreasing patterns using various representations

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Comparison of Grade 4 Mathematics *Standards of Learning* – 2016 to 2023

2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Number and Number Sense** * On the state assessment, items measuring this objective are assessed without the use of a calculator.	2023 *Standards of Learning* Knowledge and Skills (KS) **Number and Number Sense (NS)** *On the state assessment, items measuring this objective are assessed without the use of a calculator.
4.1 The student will **a) read, write, and identify the place and value of each digit in a nine-digit whole number;** ● Read nine-digit whole numbers, presented in standard form, and represent the same number in written form. (a) ● Write nine-digit whole numbers in standard form when the numbers are presented orally or in written form. (a) ● Identify and communicate, orally and in written form, the place and value for each digit in a nine-digit whole number. (a)	**4.NS.1 The student will use place value understanding to read, write, and identify the place and value of each digit in a nine-digit whole number.** a) Read nine-digit whole numbers, presented in standard form, and represent the same number in written form. b) Write nine-digit whole numbers in standard form when the numbers are presented orally or in written form. c) Apply patterns within the base 10 system to determine and communicate, orally and in written form, the place and value of each digit in a nine-digit whole number (e.g., in 568,165,724, the 8 represents 8 millions and its value is 8,000,000).
4.1 The student will **b) compare and order whole numbers expressed through millions; and** ● Compare two whole numbers expressed through millions, using the words *greater than, less than, equal to*, and *not equal to* or using the symbols >, <, =, or ≠. (b) ● Order up to four whole numbers expressed through millions. (b)	**4.NS.2 The student will demonstrate an understanding of the base 10 system to compare and order whole numbers up to seven digits.** a) Compare two whole numbers up to seven digits each, using words (*greater than, less than, equal to, not equal to)* and/or using symbols (>, <, =, ≠). b) Order up to four whole numbers up to seven digits each, from least to greatest or greatest to least.
4.1 The student will	**[Included in 4.CE.1]**

2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Number and Number Sense** * On the state assessment, items measuring this objective are assessed without the use of a calculator.	2023 *Standards of Learning* Knowledge and Skills (KS) **Number and Number Sense (NS)** *On the state assessment, items measuring this objective are assessed without the use of a calculator.
c) round whole numbers expressed through millions to the nearest thousand, ten thousand, and hundred thousand. ● Round whole numbers expressed through millions to the nearest thousand, ten thousand, and hundred thousand place. (c) ● Identify the range of numbers that round to a given thousand, ten thousand, and hundred thousand. (c)	
4.2 The student will **a) compare and order fractions and mixed numbers, with and without models;*** **b) represent equivalent fractions;* and** **c) identify the division statement that represents a fraction, with models and in context.** ● Compare and order no more than four fractions having like and unlike denominators of 12 or less, using concrete and pictorial models. (a) ● Use benchmarks (e.g., 0, $\frac{1}{2}$ or 1) to compare and order no more than four fractions having unlike denominators of 12 or less. (a) ● Compare and order no more than four fractions with like denominators of 12 or less by comparing the number of parts (numerators) (e.g., $\frac{1}{5} < \frac{3}{5}$). (a)	**4.NS.3 The student will use mathematical reasoning and justification to represent, compare, and order fractions (proper, improper, and mixed numbers with denominators 12 or less), with and without models.** a) Compare and order no more than four fractions (proper or improper), and/or mixed numbers, with like denominators by comparing the number of parts (numerators) using fractions with denominators of 12 or less (e.g., $\frac{1}{5} < \frac{3}{5}$). Justify solutions orally, in writing, or with a model.* b) Compare and order no more than four fractions (proper or improper), and/or mixed numbers, with like numerators and unlike denominators by comparing the size of the parts using fractions with denominators of 12 or less (e.g., $\frac{3}{8} < \frac{3}{5}$). Justify solutions orally, in writing, or with a model.*

2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Number and Number Sense** * On the state assessment, items measuring this objective are assessed without the use of a calculator.	2023 *Standards of Learning* Knowledge and Skills (KS) **Number and Number Sense (NS)** *On the state assessment, items measuring this objective are assessed without the use of a calculator.
• Compare and order no more than four fractions with like numerators and unlike denominators of 12 or less by comparing the size of the parts (e.g., $\frac{3}{9} < \frac{3}{5}$). (a) • Compare and order no more than four fractions (proper or improper), and/or mixed numbers, having denominators of 12 or less. (a) • Use the symbols >, <, =, and ≠ to compare fractions (proper or improper) and/or mixed numbers having denominators of 12 or less. (a) • Represent equivalent fractions through twelfths, using region/area models, set models, and measurement/length models. (b) • Identify the division statement that represents a fraction with models and in context (e.g., $\frac{3}{5}$ means the same as 3 divided by 5 or $\frac{3}{5}$ represents the amount of muffin each of five children will receive when sharing 3 muffins equally). (c)	c) Use benchmarks (e.g., 0, $\frac{1}{2}$, or 1) to compare and order no more than four fractions (proper or improper), and/or mixed numbers, with like and unlike denominators of 12 or less.* d) Compare two fractions (proper or improper) and/or mixed numbers using fractions with denominators of 12 or less using the symbols >, <, and = (e.g., $\frac{2}{3} > \frac{1}{7}$). Justify solutions orally, in writing, or with a model.* e) Represent equivalent fractions with denominators of 12 or less, with and without models.* f) Compose and decompose fractions (proper and improper) and/or mixed numbers with denominators of 12 or less, in multiple ways, with and without models.* g) Represent the division of two whole numbers as a fraction given a contextual situation and a model (e.g., $\frac{3}{5}$ means the same as 3 divided by 5 or $\frac{3}{5}$ represents the amount of muffin each of five children will receive when sharing three muffins equally).
4.3 The student will **a) read, write, represent, and identify decimals expressed through thousandths;** **c) compare and order decimals; and**	**4.NS.4 The student will use mathematical reasoning and justification to represent, compare, and order decimals through thousandths with and without models.** a) Investigate and describe the ten-to-one place value relationship for decimals through thousandths, using concrete models (e.g., place value mats/charts, decimal squares, base 10 blocks).

2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Number and Number Sense** * On the state assessment, items measuring this objective are assessed without the use of a calculator.	2023 *Standards of Learning* Knowledge and Skills (KS) **Number and Number Sense (NS)** *On the state assessment, items measuring this objective are assessed without the use of a calculator.
- Read and write decimals expressed through thousandths, using base-ten manipulatives, drawings, and numerical symbols. (a) - Represent and identify decimals expressed through thousandths, using base-ten manipulatives, pictorial representations, and numerical symbols (e.g., relate the appropriate drawing to 0.05). (a) - Investigate the ten-to-one place value relationship for decimals through thousandths, using base-ten manipulatives (e.g., place value mats/charts, decimal squares, and base-ten blocks). (a) - Identify and communicate, both orally and in written form, the position and value of a decimal through thousandths (e.g., given 0.385, the 8 is in the hundredths place and has a value of 0.08). (a) - Compare two decimals expressed through thousandths, using symbols (>, <, =, and ≠) and/or words (*greater than, less than, equal to*, and *not equal to*). (c) - Order a set of up to four decimals, expressed through thousandths, from least to greatest or greatest to least. (c)	b) Represent and identify decimals expressed through thousandths, using concrete, pictorial, and numerical representations. c) Read and write decimals expressed through thousandths, using concrete, pictorial, and numerical representations. d) Identify and communicate, both orally and in written form, the place and value of each digit in a decimal through thousandths (e.g., given 0.385, the 8 is in the hundredths place and has a value of 0.08). e) Compare using symbols (<, >, =) and/or words (*greater than, less than, equal to*) and order (least to greatest and greatest to least), a set of no more than four decimals expressed through thousandths, using multiple strategies (e.g., benchmarks, place value, number lines). Justify solutions with a model, orally, and in writing.
4.3 The student will **b) round decimals to the nearest whole number;** Round decimals expressed through thousandths to the nearest whole number. (b)	**[Included in 4.CE.4]**

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4.3 The student will **d) given a model, write the decimal and fraction equivalents. *** ● Represent fractions for halves, fourths, fifths, and tenths as decimals through hundredths, using concrete objects. (d) ● Relate fractions to decimals, using concrete objects (e.g., 10-by10 grids, meter sticks, number lines, decimal squares, decimal circles, money). (d) ● Write the decimal and fraction equivalent for a given model (e.g., $\frac{1}{4} = 0.25$ or $0.25 = \frac{1}{4}$; $1.25 = \frac{5}{4}$ or $1\frac{1}{4}$). (d)	**4.NS.5 The student will reason about the relationship between fractions and decimals (limited to halves, fourths, fifths, tenths, and hundredths) to identify and represent equivalencies.** a) Represent fractions (proper or improper) and/or mixed numbers as decimals through hundredths, using multiple representations, limited to halves, fourths, fifths, tenths, and hundredths.* b) Identify and model equivalent relationships between fractions (proper or improper) and/or mixed numbers and decimals, using halves, fourths, fifths, tenths, and hundredths.* c) Write the decimal and fraction equivalent for a given model (e.g., $\frac{1}{4} = 0.25$ or $0.25 = \frac{1}{4}$; $1.25 = \frac{5}{4}$ or $1\frac{1}{4}$; $1.02 = \frac{102}{100}$ or $1\frac{2}{100}$).*

2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Computation and Estimation** * On the state assessment, items measuring this objective are assessed without the use of a calculator.	2023 *Standards of Learning* Knowledge and Skills (KS) **Computation and Estimation (CE)** *On the state assessment, items measuring this objective are assessed without the use of a calculator.
4.4 The student will **a) demonstrate fluency with multiplication facts through 12 × 12, and the corresponding division facts;*** **b) estimate and determine sums, differences, and products of whole numbers;***	**4.CE.2 The student will estimate, represent, solve, and justify solutions to single-step and multistep problems, including those in context, using multiplication with whole numbers, and single-step problems, including those in context, using division with whole**

2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Computation and Estimation** * On the state assessment, items measuring this objective are assessed without the use of a calculator.	2023 *Standards of Learning* Knowledge and Skills (KS) **Computation and Estimation (CE)** *On the state assessment, items measuring this objective are assessed without the use of a calculator.
c) estimate and determine quotients of whole numbers, with and without remainders;* and **d) create and solve single-step and multistep practical problems involving addition, subtraction, and multiplication, and single step practical problems involving division with whole numbers.** • Demonstrate fluency with multiplication through 12×12, and the corresponding division facts. (a) • Estimate whole number sums, differences, products, and quotients, with and without context. (b, c) • Apply strategies, including place value and the properties of multiplication and/or addition, to determine the product of two whole numbers when both factors have two digits or fewer. (b) • Apply strategies, including place value and the properties of multiplication and/or addition, to determine the quotient of two whole numbers, given a one-digit divisor and a two- or three-digit dividend, with and without remainders. (c) • Refine estimates by adjusting the final amount, using terms such as *closer to*, *between*, and *a little more than*. (b, c) • Create and solve single-step and multistep practical problems involving addition, subtraction, and multiplication with whole numbers. (d) • Create and solve single-step practical problems involving division with whole numbers. (d) • Use the context in which a practical problem is situated to interpret the quotient and remainder. (d)	**numbers; and recall with automaticity the multiplication facts through 12×12 and the corresponding division facts.** a) Determine and justify whether an estimate or an exact answer is appropriate when solving contextual problems involving multiplication, and division of whole numbers. Refine estimates by adjusting the final amount, using terms such as *closer to*, *between*, and *a little more than*. b) Recall with automaticity the multiplication facts through 12×12 and the corresponding division facts.* c) Create an equation using addition, subtraction, multiplication, and division to represent the relationship between equivalent mathematical expressions (e.g., $4 \times 3 = 2 \times 6$; $10 + 8 = 36 \div 2$; $12 \times 4 = 60 - 12$). d) Identify and use the appropriate symbol to distinguish between expressions that are equal and expressions that are not equal, using addition, subtraction, multiplication, and division (e.g., $4 \times 12 = 8 \times 6$ and $64 \div 8 \neq 8 \times 8$). e) Determine all factor pairs for a whole number 1 to 100, using concrete, pictorial, and numerical representations. f) Determine common factors and the greatest common factor of no more than three numbers. g) Apply strategies (e.g., rounding, place value, properties of multiplication and/or addition) and algorithms, including the standard algorithm, to estimate and determine the product of two whole numbers when given:

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	i) a two-digit factor and a one-digit factor;* ii) a three-digit factor and a one-digit factor;* or iii) a two-digit factor and a two-digit factor.* h) Estimate, represent, solve, and justify solutions to single-step and multistep contextual problems that involve multiplication with whole numbers. i) Apply strategies (e.g., rounding, compatible numbers, place value) and algorithms, including the standard algorithm, to estimate and determine the quotient of two whole numbers, given a one-digit divisor and a two- or three-digit dividend, with and without remainders.* j) Estimate, represent, solve, and justify solutions to single-step contextual problems involving division with whole numbers. k) Interpret the quotient and remainder when solving a contextual problem.
4.4 The student will **b) estimate and determine sums, differences, and products of whole numbers;*** **d) create and solve single-step and multistep practical problems involving addition, subtraction, and multiplication, and single step practical problems involving division with whole numbers.** ● Estimate whole number sums, differences, products, and quotients, with and without context. (b, c)	**4.CE.1 The student will estimate, represent, solve, and justify solutions to single-step and multistep problems, including those in context, using addition and subtraction with whole numbers.** a) Determine and justify whether an estimate or an exact answer is appropriate when solving contextual problems involving addition and subtraction with whole numbers. Refine estimates by adjusting the final amount, using terms such as *closer to*, *between*, and *a little more than*. b) Apply strategies (e.g., rounding to the nearest 100 or 1,000, using compatible numbers, other number relationships) to

2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Computation and Estimation** * On the state assessment, items measuring this objective are assessed without the use of a calculator.	2023 *Standards of Learning* Knowledge and Skills (KS) **Computation and Estimation (CE)** *On the state assessment, items measuring this objective are assessed without the use of a calculator.
• Apply strategies, including place value and the properties of addition to determine the sum or difference of two whole numbers, each 999,999 or less. (b) • Refine estimates by adjusting the final amount, using terms such as *closer to*, *between*, and *a little more than*. (b, c) • Create and solve single-step and multistep practical problems involving addition, subtraction, and multiplication with whole numbers. (d)	estimate a solution for single-step or multistep addition or subtraction problems with whole numbers, where addends or minuends do not exceed 10,000.* c) Apply strategies (e.g., place value, properties of addition, other number relationships) and algorithms, including the standard algorithm, to determine the sum or difference of two whole numbers, where addends and minuends do not exceed 10,000.* d) Estimate, represent, solve, and justify solutions to single-step and multistep contextual problems involving addition and subtraction with whole numbers, where addends and minuends do not exceed 1,000,000.
4.5 The student will **a) determine common multiples and factors, including least common multiple and greatest common factor;** • Determine common multiples and common factors of numbers. (a) • Determine the least common multiple and greatest common factor of no more than three numbers. (a)	**[Common factors included in 4.CE.1; Common multiples moved to Grade 5]**
4.5 The student will **b) add and subtract fractions and mixed numbers having like and unlike denominators; * and** **c) solve single-step practical problems involving addition and subtraction with fractions and mixed numbers.**	**4.CE.3 The student will estimate, represent, solve, and justify solutions to single-step problems, including those in context, using addition and subtraction of fractions (proper, improper, and mixed numbers with like denominators of 2, 3, 4, 5, 6, 8, 10, and 12), with and without models; and solve single-step contextual problems**

2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Computation and Estimation** * On the state assessment, items measuring this objective are assessed without the use of a calculator.	2023 *Standards of Learning* Knowledge and Skills (KS) **Computation and Estimation (CE)** *On the state assessment, items measuring this objective are assessed without the use of a calculator.
• Determine a common denominator for fractions, using common multiples. Common denominators should not exceed 60. (b) • Estimate the sum or difference of two fractions. (b, c) • Add and subtract fractions (proper or improper) and/or mixed numbers, having like and unlike denominators limited to 2, 3, 4, 5, 6, 8, 10, and 12, and simplify the resulting fraction. (Subtraction with fractions will be limited to problems that do not require regrouping). (b) • Solve single-step practical problems that involve addition and subtraction with fractions (proper or improper) and/or mixed numbers, having like and unlike denominators limited to 2, 3, 4, 5, 6, 8, 10, and 12, and simplify the resulting fraction. (Subtraction with fractions will be limited to problems that do not require regrouping). (c)	**involving multiplication of a whole number (12 or less) and a unit fraction, with models.** a) Estimate and determine the sum or difference of two fractions (proper or improper) and/or mixed numbers, having like denominators limited to 2, 3, 4, 5, 6, 8, 10, and 12 (e.g., $\frac{3}{8} + \frac{3}{8}$, $2\frac{1}{5} + \frac{4}{5}$, $\frac{7}{4} - \frac{5}{4}$) and simplify the resulting fraction. Addition and subtraction with fractions may include regrouping.* b) Estimate, represent, solve, and justify solutions to single-step contextual problems using addition and subtraction with fractions (proper or improper) and/or mixed numbers, having like denominators limited to 2, 3, 4, 5, 6, 8, 10, and 12, and simplify the resulting fraction. Addition and subtraction with fractions may include regrouping. c) Solve single-step contextual problems involving multiplication of a whole number, limited to 12 or less, and a unit fraction (e.g., $6 \times \frac{1}{3}$, $\frac{1}{5} \times 8$, $2 \times \frac{1}{10}$), with models.* d) Apply the inverse property of multiplication in models (e.g., use a visual fraction model to represent $\frac{4}{4}$ or 1 as the product of $4 \times \frac{1}{4}$).
4.6 The student will **a) add and subtract decimals;* and** **b) solve single-step and multistep practical problems involving addition and subtraction with decimals.**	**4.CE.4 The student will estimate, represent, solve, and justify solutions to single-step and multistep problems, including those in**

2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Computation and Estimation** * On the state assessment, items measuring this objective are assessed without the use of a calculator.	2023 *Standards of Learning* Knowledge and Skills (KS) **Computation and Estimation (CE)** *On the state assessment, items measuring this objective are assessed without the use of a calculator.
- Estimate sums and differences of decimals. (a) - Add and subtract decimals through thousandths, using concrete materials, pictorial representations, and paper and pencil. (a) - Solve single-step and multistep practical problems that involve adding and subtracting with decimals through thousandths. (b)	**context, using addition and subtraction of decimals through the thousandths, with and without models.** a) Apply strategies (e.g., rounding to the nearest whole number, using compatible numbers) and algorithms, including the standard algorithm, to estimate and determine the sum or difference of two decimals through the thousandths, with and without models, in which:* i) decimals do not exceed the thousandths; and ii) addends, subtrahends, and minuends are limited to four digits. b) Estimate, represent, solve, and justify solutions to single-step and multistep contextual problems using addition and subtraction of decimals through the thousandths.

2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Measurement and Geometry**	2023 *Standards of Learning* Knowledge and Skills (KS) **Measurement and Geometry (MG)**
4.7 The student will solve practical problems that involve determining perimeter and area in U.S. Customary and metric units. - Determine the perimeter of a polygon with no more than eight sides, when the lengths of the sides are given, with diagrams.	**4.MG.3 The student will use multiple representations to develop and use formulas to solve problems, including those in context, involving area and perimeter limited to rectangles and squares (in both U.S. Customary and metric units).**

2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Measurement and Geometry**	2023 *Standards of Learning* Knowledge and Skills (KS) **Measurement and Geometry (MG)**
• Determine the perimeter and area of a rectangle when given the measure of two adjacent sides, with and without diagrams. • Determine the perimeter and area of a square when the measure of one side is given, with and without diagrams. • Solve practical problems that involve determining perimeter and area in U.S. Customary and metric units.	a) Use concrete materials and pictorial models to develop a formula for the area and perimeter of a rectangle (including a square). b) Determine the area and perimeter of a rectangle when given the measure of two adjacent sides (in whole number units), with and without models. c) Determine the area and perimeter of a square when given the measure of one side (in whole number units), with and without models. d) Use concrete materials and pictorial models to explore the relationship between area and perimeter of rectangles. e) Identify and represent rectangles with the same perimeter and different areas or with the same area and different perimeters. f) Solve contextual problems involving area and perimeter of rectangles and squares.
4.8 The student will **a) estimate and measure length and describe the result in U.S. Customary and metric units;** **b) estimate and measure weight/mass and describe the result in U.S. Customary and metric units;** **c) given the equivalent measure of one unit, identify equivalent measures of length, weight/mass, and liquid volume between units within the U.S. Customary system; and** **d) solve practical problems that involve length, weight/mass, and liquid volume in U.S. Customary units.**	**4.MG.1 The student will reason mathematically to solve problems, including those in context, that involve length, weight/mass, and liquid volume using U.S. Customary and metric units.** a) Determine an appropriate unit of measure to use when measuring: 　i) length in both U.S. Customary (inch, foot, yard, mile) and metric units (millimeter, centimeter, meter); 　ii) weight/mass in both U.S. Customary (ounce, pound) and metric units (gram, kilogram); and 　iii) liquid volume in both U.S. Customary (cup, pint, quart, gallon) and metric unit (milliliter, liter). b) Estimate and measure:

2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Measurement and Geometry**	2023 *Standards of Learning* Knowledge and Skills (KS) **Measurement and Geometry (MG)**

• Determine an appropriate unit of measure (inch, foot, yard, mile, millimeter, centimeter, and meter) to use when measuring length in both U.S. Customary and metric units. (a)
 	
• Estimate and measure length in U.S. Customary and metric units, measuring to the nearest part of an inch ($\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$), and to the nearest foot, yard, millimeter, centimeter, or meter, and record the length including the unit of measure (e.g., 24 inches). (a)
 	
• Compare estimates of the length with the actual measurement of the length. (a)
 	
• Determine an appropriate unit of measure (ounce, pound, gram, and kilogram) to use when measuring the weight/mass of everyday objects in both U.S. Customary and metric units. (b)
 	
• Estimate and measure the weight/mass of objects in both U.S. Customary and metric units (ounce, pound, gram, or kilogram) to the nearest appropriate measure, using a variety of measuring instruments. (b)
 	
• Record the weight/mass of an object with the unit of measure (e.g., 24 grams). (b)
 	
• Given the equivalent measure of one unit, identify equivalent measures between units within the U.S. Customary system for:
  o length (inches and feet, feet and yards, inches, and yards); yards and miles;
  o weight/mass (ounces and pounds); and
  o liquid volume (cups, pints, quarts, and gallons). (c)
 	
• Solve practical problems that involve length, weight/mass, and liquid volume in U.S. Customary units. (d)
	i) length of an object to the nearest U.S. Customary unit ($\frac{1}{2}$ inch, $\frac{1}{4}$ inch, $\frac{1}{8}$ inch, foot, yard) and nearest metric unit (millimeter, centimeter, or meter);
	  ii) weight/mass of an object to the nearest U.S. Customary unit (ounce, pound) and nearest metric unit (gram, kilogram); and
	  iii) liquid volume to the nearest U.S. Customary unit (cup, pint, quart, gallon) and nearest metric unit (milliliter, liter).
	
	c) Compare estimates of length, weight/mass, or liquid volume with the actual measurements.
	
	d) Given the equivalent measure of one unit, solve problems, including those in context, by determining the equivalent measures within the U.S. Customary system for:
	  i) length (inches and feet, feet and yards, inches and yards);
	  ii) weight/mass (ounces and pounds); and
	  iii) liquid volume (cups, pints, quarts, and gallons).

2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Measurement and Geometry**	2023 *Standards of Learning* Knowledge and Skills (KS) **Measurement and Geometry (MG)**

4.9 The student will solve practical problems related to elapsed time in hours and minutes within a 12-hour period.
 	
● Solve practical problems related to elapsed time in hours and minutes, within a 12-hour period (within a.m., within p.m., and across a.m. and p.m.):
  o when given the beginning time and the ending time, determine the time that has elapsed;
  o when given the beginning time and amount of elapsed time in hours and minutes, determine the ending time; and
  o when given the ending time and the elapsed time in hours and minutes, determine the beginning time.
	4.MG.2 The student will solve single-step and multistep contextual problems involving elapsed time (limited to hours and minutes within a 12-hour period).
	
	a) Solve single-step and multistep contextual problems involving elapsed time in hours and minutes, within a 12-hour period (within a.m., within p.m., and across a.m. and p.m.) when given:
	  i) the starting time and the ending time, determine the amount of time that has elapsed in hours and minutes;
	  ii) the starting time and amount of elapsed time in hours and minutes, determine the ending time; or
	  iii) the ending time and the amount of elapsed time in hours and minutes, determine the starting time.

4.10 The student will **a) identify and describe points, lines, line segments, rays, and angles, including endpoints and vertices; and** **b) identify and describe intersecting, parallel, and perpendicular lines.** ● Identify and describe points, lines, line segments, rays, and angles, including endpoints and vertices. (a) ● Use symbolic notation to name points, lines, line segments, rays, and angles. (a) ● Identify parallel, perpendicular, and intersecting line segments in plane and solid figures. (b) ● Identify practical situations that illustrate parallel, intersecting, and perpendicular lines. (b)	**4.MG.4 The student will identify, describe, and draw points, rays, line segments, angles, and lines, including intersecting, parallel, and perpendicular lines.** a) Identify and describe points, lines, line segments, rays, and angles, including endpoints and vertices. b) Describe endpoints and vertices in relation to lines, line segments, rays, and angles. c) Draw representations of points, line segments, rays, angles, and lines, using a ruler or straightedge. d) Identify parallel, perpendicular, and intersecting lines and line segments in plane and solid figures, including those in context. e) Use symbolic notation to name points, lines, line segments, rays, angles, and to describe parallel and perpendicular lines.

2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Measurement and Geometry**	2023 *Standards of Learning* Knowledge and Skills (KS) **Measurement and Geometry (MG)**
● Use symbolic notation to describe parallel lines and perpendicular lines. (b)	
4.11 The student will identify, describe, compare, and contrast plane and solid figures according to their characteristics (number of angles, vertices, edges, and the number and shape of faces) using concrete models and pictorial representations. ● Identify concrete models and pictorial representations of solid figures (cube, rectangular prism, square pyramid, sphere, cone, and cylinder). ● Identify and describe solid figures (cube, rectangular prism, square pyramid, and sphere) according to their characteristics (number of angles, vertices, edges, and by the number and shape of faces). ● Compare and contrast plane and solid figures (circle/sphere, square/cube, triangle/square pyramid, and rectangle/ rectangular prism) according to their characteristics (number of sides, angles, vertices, edges, and the number and shape of faces).	**4.MG.6 The student will identify, describe, compare, and contrast plane and solid figures according to their characteristics (number of angles, vertices, edges, and the number and shape of faces), with and without models.** a) Identify concrete models and pictorial representations of solid figures (cube, rectangular prism, square pyramid, sphere, cone, and cylinder). b) Identify and describe solid figures (cube, rectangular prism, square pyramid, and sphere) according to their characteristics (number of angles, vertices, edges, and the number and shape of faces). c) Compare and contrast plane and solid figures (limited to circles, squares, triangles, rectangles, spheres, cubes, square pyramids, and rectangular prisms) according to their characteristics (number of sides, angles, vertices, edges, and the number and shape of faces).
4.12 The student will classify quadrilaterals as parallelograms, rectangles, squares, rhombi, and/or trapezoids. ● Develop definitions for parallelograms, rectangles, squares, rhombi, and trapezoids. ● Identify properties of quadrilaterals including parallel, perpendicular, and congruent sides.	**4.MG.5 The student will classify and describe quadrilaterals (parallelograms, rectangles, squares, rhombi, and/or trapezoids) using specific properties and attributes.** a) Develop definitions for parallelograms, rectangles, squares, rhombi, and trapezoids through the exploration of properties and attributes.

2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Measurement and Geometry**	2023 *Standards of Learning* Knowledge and Skills (KS) **Measurement and Geometry (MG)**
● Classify quadrilaterals as parallelograms, rectangles, squares, rhombi, and/or trapezoids. ● Compare and contrast the properties of quadrilaterals. ● Identify parallel sides, congruent sides, and right angles using geometric markings to denote properties of quadrilaterals.	b) Identify and describe points, line segments, angles, and vertices in quadrilaterals. c) Identify and describe parallel, intersecting, perpendicular, and congruent sides in quadrilaterals. d) Compare, contrast, and classify quadrilaterals (parallelograms, rectangles, squares, rhombi, and/or trapezoids) based on the following properties and attributes: i) parallel sides; ii) perpendicular sides; iii) congruence of sides; and iv) number of right angles. e) Denote properties of quadrilaterals and identify parallel sides, congruent sides, and right angles by using geometric markings. f) Use symbolic notation to name line segments and angles in quadrilaterals.

2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Probability and Statistics**	2023 *Standards of Learning* Knowledge and Skills (KS) **Probability and Statistics (PS)**
4.13 The student will **a) determine the likelihood of an outcome of a simple event;** **b) represent probability as a number between 0 and 1, inclusive; and** **c) create a model or practical problem to represent a given probability.**	**4.PS.2 The student will model and determine the probability of an outcome of a simple event.** a) Describe probability as the degree of likelihood of an outcome occurring using terms such as *impossible*, *unlikely*, *equally likely*, *likely*, and *certain*.

2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Probability and Statistics**	2023 *Standards of Learning* Knowledge and Skills (KS) **Probability and Statistics (PS)**
• Model and determine all possible outcomes of a given simple event where there are no more than 24 possible outcomes, using a variety of manipulatives (e.g., coins, number cubes, and spinners). (a) • Determine the outcome of an event that is least likely to occur or most likely to occur where there are no more than 24 possible outcomes. (a) • Write the probability of a given simple event as a fraction, where there are no more than 24 possible outcomes. (b) • Determine the likelihood of an event occurring and relate it to its whole number or fractional representation (e.g., impossible or zero; equally likely; certain or one). (a, b) • Create a model or practical problem to represent a given probability. (c)	b) Model and determine all possible outcomes of a given simple event where there are no more than 24 possible outcomes, using a variety of manipulatives (e.g., coins, two-sided counters, number cubes, spinners). c) Write the probability of a given simple event as a fraction between 0 and 1, where there are no more than 24 possible outcomes. d) Determine the likelihood of an event occurring and relate it to its whole number or fractional representation (e.g., impossible or zero; equally likely; certain or one). e) Create a model or contextual problem to represent a given probability.
4.14 The student will a) **collect, organize, and represent data in bar graphs and line graphs;** b) **interpret data represented in bar graphs and line graphs; and** c) **compare two different representations of the same data (e.g., a set of data displayed on a chart and a bar graph, a chart and a line graph, or a pictograph and a bar graph).** • Collect data, using, for example, observations, measurement, surveys, scientific experiments, polls, or questionnaires. (a) • Organize data into a chart or table. (a)	**4.PS.1 The student will apply the data cycle (formulate questions; collect or acquire data; organize and represent data; and analyze data and communicate results) with a focus on line graphs.** a) Formulate questions that require the collection or acquisition of data. b) Determine the data needed to answer a formulated question and collect or acquire existing data (limited to 10 or fewer data points) using various methods (e.g., observations, measurements, experiments).

2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Probability and Statistics**	**2023 *Standards of Learning*** Knowledge and Skills (KS) **Probability and Statistics (PS)**

- Represent data in bar graphs, labeling one axis with equal whole number increments of one or more (numerical data) (e.g., 2, 5, 10, or 100) and the other axis with categories related to the title of the graph (categorical data) (e.g., swimming, fishing, boating, and water skiing as the categories of "Favorite Summer Sports"). (a) - Represent data in line graphs, labeling the vertical axis with equal whole number increments of one or more and the horizontal axis with continuous data commonly related to time (e.g., hours, days, months, years. Line graphs will have no more than 10 identified points along a continuum for continuous data. (a) - Title the graph or identify an appropriate title. Label the axes or identify the appropriate labels. (a) - Interpret data by making observations from bar graphs and line graphs by describing the characteristics of the data and the data as a whole (e.g., the time period when the temperature increased the most, the category with the greatest/least, categories with the same number of responses, similarities and differences, the total number). One set of data will be represented on a graph. (b) - Interpret data by making inferences from bar graphs and line graphs. (b) - Interpret the data to answer the question posed, and compare the answer to the prediction (e.g., "The summer sport preferred by most is swimming, which is what I predicted before collecting the data."). (b) - Write at least one sentence to describe the analysis and interpretation of the data, identifying parts of the data that have special characteristics, including categories with the greatest, the least, or the same. (b)	c) Organize and represent a data set using line graphs with a title and labeled axes with whole number increments, with and without the use of technology tools.
	
	d) Analyze data represented in line graphs and communicate results orally and in writing:
	  i) describe the characteristics of the data represented in a line graph and the data as a whole (e.g., the time period when the temperature increased the most);
	  ii) identify parts of the data that have special characteristics and explain the meaning of the greatest, the least, or the same (e.g., the highest temperature shows the warmest day);
	  iii) make inferences about data represented in line graphs;
	  iv) draw conclusions about the data and make predictions based on the data to answer questions; and
	  v) solve single-step and multistep addition and subtraction problems using data from line graphs.

2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Probability and Statistics**	2023 *Standards of Learning* Knowledge and Skills (KS) **Probability and Statistics (PS)**
● Compare two different representations of the same data (e.g., a set of data displayed on a chart and a bar graph; a chart and a line graph; a pictograph and a bar graph). (c)	

2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Patterns, Functions, and Algebra**	2023 *Standards of Learning* Knowledge and Skills (KS) **Patterns, Functions, and Algebra (PFA)**
4.15 The student will identify, describe, create, and extend patterns found in objects, pictures, numbers, and tables. ● Identify and describe patterns, using words, objects, pictures, numbers, and tables. ● Create patterns using objects, pictures, numbers, and tables. ● Extend patterns, using objects, pictures, numbers, and tables. ● Solve practical problems that involve identifying, describing, and extending single-operation input and output rules, limited to addition, subtraction, and multiplication of whole numbers and addition and subtraction of fractions with like denominators of 12 or less. ● Identify the rule in a single-operation numerical pattern found in a list or table, limited to addition, subtraction, and multiplication of whole numbers.	**4.PFA.1 The student will identify, describe, extend, and create increasing and decreasing patterns (limited to addition, subtraction, and multiplication of whole numbers), including those in context, using various representations.** a) Identify, describe, extend, and create increasing and decreasing patterns using various representations (e.g., objects, pictures, numbers, number lines, input/output tables, and function machines). b) Analyze an increasing or decreasing single-operation numerical pattern found in lists, input/output tables, or function machines and generalize the change to identify the rule, extend the pattern, or identify missing terms. c) Given a rule, create increasing and decreasing patterns using numbers and input/output tables (including function machines). d) Solve contextual problems that involve identifying, describing, and extending increasing and decreasing patterns using single-operation input and output rules.
4.16 The student will recognize and demonstrate the meaning of equality in an equation. ● Write an equation to represent the relationship between equivalent mathematical expressions (e.g., 4 x 3 = 2 x 6; 10 + 8 = 36 ÷ 2; 12 x 4 = 60 - 12). ● Identify and use the appropriate symbol to distinguish between expressions that are equal and expressions that are not equal, using addition, subtraction, multiplication, and division (e.g., $4 \times 12 = 8 \times 6$ and $64 \div 8 \neq 8 \times 8$).	**[Included in 4.CE.2]**

2023 Grade 4 *Mathematics SOL* – **Summary of Changes**

Grade 4 (2016 SOL to 2023 SOL Numbering)	Parameter Changes/Clarification (2023 SOL)
4.1a → 4.NS.1 4.1b → 4.NS.2 4.1c → [Included in 4.CE.1] 4.2a-c → 4.NS.3 4.3a,c → 4.NS.4 4.3b → [Included in 4.CE.3] 4.3d → 4.NS.5 4.4a-d → 4.CE.1 and 4.CE.2 4.5a → [Common factors included in 4.CE.2; Common multiples moved to Grade 5] 4.5b-c → 4.CE.3 4.6a-b → 4.CE.4 4.7 → 4.MG.3 4.8a-d → 4.MG.1 4.9 → 4.MG.2 4.10a-b → 4.MG.4 4.11 → 4.MG.6 4.12 → 4.MG.5 4.13a-c → 4.PS.2 4.14a-c → 4.PS.1 4.15 → 4.PFA.1 4.16 → [Included in 4.CE.2]	4.NS.1c Apply patterns within the base 10 system to determine and communicate, orally and in written form, the place and value of each digit in a nine-digit whole number 4.NS.3a – Represent equivalent fractions with and without models (previously required a model) 4.NS.5a,b - Hundredths included when identifying and representing fractions as decimals 4.CE.1 and 4.CE.2 - "Create and solve…" reworded to "Estimate, represent, solve, and justify…" 4.CE.1b - Rounding numbers included as a strategy for estimation 4.CE.1b,c,d - Addition and subtraction problems (without access to a calculator) involve addends and minuends that do not exceed 10,000; contextual addition and subtraction problems (with access to a calculator) involve addends and minuends that do not exceed 1,000,000 4.CE.2a - "Demonstrate fluency with…" expanded to include "Recall with automaticity" the multiplication facts through 12×12 and the corresponding division facts 4.CE.3a,b - Add and subtract two fractions (proper and improper) and/or mixed numbers, having like denominators limited to 2, 3, 4, 5, 6, 8, 10, and 12, and simplify the resulting fraction; problems may include regrouping (previously add or subtract two fractions with like or unlike denominators of 12 or less; subtraction problems did not include regrouping) 4.CE.4a – Rounding decimals to the nearest whole number included as an estimation strategy 4.MG.1c - Compare estimates with actual measurements expanded from only length to also include weight/mass, or liquid volume

Grade 4 (2016 SOL to 2023 SOL Numbering)	Parameter Changes/Clarification (2023 SOL)
	4.MG.2 – Clarified to include single-step and multistep contextual problems for elapsed time 4.MG.5 – Classify and describe quadrilaterals including points, line segments, angles, vertices, parallel, intersecting, perpendicular, and congruent sides 4.MG.5f - Use symbolic notation to name line segments and angles in quadrilaterals 4.MG.6 – Identify, describe, compare, and contrast plane and solid figures with and without models (previously using concrete materials and pictorial representations) 4.PFA.1c - Given a rule, create patterns using numbers and input/output tables (including function machines)

Deletions from Grade 4 (2016 SOL)	Additions to Grade 4 (2023 SOL)
4.1c [EKS] - Identify the range of numbers that round to a given thousand, ten thousand, and hundred thousand 4.4d - Create single-step and multistep practical problems involving addition, subtraction, multiplication, and division with whole numbers 4.5a - Determine common multiples and the least common multiple of no more than three numbers [Moved to Grade 5] 4.5b - Add and subtract fractions with unlike denominators [Included in Grade 5] 4.7 [EKS] - Determine the perimeter of a polygon with no more than eight sides 4.14a-b - Collect, organize, represent, and interpret data in bar graphs 4.14c - Compare two representations of the same data 4.15 - Patterns that include the addition and subtraction of fractions with like denominators of 12 or less [Included in Grade 5]	4.NS.3f - Compose and decompose fractions (proper and improper) and mixed numbers with denominators of 12 or less, in multiple ways, with and without models 4.CE.1a and 4.CE.2a - Determine and justify whether an estimate or an exact answer is appropriate when solving contextual problems 4.CE.2e - Determine all factor pairs for a whole number 1 to 100, using concrete, pictorial, and numerical representations 4.CE.2g - Estimate and determine the product of two whole numbers (a three-digit factor and a one-digit factor) 4.CE.3c,d - Solve single-step contextual problems involving multiplication of a whole number, limited to 12 or less, and a unit fraction, with models; apply the inverse property of multiplication in models [Moved from Grade 5] 4.MG.1a,b – Determine appropriate unit to measure liquid volume; estimate and measure liquid volume 4.MG.3a,d - Use models to explore the relationship between area and perimeter of rectangles; develop a formula for the area and perimeter of rectangles (including squares) 4.MG.3e - Identify and represent rectangles with the same perimeter and different areas or with the same area and different perimeters 4.MG.4c - Draw representations of points, line segments, rays, angles, and lines [Moved from Grade 3] 4.PS.1 - Additional data analysis knowledge and skills representing the data cycle have been included 4.PS.2a - Describe probability as the degree of likelihood of an outcome occurring using terms such as *impossible, unlikely, equally likely, likely,* and *certain* [Moved from Grade 3]

KEY: NS = Number and Number Sense; CE = Computation and Estimation; MG = Measurement and Geometry; PS = Probability and Statistics; PFA = Patterns, Functions, and Algebra; EKS = Essential Knowledge and Skills (2016); KS = Knowledge and Skills (2023); US = Understanding the Standard

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Grade 5 Mathematics *Standards of Learning* - 2023 Overview of Revisions

This overview includes a summary of the content embedded in five content strands.

Number and Number Sense: *Exploring relationships between fractions and decimals and their representations provides meaning and structure and allows us to quantify, measure and make decisions in life.*
- Identify and represent equivalency between fractions and decimals, and compare and order sets of fractions and/or decimals
- Demonstrate an understanding of the number characteristics prime and composite; determine prime factorization for whole numbers up to 100

Computation and Estimation: *The operations of addition, subtraction, multiplication, and division, and estimation, allow us to model, represent, and solve different types of problems with whole numbers and rational numbers (not including integers).*
- Estimate, represent, solve, and justify solutions to single-step and multistep contextual problems using addition, subtraction, multiplication, and division with whole numbers
- Add and subtract fractions with like and unlike denominators, and solve single-step and multistep contextual problems
- Solve single-step contextual problems involving multiplication of a whole number and a proper fraction
- Add, subtract, multiply, and divide with decimal numbers and solve single-step and multistep contextual problems
- Simplify whole number numerical expressions using the order of operations

Measurement and Geometry: *Analyzing and describing geometric objects, the relationships and structures among them, or the space that they occupy can be used to classify, quantify, measure, or count one or more attributes.*
- Solve problems, including those in context, that involve length, mass, and liquid volume using metric units
- Solve problems involving area, perimeter, and volume
- Classify and measure angles and triangles

Probability and Statistics: *The world can be investigated through posing questions and collecting, representing, analyzing, and interpreting data to describe and predict events and real-world phenomena.*
- Apply the data cycle (formulate questions; collect or acquire data; organize and represent data; and analyze data and communicate results) with a focus on line plots (dot plots) and stem-and-leaf plots
- Solve contextual problems using measures of center and the range
- Determine the probability of a simple event by constructing a model of a sample space and using the Fundamental (Basic) Counting Principle

Patterns, Functions, and Algebra: *Relationships can be described and generalizations can be made using patterns and relations.*
- Identify, describe, extend, and create increasing and decreasing patterns with whole numbers, fractions, and decimals using various representations

- Investigate and use variables in contextual problems

Comparison of Grade 5 *Mathematics Standards of Learning* – 2016 to 2023

2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Number and Number Sense** * On the state assessment, items measuring this objective are assessed without the use of a calculator.	2023 *Standards of Learning* Knowledge and Skills (KS) **Number and Number Sense (NS)** *On the state assessment, items measuring this objective are assessed without the use of a calculator.
5.1 The student, given a decimal through thousandths, will round to the nearest whole number, tenth, or hundredth. ● Given a decimal through thousandths, round to the nearest whole number, tenth, or hundredth.	**[Included in 5.CE.3]**
5.2 The student will **a) represent and identify equivalencies among fractions and decimals, with and without models;* and** **b) compare and order fractions, mixed numbers, and/or decimals, in a given set, from least to greatest and greatest to least.*** ● Represent fractions with denominators that are thirds, eighths, and factors of 100 in their equivalent decimal form with concrete or pictorial models. (a) ● Represent decimals in their equivalent fraction form (thirds, eighths, and factors of 100) with concrete or pictorial models. (a) ● Identify equivalent relationships between decimals and fractions with denominators that are thirds, eighths, and factors of 100 in their equivalent decimal form without models. (a) ● Compare and order from least to greatest and greatest to least a given set of no more than four decimals, fractions (proper or improper), and/or mixed numbers with denominators of 12 or less. (b)	**5.NS.1 The student will use reasoning and justification to identify and represent equivalency between fractions (with denominators that are thirds, eighths, and factors of 100) and decimals; and compare and order sets of fractions (proper, improper, and/or mixed numbers having denominators of 12 or less) and decimals (through thousandths).** a) Use concrete and pictorial models to represent fractions with denominators that are thirds, eighths, and factors of 100 in their equivalent decimal form.* b) Use concrete and pictorial models to represent decimals in their equivalent fraction form (with denominators that are thirds, eighths, and factors of 100).* c) Identify equivalent relationships between decimals and fractions with denominators that are thirds, eighths, and factors of 100 in their equivalent decimal form, with and without models.* d) Compare (using symbols $<$, $>$, $=$) and order (least to greatest and greatest to least) a set of no more than four decimals, fractions, and/or mixed numbers using multiple strategies (e.g.,

2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Number and Number Sense** * On the state assessment, items measuring this objective are assessed without the use of a calculator.	2023 *Standards of Learning* Knowledge and Skills (KS) **Number and Number Sense (NS)** *On the state assessment, items measuring this objective are assessed without the use of a calculator.
● Use the symbols >, <, =, and ≠ to compare decimals through thousandths, fractions (proper or improper fractions), and/or mixed numbers, having denominators of 12 or less. (b)	benchmarks, place value, number line, etc.). Justify solutions orally, in writing, or with a model.*
5.3 The student will **a) identify and describe the characteristics of prime and composite numbers; and** **b) identify and describe the characteristics of even and odd numbers.** ● Identify prime numbers less than or equal to 100. (a) ● Identify composite numbers less than or equal to 100. (a) ● Demonstrate with concrete or pictorial representations and explain orally or in writing why a number is prime or composite. (a) ● Identify which numbers are even or odd. (b) ● Demonstrate with concrete or pictorial representations and explain orally or in writing why a number is even or odd. (b) ● Demonstrate with concrete or pictorial representations and explain orally or in writing why the sum or difference of two numbers is even or odd. (b)	**5.NS.2 The student will demonstrate an understanding of prime and composite numbers, and determine the prime factorization of a whole number up to 100.** a) Given a whole number up to 100, create a concrete or pictorial representation to demonstrate whether the number is prime or composite, and justify reasoning. b) Classify, compare, and contrast whole numbers up to 100 using the characteristics prime and composite. c) Determine the prime factorization for a whole number up to 100.

2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Computation and Estimation** *On the state assessment, items measuring this objective are assessed without the use of a calculator.	2023 *Standards of Learning* Knowledge and Skills (KS) **Computation and Estimation (CE)** *On the state assessment, items measuring this objective are assessed without the use of a calculator.

5.4 The student will create and solve single-step and multistep practical problems involving addition, subtraction, multiplication, and division of whole numbers.
 	
● Create single-step and multistep practical problems involving addition, subtraction, multiplication, and division of whole numbers, with and without remainders.
● Estimate the sum, difference, product, and quotient of whole numbers.
● Apply strategies, including place value and application of the properties of addition and multiplication, to solve single-step and multistep practical problems involving addition, subtraction, multiplication, and division of whole numbers, with and without remainders, in which:
   o sums, differences, and products do not exceed five digits;
   o factors do not exceed two digits by three digits;
   o divisors do not exceed two digits; or
   o dividends do not exceed four digits.
● Use the context of a practical problem to interpret the quotient and remainder.
	5.CE.1 The student will estimate, represent, solve, and justify solutions to single-step and multistep contextual problems using addition, subtraction, multiplication, and division with whole numbers.
	
	a) Estimate the sum, difference, product, and quotient of whole numbers in contextual problems.
	b) Represent, solve, and justify solutions to single-step and multistep contextual problems by applying strategies (e.g., estimation, properties of addition and multiplication) and algorithms, including the standard algorithm, involving addition, subtraction, multiplication, and division of whole numbers, with and without remainders, in which:
	   i) sums, differences, and products do not exceed five digits;
	   ii) factors do not exceed two digits by three digits;
	   iii) divisors do not exceed two digits; or
	   iv) dividends do not exceed four digits.
	c) Interpret the quotient and remainder when solving a contextual problem.

5.5 The student will **a) estimate and determine the product and quotient of two numbers involving decimals* and** **b) create and solve single-step and multistep practical problems involving addition, subtraction, and multiplication of decimals,**	**5.CE.3 The student will estimate, represent, solve, and justify solutions to single-step and multistep problems, including those in context, using addition, subtraction, multiplication, and division with decimal numbers.**

2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Computation and Estimation** *On the state assessment, items measuring this objective are assessed without the use of a calculator.*	2023 *Standards of Learning* Knowledge and Skills (KS) **Computation and Estimation (CE)** *On the state assessment, items measuring this objective are assessed without the use of a calculator.*

and create and solve single-step practical problems involving division of decimals.
 	
● Estimate and determine the product of two numbers in which:
  ○ the factors do not exceed two digits by two digits (e.g., 2.3 × 4.5, 0.08 × 0.9, 0.85 × 2.3, 1.8 × 5); and
  ○ the products do not exceed the thousandths place. (Leading zeroes will not be considered when counting digits.) (a)
● Estimate and determine the quotient of two numbers in which
  ○ quotients do not exceed four digits with or without a decimal point;
  ○ quotients may include whole numbers, tenths, hundredths, or thousandths;
  ○ divisors are limited to a single digit whole number or a decimal expressed as tenths; and
  ○ no more than one additional zero will need to be annexed. (a)
● Use multiple representations to model multiplication and division of decimals and whole numbers. (a)
● Create and solve single-step and multistep practical problems involving addition, subtraction, and multiplication of decimals. (b)
● Create and solve single-step practical problems involving division of decimals. (b)
	a) Apply estimation strategies (e.g., rounding to the nearest whole number, tenth or hundredth; compatible numbers, place value) to determine a reasonable solution for single-step and multistep contextual problems involving addition, subtraction, and multiplication of decimals, and single-step contextual problems involving division of decimals.
	b) Estimate and determine the product of two numbers using strategies and algorithms, including the standard algorithm, when given:
	  i) a two-digit factor and a one-digit factor (e.g., 2.3 × 4; 0.08 × 0.9; .16 × 5);*
	  ii) a three-digit factor and a one-digit factor (e.g., 0.156 × 4, 3.28 × 7, 8.09 × 0.2);* and
	  iii) a two-digit factor and a two-digit factor (e.g., 0.85 × 3.7, 14 × 1.6, 9.2 × 3.5).*
	
	(Products will not exceed the thousandths place, and leading zeroes will not be considered when counting digits.)
	
	c) Estimate and determine the quotient of two numbers using strategies and algorithms, including the standard algorithm, in which*:
	  i) quotients do not exceed four digits with or without a decimal point;
	  ii) quotients may include whole numbers, tenths, hundredths, or thousandths;
	  iii) divisors are limited to a single digit whole number a decimal expressed as tenths; and

2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Computation and Estimation** *On the state assessment, items measuring this objective are assessed without the use of a calculator.	2023 *Standards of Learning* Knowledge and Skills (KS) **Computation and Estimation (CE)** *On the state assessment, items measuring this objective are assessed without the use of a calculator.
	iv) no more than one additional zero will need to be annexed. d) Solve single-step and multistep contextual problems involving addition, subtraction, and multiplication of decimals by applying strategies (e.g., estimation, modeling) and algorithms, including the standard algorithm. e) Solve single-step contextual problems involving division with decimals by applying strategies (e.g., estimation, modeling) and algorithms, including the standard algorithm.
5.6 The student will **a) solve single-step and multistep practical problems involving addition and subtraction with fractions and mixed numbers; and** **b) solve single-step practical problems involving multiplication of a whole number, limited to 12 or less, and a proper fraction, with models.*** • Solve single-step and multistep practical problems involving addition and subtraction with fractions (proper or improper) having like and unlike denominators and/or mixed numbers. Denominators in the problems should be limited to 12 or less (e.g., $\frac{5}{8} + \frac{1}{4}$, $\frac{5}{6} - \frac{2}{3}$, $3\frac{3}{4} + 2\frac{5}{12}$) and answers should be expressed in simplest form. (a) • Solve single-step practical problems involving multiplication of a whole number, limited to 12 or less, and a proper fraction (e.g., 6 $\times \frac{1}{3}$, $\frac{1}{4} \times 8$, $9 \times \frac{2}{3}$), with models. The denominator will be a factor	**5.CE.2 The student will estimate, represent, solve, and justify solutions to single-step and multistep problems, including those in context, using addition and subtraction of fractions with like and unlike denominators (with and without models), and solve single-step contextual problems involving multiplication of a whole number and a proper fraction, with models.** a) Determine the least common multiple of two numbers to find the least common denominator for two fractions. b) Estimate and determine the sum or difference of two fractions (proper or improper) and/or mixed numbers, having like and unlike denominators limited to 2, 3, 4, 5, 6, 8, 10, and 12 (e.g., $\frac{5}{8} + \frac{1}{4}$, $\frac{4}{5} - \frac{2}{3}$, $3\frac{3}{4} + 2\frac{5}{12}$), and simplify the resulting fraction.* c) Estimate and solve single-step and multistep contextual problems involving addition and subtraction with fractions (proper or improper) and/or mixed numbers having like and unlike denominators, with and without models. Denominators

2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Computation and Estimation** *On the state assessment, items measuring this objective are assessed without the use of a calculator.	2023 *Standards of Learning* Knowledge and Skills (KS) **Computation and Estimation (CE)** *On the state assessment, items measuring this objective are assessed without the use of a calculator.
of the whole number and answers should be expressed in simplest form. (b) ● Apply the inverse property of multiplication in models. (For example, use a visual fraction model to represent $\frac{4}{4}$ or as the product of $4 \times \frac{1}{4}$). (b)	should be limited to 2, 3, 4, 5, 6, 8, 10, 12. Answers should be expressed in simplest form. d) Solve single-step practical problems involving multiplication of a whole number, limited to 12 or less, and a proper fraction (e.g., $9 \times \frac{2}{3}$, $8 \times \frac{3}{4}$), with models. The denominator will be a factor of the whole number and answers should be expressed in simplest form.*

5.7 The student will simplify whole number numerical expressions using the order of operations.*
 	
● Use the order of operations to simplify whole number numerical expressions, limited to addition, subtraction, multiplication, and division. Expressions may contain parentheses.
● Given a whole number numerical expression involving more than one operation, describe which operation is completed first, which is second, etc.
	5.CE.4 The student will simplify numerical expressions with whole numbers using the order of operations.
	
	a) Use order of operations to simplify numerical expressions with whole numbers, limited to addition, subtraction, multiplication, and division in which:*
	   i) expressions may contain no more than one set of parentheses;
	   ii) simplification will be limited to five whole numbers and four operations in any combination of addition, subtraction, multiplication, or division;
	   iii) whole numbers will be limited to two digits or less; and
	   iv) expressions should not include braces, brackets, or fraction bars.
	b) Given a whole number numerical expression involving more than one operation, describe which operation is completed first, which is second, and which is third.*

2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Measurement and Geometry**	2023 *Standards of Learning* Knowledge and Skills (KS) **Measurement and Geometry (MG)**
5.8 The student will **a) solve practical problems that involve perimeter, area, and volume in standard units of measure; and** **b) differentiate among perimeter, area, and volume and identify whether the application of the concept of perimeter, area, or volume is appropriate for a given situation.** ● Solve practical problems that involve perimeter, area, and volume in standard units of measure. (a) ● Determine the perimeter of a polygon, with or without diagrams, when 　o the lengths of all sides of a polygon that is not a rectangle or a square are given; 　o the length and width of a rectangle are given; or 　o the length of a side of a square is given. (a) ● Estimate and determine the area of a square and rectangle using whole number measurements given in metric or U.S. Customary units, and record the solution with the appropriate unit of measure (e.g., 24 square inches). (a) ● Develop a procedure for determining the area of a right triangle using only whole number measurements given in metric or U.S. Customary units, and record the solution with the appropriate unit of measure (e.g., 12 square inches). (a) ● Estimate and determine the area of a right triangle, with diagrams, when the base and the height are given. (a) ● Develop a procedure for determining volume using manipulatives (e.g., cubes). (a)	**5.MG.2 The student will use multiple representations to solve problems, including those in context, involving perimeter, area, and volume.** a) Investigate and develop a formula for determining the area of a right triangle. b) Estimate and determine the area of a right triangle, with diagrams, when the base and the height are given in whole number units, in metric or U.S. Customary units, and record the solution with the appropriate unit of measure (e.g., 16 square inches). c) Describe volume as a measure of capacity and give examples of volume as a measurement in contextual situations. d) Investigate and develop a formula for determining the volume of rectangular prisms using concrete objects. e) Solve problems, including those in context, to estimate and determine the volume of a rectangular prism using concrete objects, diagrams, and formulas when the length, width, and height are given in whole number units. Record the solution with the appropriate unit of measure (e.g., 12 cubic inches). f) Identify whether the application of the concept of perimeter, area, or volume is appropriate for a given situation. g) Solve contextual problems that involve perimeter, area, and volume in standard units of measure.

2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Measurement and Geometry**	2023 *Standards of Learning* Knowledge and Skills (KS) **Measurement and Geometry (MG)**
• Estimate and determine the volume of a rectangular prism with diagrams, when the length, width, and height are given, using whole number measurements. Record the solution with the appropriate unit of measure (e.g., 12 cubic inches). (a) • Describe practical situations where perimeter, area, and volume are appropriate measures to use, and justify orally or in writing. (b) • Identify whether the application of the concept of perimeter, area, or volume is appropriate for a given situation. (b)	

5.9 The student will
a) given the equivalent measure of one unit, identify equivalent measurements within the metric system; and
b) solve practical problems involving length, mass, and liquid volume using metric units.
 	
• Given the equivalent measure of one unit, identify equivalent measurements within the metric system for the following:
  o length (millimeters, centimeters, meters, and kilometers);
  o mass (grams and kilograms); and
  o liquid volume (milliliters and liters). (a)
• Estimate and measure to solve practical problems that involve metric units:
  o length (millimeters, centimeters, meters, and kilometers);
  o mass (grams and kilograms); and
  o liquid volume (milliliters, and liters). (b)
	5.MG.1 The student will reason mathematically to solve problems, including those in context, that involve length, mass, and liquid volume using metric units.
	
	a) Determine the most appropriate unit of measure to use in a contextual problem that involves metric units:
	  i) length (millimeters, centimeters, meters, and kilometers);
	  ii) mass (grams and kilograms); and
	  iii) liquid volume (milliliters and liters).
	b) Estimate and measure to solve contextual problems that involve metric units:
	  i) length (millimeters, centimeters, and meters);
	  ii) mass (grams and kilograms); and
	  iii) liquid volume (milliliters and liters).
	c) Given the equivalent metric measure of one unit, in a contextual problem, determine the equivalent measurement within the metric system for the following:
	  i) length (millimeters, centimeters, meters, and kilometers);

2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Measurement and Geometry**	2023 *Standards of Learning* Knowledge and Skills (KS) **Measurement and Geometry (MG)**
	ii) mass (grams and kilograms); and iii) liquid volume (milliliters and liters).

5.10 The student will identify and describe the diameter, radius, chord, and circumference of a circle.
 	
● Identify and describe the diameter, radius, chord, and circumference of a circle.
● Investigate and describe the relationship between
   o diameter and radius;
   o diameter and chord;
   o radius and circumference; and
   o diameter and circumference.
	[Included in Grade 6]

5.11 The student will solve practical problems related to elapsed time in hours and minutes within a 24-hour period.
 	
● Solve practical problems related to elapsed time in hours and minutes within a 24-hour period:
   o when given the beginning time and the ending time, determine the time that has elapsed;
   o when given the beginning time and amount of elapsed time in hours and minutes, determine the ending time; or
   o when given the ending time and the elapsed time in hours and minutes, determine the beginning time.
	[Deleted]

5.12 The student will classify and measure right, acute, obtuse, and straight angles.	**5.MG.3 The student will classify and measure angles and triangles, and solve problems, including those in context.**

2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Measurement and Geometry**	2023 *Standards of Learning* Knowledge and Skills (KS) **Measurement and Geometry (MG)**
• Classify angles as right, acute, obtuse, or straight. • Identify the appropriate tools (e.g., protractor and straightedge or angle ruler as well as available software) used to measure and draw angles. • Measure right, acute, obtuse, and straight angles, using appropriate tools, and identify their measures in degrees. • Solve addition and subtraction problems to determine unknown angle measures on a diagram in practical problems.	a) Classify angles as right, acute, obtuse, or straight and justify reasoning. b) Classify triangles as right, acute, or obtuse and equilateral, scalene, or isosceles and justify reasoning. c) Identify congruent sides and right angles using geometric markings to denote properties of triangles. d) Compare and contrast the properties of triangles. e) Identify the appropriate tools (e.g., protractor, straightedge, angle ruler, available technology) to measure and draw angles. f) Measure right, acute, obtuse, and straight angles, using appropriate tools, and identify measures in degrees. g) Use models to prove that the sum of the interior angles of a triangle is 180 degrees and use the relationship to determine an unknown angle measure in a triangle. h) Solve addition and subtraction contextual problems to determine unknown angle measures on a diagram.
5.13 The student will **a) classify triangles as right, acute, or obtuse and equilateral, scalene, or isosceles; and** **b) investigate the sum of the interior angles in a triangle and determine an unknown angle measure.** • Classify triangles as right, acute, or obtuse. (a) • Classify triangles as equilateral, scalene, or isosceles. (a) • Compare and contrast the properties of triangles. (a)	**[Included in 5.MG.3]**

2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Measurement and Geometry**	2023 *Standards of Learning* Knowledge and Skills (KS) **Measurement and Geometry (MG)**
● Identify congruent sides and right angles using geometric markings to denote properties of triangles. (a) ● Use models to prove that the sum of the interior angles of a triangle is 180 degrees, and use that relationship to determine an unknown angle measure in a triangle. (b)	
5.14 The student will **a) recognize and apply transformations, such as translation, reflection, and rotation; and** ● Apply transformations to polygons in order to determine congruence. (a) ● Recognize that translations, reflections, and rotations preserve congruency. (a) ● Identify the image of a polygon resulting from a single transformation (translation, reflection, or rotation). (a)	**[Included in Grades 7 and 8]**
5.14 The student will **b) investigate and describe the results of combining and subdividing polygons.** ● Investigate and describe the results of combining and subdividing polygons. (b) ● Compare and contrast the characteristics of a given polygon that has been subdivided with the characteristics of the resulting parts. (b)	**[Deleted]**

2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Probability and Statistics**	2023 *Standards of Learning* Knowledge and Skills (KS) **Probability and Statistics (PS)**
5.15 The student will determine the probability of an outcome by constructing a sample space or using the Fundamental (Basic) Counting Principle. ● Construct a sample space, using a tree diagram to identify all possible outcomes. ● Construct a sample space, using a list or chart to represent all possible outcomes. ● Determine the probability of an outcome by constructing a sample space. The sample space will have a total of 24 or fewer equally likely possible outcomes. ● Determine the number of possible outcomes by using the Fundamental (Basic) Counting Principle.	**5.PS.3 The student will determine the probability of an outcome by constructing a model of a sample space and using the Fundamental (Basic) Counting Principle.** a) Determine the probability of an outcome by constructing a sample space (with a total of 24 or fewer equally likely possible outcomes), using a tree diagram, list, or chart to represent and determine all possible outcomes. b) Determine the number of possible outcomes by using the Fundamental (Basic) Counting Principle.
5.16 The student, given a practical problem, will **a) represent data in line plots and stem-and-leaf plots;** **b) interpret data represented in line plots and stem-and-leaf plots; and** **c) compare data represented in a line plot with the same data represented in a stem-and-leaf plot.** ● Collect data, using observations (e.g., weather), measurement (e.g., shoe sizes), surveys (e.g., hours watching television), or experiments (e.g., plant growth). (a) ● Organize the data into a chart or table. (a) ● Represent data in a line plot. Line plots will have no more than 30 data points. (a)	**5.PS.1 The student will apply the data cycle (formulate questions; collect or acquire data; organize and represent data; and analyze data and communicate results) with a focus on line plots (dot plots) and stem-and-leaf plots.** a) Formulate questions that require the collection or acquisition of data. b) Determine the data needed to answer a formulated question and collect or acquire existing data (limited to 30 or fewer data points) using various methods (e.g., polls, observations, measurements, experiments). c) Organize and represent a data set using a line plot (dot plot) with a title, labeled axes, and a key, with and without the use of technology tools. Lines plot (dot plots) may contain whole numbers, fractions, or decimals.

2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Probability and Statistics**	2023 *Standards of Learning* Knowledge and Skills (KS) **Probability and Statistics (PS)**

● Represent data in a stem-and-leaf plot where the stem is listed in ascending order and the leaves are in ascending order, with or without commas between leaves. Stem-and-leaf plots will be limited to no more than 30 data points. (a)
 	
● Title the given graph or identify an appropriate title. (a)
 	
● Interpret data by making observations from line plots and stem-and-leaf plots, describing the characteristics of the data, and describing the data as a whole. One set of data will be represented on a graph. (b)
 	
● Interpret data by making inferences from line plots and stem-and-leaf plots. (b)
 	
● Compare data represented in a line plot with the same data represented in a stem-and-leaf plot. (c)
	d) Organize and represent numerical data using a stem-and-leaf plot with a title and key, where the stems are listed in ascending order and the leaves are in ascending order, with or without commas between the leaves.
	
	e) Analyze data represented in line plots (dot plots) and stem-and-leaf plots and communicate results orally and in writing:
	  i) describe the characteristics of the data represented in a line plot (dot plot) and stem-and-leaf plot as a whole (e.g., the shape and spread of the data);
	  ii) make inferences about data represented in line plots (dot plots) and stem-and-leaf plots (e.g., based on a line plot (dot plot) of the number of books students in a bus line have in their backpack, every student will have from two to four books in their backpack);
	  iii) identify parts of the data that have special characteristics and explain the meaning of the greatest, the least, or the same (e.g., the stem-and-leaf plot shows that the same number of students scored in the 90s as scored in the 70s);
	  iv) draw conclusions about the data and make predictions based on the data to answer questions; and
	  v) solve single-step and multistep addition and subtraction problems using data from line plots (dot plots) and stem-and-leaf plots.

5.17 The student, given a practical context, will **a) describe mean, median, and mode as measures of center;** **b) describe mean as fair share;** **c) describe the range of a set of data as a measure of spread; and** **d) determine the mean, median, mode, and range of a set of data.**	**5.PS.2 The student will solve contextual problems using measures of center and the range.** a) Describe mean as fair share.

2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Probability and Statistics**	2023 *Standards of Learning* Knowledge and Skills (KS) **Probability and Statistics (PS)**
● Describe and determine the mean of a group of numbers representing data from a given context as a measure of center. (a, d) ● Describe and determine the median of a group of numbers representing data from a given context as a measure of center. (a, d) ● Describe and determine the mode of a group of numbers representing data from a given context as a measure of center. (a, d) ● Describe mean as fair share. (b) ● Describe and determine the range of a group of numbers representing data from a given context as a measure of spread. (c, d)	b) Describe and determine the mean of a set of data values representing data from a given context as a measure of center. c) Describe and determine the median of a set of data values representing data from a given context as a measure of center. d) Describe and determine the mode of a set of data values representing data from a given context as a measure of center. e) Describe and determine the range of a set of data values representing data from a given context as a measure of spread.

2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Patterns, Functions, and Algebra**	2023 *Standards of Learning* Knowledge and Skills (KS) **Patterns, Functions, and Algebra (PFA)**
5.18 The student will identify, describe, create, express, and extend number patterns found in objects, pictures, numbers, and tables. ● Identify, create, describe, and extend patterns using concrete materials, number lines, tables, or pictures.	**5.PFA.1 The student will identify, describe, extend, and create increasing and decreasing patterns with whole numbers, fractions, and decimals, including those in context, using various representations.**

2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Patterns, Functions, and Algebra**	2023 *Standards of Learning* Knowledge and Skills (KS) **Patterns, Functions, and Algebra (PFA)**
• Describe and express the relationship found in patterns, using words, tables, and symbols. • Solve practical problems that involve identifying, describing, and extending single-operation input and output rules (limited to addition, subtraction, and multiplication of whole numbers; addition and subtraction of fractions, with denominators of 12 or less; and addition and subtraction of decimals expressed in tenths or hundredths). • Identify the rule in a single-operation numerical pattern found in a list or table (limited to addition, subtraction, and multiplication of whole numbers; addition and subtraction of fractions, with denominators of 12 or less; and addition and subtraction of decimals expressed in tenths or hundredths).	a) Identify, describe, extend, and create increasing and decreasing patterns using various representations (e.g., objects, pictures, numbers, number lines, input/output tables, function machines). b) Analyze an increasing or decreasing single-operation numerical pattern found in lists, input/output tables, and function machines, and generalize the change to identify the rule, extend the pattern, or identify missing terms. (Patterns will be limited to addition, subtraction, multiplication, and division of whole numbers; addition and subtraction of fractions with like denominators of 12 or less; and addition and subtraction of decimals expressed in tenths or hundredths). c) Solve contextual problems that involve identifying, describing, and extending increasing and decreasing patterns using single-operation input and output rules (limited to addition, subtraction, multiplication, and division of whole numbers; addition and subtraction of fractions with like denominators of 12 or less; and addition and subtraction of decimals expressed in tenths or hundredths).
5.19 The student will **a) investigate and describe the concept of variable;** **b) write an equation to represent a given mathematical relationship, using a variable;** **c) use an expression with a variable to represent a given verbal expression involving one operation; and** **d) create a problem situation based on a given equation, using a single variable and one operation.**	**5.PFA.2 The student will investigate and use variables in contextual problems.** a) Describe the concept of a variable (presented as a box, letter, or other symbol) as a representation of an unknown quantity. b) Write an equation (with a single variable that represents an unknown quantity and one operation) from a contextual situation, using addition, subtraction, multiplication, or division.

2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Patterns, Functions, and Algebra**	2023 *Standards of Learning* Knowledge and Skills (KS) **Patterns, Functions, and Algebra (PFA)**
• Describe the concept of a variable (presented as boxes, letters, or other symbols) as a representation of an unknown quantity. (a) • Write an equation with addition, subtraction, multiplication, or division, using a variable to represent an unknown quantity. (b) • Use an expression with a variable to represent a given verbal expression involving one operation (e.g., "5 more than a number" can be represented by $y + 5$). (c) • Create and write a word problem to match a given equation with a single variable and one operation. (d)	c) Use an expression with a variable to represent a given verbal expression involving one operation (e.g., "5 more than a number" can be represented by $y + 5$). d) Create and write a word problem to match a given equation with a single variable and one operation.

2023 Grade 5 Mathematics SOL – Summary of Changes

Grade 5 (2016 SOL to 2023 SOL Numbering)	Parameter Changes/Clarification (2023 SOL)
5.1 → [Included in 5.CE.3] 5.2a-b → 5.NS.1 5.3a-b → 5.NS.2 5.4 → 5.CE.1 5.5a-b → 5.CE.3 5.6a-b → 5.CE.2 5.7 → 5.CE.4 5.8a-b → 5.MG.2 5.9a-b → 5.MG.1 5.10 → [Included in Grade 6] 5.11 → [Deleted] 5.12 → 5.MG.3 5.13a-b → [Included in 5.MG.3] 5.14a → [Included in Grades 7 and 8] 5.14b → [Deleted] 5.15 → 5.PS.3 5.16a-c → 5.PS.1 5.17a-d → 5.PS.2 5.18 → 5.PFA.1 5.19a-d → 5.PFA.2	5.CE.1 - "Create and solve…" reworded to "Estimate, represent, solve, and justify…" for whole number problems 5.CE.2, and 5.CE.3 – Standard now states "estimate, represent, solve, and justify…" for fraction and decimal problems 5.CE.2b,c - Fraction denominators limited to 2, 3, 4, 5, 6, 8, 10, and 12 (previously 12 or less) 5.CE.3a - Rounding to the nearest whole, tenth, or hundredth included as an estimation strategy for decimal computation 5.CE.4a - Order of operations limited to expressions containing no more than one set of parentheses; simplification will be limited to five whole numbers and four operations in any combination of addition, subtraction, multiplication, or division; whole numbers will be limited to two digits or less; expressions should not include braces, brackets, or fraction bars 5.MG.2 – Use of multiple representations when solving area, perimeter, and volume problems 5.MG.2a,d – Investigate and develop formulas for determining the area of a right triangle and volume of a rectangular prism (previously develop a procedure) 5.MG.3 – Solve problems, including those in context, with angles and triangles 5.PFA.1 – Increasing and decreasing patterns with whole numbers, fractions with like denominators of 12 or less, and decimals expressed in tenths or hundredths 5.PFA.2 – Investigate and use variables in contextual problems

Deletions from Grade 5 (2016 SOL)	Additions to Grade 5 (2023 SOL)
5.3b [EKS] – Identify which numbers are even and odd 5.3 [EKS] - Demonstrate with concrete or pictorial representations and explain orally or in writing why the sum or difference of two numbers is even or odd 5.4 - Create single-step and multistep practical problems involving addition, subtraction, multiplication, and division of whole numbers, with and without remainders 5.5b [EKS] – Create single-step and multistep practical problems involving addition, subtraction, and multiplication of decimals and create single-step practical problems involving division of decimals 5.6 [EKS] - Apply the inverse property of multiplication in models. (For example, use a visual fraction model to represent $\frac{4}{4}$ or as the product of $4 \times \frac{1}{4}$) [Moved to Grade 4] 5.8 [EKS] - Determine the perimeter of a polygon when the lengths of all sides are given 5.8 [EKS] – Estimate and determine the area of a square and rectangle using whole number measurements [Moved to Grade 4] 5.9 [EKS] - Estimate and measure to solve practical problems that involve length (kilometers removed) 5.10 - Identify and describe the diameter, radius, chord, and circumference of a circle [Included in Grade 6] 5.11 - Solve practical problems related to elapsed time in hours and minutes within a 24-hour period 5.14a - Recognize and apply transformations, such as translation, reflection, and rotation [Included in Grades 7 and 8] 5.14b - Investigate and describe the results of combining and subdividing polygons	5.NS.2c – Determine the prime factorization of a whole number up to 100 5.CE.2a - Determine the least common multiple of two numbers to find the least common denominator for two fractions [Moved from Grade 4] 5.CE.3b – multiplication of decimals expanded to include a three-digit factor and a one-digit factor 5.MG.1a - Determine the most appropriate unit of measure to use in a contextual problem that involves metric units for length mass, and liquid volume 5.MG.2c - Describe volume as a measure of capacity 5.PS.1 - Additional data analysis knowledge and skills representing the data cycle have been included 5.PFA.1b,c – Patterns may include division

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KEY: NS = Number and Number Sense; CE = Computation and Estimation; MG = Measurement and Geometry; PS = Probability and Statistics; PFA = Patterns, Functions, and Algebra; EKS = Essential Knowledge and Skills (2016); KS = Knowledge and Skills (2023); US = Understanding the Standard

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Grade 6 Mathematics *Standards of Learning* - 2023 Overview of Revisions

This overview includes a summary of the content embedded in five content strands.

Number and Number Sense: *There are multiple representations of numbers and relationships among numbers that provide meaning and structure and allow us to make sense of the world around us.*
- Express equivalency, compare, and order numbers written as fractions, mixed numbers, decimals, and percents
- Represent, compare, and order integers
- Recognize and represent patterns with exponents and perfect squares

Computation and Estimation: *Estimation and the operations of addition, subtraction, multiplication, and division, allow us to model, represent, and solve different types of problems with rational numbers.*
- Represent and solve problems using operations with fractions and mixed numbers
- Represent and solve problems using operations with integers

Measurement and Geometry: *Analyzing and describing geometric objects, the relationships and structures among them, or the space that they occupy can be used to classify, quantify, measure, or count one or more attributes.*
- Solve problems involving area and circumference of circles
- Solve problems involving the area and perimeter of triangles and parallelograms.
- Describe characteristics of the coordinate plane and graph ordered pairs
- Determine congruence of segments, angles, and polygons

Probability and Statistics: *The world can be investigated through posing questions and collecting, representing, analyzing, and interpreting data to describe and predict events and real-world phenomena.*
- Apply the data cycle (formulate questions; collect or acquire data; organize and represent data; and analyze data and communicate results) with a focus on circle graphs.
- Represent the mean as a balance point and describe how statistical measures are affected when a data value is added, removed, or changed

Patterns, Functions, and Algebra: *Proportional relationships can be described, and generalizations can be made using patterns, relations, and functions. Algebraic equations and inequalities can be used to represent and solve real world problems.*
- Use ratios to represent relationships between quantities
- Identify and represent proportional relationships between two quantities

- Create and solve one-step linear equations in one variable
- Represent a contextual situation using a linear inequality in one variable with symbols and graphs on a number line

Comparison of Grade 6 Mathematics *Standards of Learning* – 2016 to 2023

2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Number and Number Sense** * On the state assessment, items measuring this objective are assessed without the use of a calculator.	**2023 *Standards of Learning*** Knowledge and Skills (KS) **Number and Number Sense (NS)** * On the state assessment, items measuring this knowledge and skill are assessed without the use of a calculator.
6.1 The student will represent relationships between quantities using ratios, and will use appropriate notations, such as $\frac{a}{b}$, *a* to *b*, and *a:b*. ● Represent a relationship between two quantities using ratios. ● Represent a relationship in words that makes a comparison by using the notations $\frac{a}{b}$, *a:b, and a to b.* ● Create a relationship in words for a given ratio expressed symbolically.	**[Moved to 6.PFA.1]**
6.2 The student will **a) represent and determine equivalencies among fractions, mixed numbers, decimals, and percents; * and** **b) compare and order positive rational numbers. *** **c) represent and determine equivalencies among fractions, mixed numbers, decimals, and percents; * and** **d) compare and order positive rational numbers. *** ● Represent ratios as fractions (proper or improper), mixed numbers, decimals, and/or percents. (a) ● Determine the decimal and percent equivalents for numbers written in fraction form (proper or improper) or as a mixed number, including repeating decimals. (a)	**6.NS.1 The student will reason and use multiple strategies to express equivalency, compare, and order numbers written as fractions, mixed numbers, decimals, and percents.** a) Estimate and determine the percent represented by a given model (e.g., number line, picture, verbal description), including percents greater than 100% and less than 1%.* b) Represent and determine equivalencies among decimals (through the thousandths place) and percents incorporating the use of number lines, and concrete and pictorial models.* c) Represent and determine equivalencies among fractions (proper or improper) and mixed numbers that have denominators that are 12 or less or factors of 100 and percents incorporating the use of number lines, and concrete and pictorial models.*

2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Number and Number Sense** * On the state assessment, items measuring this objective are assessed without the use of a calculator.	2023 *Standards of Learning* Knowledge and Skills (KS) **Number and Number Sense (NS)** * On the state assessment, items measuring this knowledge and skill are assessed without the use of a calculator.
• Represent and determine equivalencies among decimals, percents, fractions (proper or improper), and mixed numbers that have denominators that are 12 or less or factors of 100. (a) • Compare two percents using pictorial representations and symbols ($<, \leq, \geq, >, =$). (b) • Order no more than four positive rational numbers expressed as fractions (proper or improper), mixed numbers, decimals, and percents (decimals through thousandths, fractions with denominators of 12 or less or factors of 100). Ordering may be in ascending or descending order. (b)	d) Represent and determine equivalencies among decimals, percents, fractions (proper or improper), and mixed numbers that have denominators that are 12 or less or factors of 100 incorporating the use of number lines, and concrete and pictorial models.* e) Use multiple strategies (e.g., benchmarks, number line, equivalency) to compare and order no more than four positive rational numbers expressed as fractions (proper or improper), mixed numbers, decimals, and percents (decimals through thousandths, fractions with denominators of 12 or less or factors of 100) with and without models. Justify solutions orally, in writing or with a model. Ordering may be in ascending or descending order.*
6.3 The student will **a) identify and represent integers;** **b) compare and order integers; and** **c) identify and describe absolute value of integers.** • Model integers, including models derived from practical situations. (a) • Identify an integer represented by a point on a number line. (a) • Compare and order integers using a number line. (b) • Compare integers, using mathematical symbols ($<, >, =$). (b) • Identify and describe the absolute value of an integer. (c)	**6.NS.2 The student will reason and use multiple strategies to represent, compare, and order integers.** a) Represent integers (e.g., number lines, concrete materials, pictorial models), including models derived from contextual situations, and identify an integer represented by a point on a number line. b) Compare and order integers using a number line. c) Compare integers, using mathematical symbols ($<, >, =$). d) Identify and describe the absolute value of an integer as the distance from zero on the number line.

2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Number and Number Sense** * On the state assessment, items measuring this objective are assessed without the use of a calculator.	2023 *Standards of Learning* Knowledge and Skills (KS) **Number and Number Sense (NS)** * On the state assessment, items measuring this knowledge and skill are assessed without the use of a calculator.
6.4 The student will recognize and represent patterns with whole number exponents and perfect squares. ● Recognize and represent patterns with bases and exponents that are whole numbers. ● Recognize and represent patterns of perfect squares not to exceed 20^2, by using grid paper, square tiles, tables, and calculators. ● Recognize powers of 10 with whole number exponents by examining patterns in place value.	**6.NS.3 The student will recognize and represent patterns with whole number exponents and perfect squares.** a) Recognize and represent patterns with bases and exponents that are whole numbers. b) Recognize and represent patterns of perfect squares not to exceed 20^2, by using concrete and pictorial models. c) Justify if a number between 0 and 400 is a perfect square through modeling or mathematical reasoning. d) Recognize and represent powers of 10 with whole number exponents by examining patterns in place value.

2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Computation and Estimation**	2023 *Standards of Learning* Knowledge and Skills (KS) **Computation and Estimation (CE)** * On the state assessment, items measuring this knowledge and skill are assessed without the use of a calculator.
6.5 The student will a) **multiply and divide fractions and mixed numbers; *** b) **solve single-step and multistep practical problems involving addition, subtraction, multiplication, and division of fractions and mixed numbers; and**	**6.CE.1 The student will estimate, demonstrate, solve, and justify solutions to problems using operations with fractions and mixed numbers, including those in context.** a) Demonstrate/model multiplication and division of fractions (proper or improper) and mixed numbers using multiple representations.*

2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Computation and Estimation**	2023 *Standards of Learning* Knowledge and Skills (KS) **Computation and Estimation (CE)** * On the state assessment, items measuring this knowledge and skill are assessed without the use of a calculator.
• Demonstrate/model multiplication and division of fractions (proper or improper) and mixed numbers using multiple representations. (a) • Multiply and divide fractions (proper or improper) and mixed numbers. Answers are expressed in simplest form. (a) • Solve single-step and multistep practical problems that involve addition and subtraction with fractions (proper or improper) and mixed numbers, with and without regrouping, that include like and unlike denominators of 12 or less. Answers are expressed in simplest form. (b) • Solve single-step and multistep practical problems that involve multiplication and division with fractions (proper or improper) and mixed numbers that include denominators of 12 or less. Answers are expressed in simplest form. (b)	b) Multiply and divide fractions (proper or improper) and mixed numbers that include denominators of 12 or less. Answers are expressed in simplest form.* c) Investigate and explain the effect of multiplying or dividing a fraction, whole number, or mixed number by a number between zero and one.* d) Estimate, determine, and justify the solution to single-step and multistep problems in context that involve addition and subtraction with fractions (proper or improper) and mixed numbers, with and without regrouping, that include like and unlike denominators of 12 or less. Answers are expressed in simplest form.
6.5 The student will **c) solve multistep practical problems involving addition, subtraction, multiplication, and division of decimals.** • Solve multistep practical problems involving addition, subtraction, multiplication, and division with decimals. Divisors are limited to a three-digit number, with decimal divisors limited to hundredths. (c)	**[Included in Grade 5 and Grade 7]**

2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Computation and Estimation**	2023 *Standards of Learning* Knowledge and Skills (KS) **Computation and Estimation (CE)** * On the state assessment, items measuring this knowledge and skill are assessed without the use of a calculator.
6.6 The student will a) **add, subtract, multiply, and divide integers; *** b) **solve practical problems involving operations with integers; and** • Model addition, subtraction, multiplication, and division of integers using pictorial representations or concrete manipulatives. (a) • Add, subtract, multiply, and divide two integers. (a) • Solve practical problems involving addition, subtraction, multiplication, and division with integers. (b)	**6.CE.2 The student will estimate, demonstrate, solve, and justify solutions to problems using operations with integers, including those in context.** a) Demonstrate/model addition, subtraction, multiplication, and division of integers using pictorial representations or concrete manipulatives.* b) Add, subtract, multiply, and divide two integers.* c) Simplify an expression that contains absolute value bars | | and an operation with two integers (e.g., $-\|5-8\|$ or $\frac{\|-12\|}{8}$) and represent the result on a number line. d) Estimate, determine, and justify the solution to one and two-step contextual problems, involving addition, subtraction, multiplication, and division with integers.
6.6 The student will c) **simplify numerical expressions involving integers. *** • Use the order of operations and apply the properties of real numbers to simplify numerical expressions involving more than two integers. Expressions should not include braces { } or brackets [ ], but may contain absolute value bars. Simplification will be limited to three operations, which may include simplifying a whole number raised to an exponent of 1, 2 or 3. (c)	**[Included in 6.CE.2 and Grade 7]**

2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Measurement and Geometry**	2023 *Standards of Learning* Knowledge and Skills (KS) **Measurement and Geometry (MG)**

6.7 The student will
 a) derive π (pi);
 b) solve problems, including practical problems, involving circumference and area of a circle; and
 	
 • Derive an approximation for pi (3.14 or $\frac{22}{7}$) by gathering data and comparing the circumference to the diameter of various circles, using concrete materials or computer models. (a)
 • Solve problems, including practical problems, involving circumference and area of a circle when given the length of the diameter or radius. (b)
	6.MG.1 The student will identify the characteristics of circles and solve problems, including those in context, involving circumference and area.
	
	 a) Identify and describe chord, diameter, radius, circumference, and area of a circle.
	 b) Investigate and describe the relationship between:
	   i) diameter and radius;
	   ii) radius and circumference; and
	   iii) diameter and circumference.
	 c) Develop an approximation for pi (3.14) by gathering data and comparing the circumference to the diameter of various circles, using concrete manipulatives or technological models.
	 d) Develop the formula for circumference using the relationship between diameter, radius, and pi.
	 e) Solve problems, including those in context, involving circumference and area of a circle when given the length of the diameter or radius.

6.7 The student will **c) solve problems, including practical problems, involving area and perimeter of triangles and rectangles.** • Solve problems, including practical problems, involving area and perimeter of triangles and rectangles. (c)	**6.MG.2 The student will reason mathematically to solve problems, including those in context, that involve the area and perimeter of triangles and parallelograms.** a) Develop the formula for determining the area of parallelograms and triangles using pictorial representations and concrete manipulatives (e.g., two-dimensional diagrams, grid paper). b) Solve problems, including those in context, involving the perimeter and area of triangles and parallelograms.

2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Measurement and Geometry**	2023 *Standards of Learning* Knowledge and Skills (KS) **Measurement and Geometry (MG)**
	[Rectangles included in Grade 4 and Grade 5]
6.8 The student will **a) identify the components of the coordinate plane; and** **b) identify the coordinates of a point and graph ordered pairs in a coordinate plane.** ● Identify and label the axes, origin, and quadrants of a coordinate plane. (a) ● Identify the quadrant or the axis on which a point is positioned by examining the coordinates (ordered pair) of the point. Ordered pairs will be limited to coordinates expressed as integers. (a) ● Graph ordered pairs in the four quadrants and on the axes of a coordinate plane. Ordered pairs will be limited to coordinates expressed as integers. (b) ● Identify ordered pairs represented by points in the four quadrants and on the axes of the coordinate plane. Ordered pairs will be limited to coordinates expressed as integers. (b) ● Relate the coordinates of a point to the distance from each axis and relate the coordinates of a single point to another point on the same horizontal or vertical line. Ordered pairs will be limited to coordinates expressed as integers. (b) ● Draw polygons in the coordinate plane given coordinates for the vertices; use coordinates to determine the length of a side joining points with the same first coordinate or the same second coordinate. Ordered pairs will be limited to coordinates expressed as integers. Apply these techniques in the context of solving practical and mathematical problems. (b)	**6.MG.3 The student will describe the characteristics of the coordinate plane and graph ordered pairs.** a) Identify and label the axes, origin, and quadrants of a coordinate plane. b) Identify and describe the location (quadrant or the axis) of a point given as an ordered pair. Ordered pairs will be limited to coordinates expressed as integers. c) Graph ordered pairs in the four quadrants and on the axes of a coordinate plane. Ordered pairs will be limited to coordinates expressed as integers. d) Identify ordered pairs represented by points in the four quadrants and on the axes of the coordinate plane. Ordered pairs will be limited to coordinates expressed as integers. e) Relate the coordinates of a point to the distance from each axis and relate the coordinates of a single point to another point on the same horizontal or vertical line. Ordered pairs will be limited to coordinates expressed as integers. f) Draw polygons in the coordinate plane given coordinates for the vertices; use coordinates to determine the length of a side joining points with the same first coordinate or the same second coordinate. Ordered pairs will be limited to coordinates expressed as integers. Apply these techniques in the context of solving contextual and mathematical problems.

2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Measurement and Geometry**	2023 *Standards of Learning* Knowledge and Skills (KS) **Measurement and Geometry (MG)**
6.9 The student will determine congruence of segments, angles, and polygons. ● Identify regular polygons. ● Draw lines of symmetry to divide regular polygons into two congruent parts. ● Determine the congruence of segments, angles, and polygons given their properties. ● Determine whether polygons are congruent or noncongruent according to the measures of their sides and angles.	**6.MG.4 The student will determine congruence of segments, angles, and polygons.** a) Identify regular polygons. b) Draw lines of symmetry to divide regular polygons into two congruent parts. c) Determine the congruence of segments, angles, and polygons given their properties. d) Determine whether polygons are congruent or noncongruent according to the measures of their sides and angles.

2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Probability and Statistics**	2023 *Standards of Learning* Knowledge and Skills (KS) **Probability and Statistics (PS)**
6.10 The student, given a practical situation, will **a) represent data in a circle graph;** **b) make observations and inferences about data represented in a circle graph; and** **c) compare circle graphs with the same data represented in bar graphs, pictographs, and line plots.** ● Collect, organize, and represent data in a circle graph. The number of data values should be limited to allow for comparisons that have denominators of 12 or less or those that are factors of 100 (e.g., in	**6.PS.1 The student will apply the data cycle (formulate questions; collect or acquire data; organize and represent data; and analyze data and communicate results) with a focus on circle graphs.** a) Formulate questions that require the collection or acquisition of data with a focus on circle graphs. b) Determine the data needed to answer a formulated question and collect the data (or acquire existing data) using various methods (e.g., observations, measurement, surveys, experiments).

2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Probability and Statistics**	2023 *Standards of Learning* Knowledge and Skills (KS) **Probability and Statistics (PS)**
a class of 20 students, 7 choose apples as a favorite fruit, so the comparison is 7 out of 20, $\frac{7}{20}$, or 35%). (a) • Make observations and inferences about data represented in a circle graph. (b) • Compare data represented in a circle graph with the same data represented in bar graphs, pictographs, and line plots. (c)	c) Determine the factors that will ensure that the data collected is a sample that is representative of a larger population. d) Organize and represent data using circle graphs, with and without the use of technology tools. The number of data values should be limited to allow for comparisons that have denominators of 12 or less or those that are factors of 100 (e.g., in a class of 20 students, 7 choose apples as a favorite fruit, so the comparison is 7 out of 20, $\frac{7}{20}$, or 35%). e) Analyze data represented in a circle graph by making observations and drawing conclusions. f) Compare data represented in a circle graph with the same data represented in other graphs, including but not limited to bar graphs, pictographs, and line plots (dot plots), and justify which graphical representation best represents the data.
6.11 The student will a) **represent the mean of a data set graphically as the balance point; and** b) **determine the effect on measures of center when a single value of a data set is added, removed, or changed.** • Represent the mean of a set of data graphically as the balance point represented in a line plot. (a) • Determine the effect on measures of center when a single value of a data set is added, removed, or changed. (b)	**6.PS.2 The student will represent the mean as a balance point and determine the effect on statistical measures when a data point is added, removed, or changed.** a) Represent the mean of a set of data graphically as the balance point represented in a line plot (dot plot). b) Determine the effect on measures of center when a single value of a data set is added, removed, or changed. c) Observe patterns in data to identify outliers and determine their effect on mean, median, mode, or range.

2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Patterns, Functions, and Algebra**	2023 *Standards of Learning* Knowledge and Skills (KS) **Patterns, Functions, and Algebra (PFA)**
6.12 The student will a) **represent a proportional relationship between two quantities, including those arising from practical situations;** ● Make a table of equivalent ratios to represent a proportional relationship between two quantities, when given a ratio. (a) ● Make a table of equivalent ratios to represent a proportional relationship between two quantities, when given a practical situation. (a)	**6.PFA.1 The student will use ratios to represent relationships between quantities, including those in context.** a) Represent a relationship between two quantities using ratios. b) Represent a relationship in context that makes a comparison by using the notations $\frac{a}{b}$, *a:b*, and *a* to *b*. c) Represent different comparisons within the same quantity or between different quantities (e.g., part to part, part to whole, whole to whole). d) Create a relationship in words for a given ratio expressed symbolically. e) Create a table of equivalent ratios to represent a proportional relationship between two quantities, when given a ratio. f) Create a table of equivalent ratios to represent a proportional relationship between two quantities, when given a contextual situation.
6.12 The student will b) **determine the unit rate of a proportional relationship and use it to find a missing value in a ratio table;** c) **determine whether a proportional relationship exists between two quantities; and** d) **make connections between and among representations of a proportional relationship between two quantities using verbal descriptions, ratio tables, and graphs.** ● Identify the unit rate of a proportional relationship represented by a table of values or a verbal description, including those	**6.PFA.2 The student will identify and represent proportional relationships between two quantities, including those in context (unit rates are limited to positive values).** a) Identify the unit rate of a proportional relationship represented by a table of values, a contextual situation, or a graph. b) Determine a missing value in a ratio table that represents a proportional relationship between two quantities using a unit rate. c) Determine whether a proportional relationship exists between two quantities, when given a table of values, context, or graph.

2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Patterns, Functions, and Algebra**	2023 *Standards of Learning* Knowledge and Skills (KS) **Patterns, Functions, and Algebra (PFA)**
represented in a practical situation. Unit rates are limited to positive values. (b) ● Determine a missing value in a ratio table that represents a proportional relationship between two quantities using a unit rate. Unit rates are limited to positive values. (b) ● Determine whether a proportional relationship exists between two quantities, when given a table of values or a verbal description, including those represented in a practical situation. Unit rates are limited to positive values. (c) ● Determine whether a proportional relationship exists between two quantities given a graph of ordered pairs. Unit rates are limited to positive values. (c) ● Make connections between and among multiple representations of the same proportional relationship using verbal descriptions, ratio tables, and graphs. Unit rates are limited to positive values. (d)	d) When given a contextual situation representing a proportional relationship, find the unit rate and create a table of values or a graph. e) Make connections between and among multiple representations of the same proportional relationship using verbal descriptions, ratio tables, and graphs.
6.13 The student will solve one-step linear equations in one variable, including practical problems that require the solution of a one-step linear equation in one variable. ● Identify examples of the following algebraic vocabulary: equation, variable, expression, term, and coefficient. ● Represent and solve one-step linear equations in one variable, using a variety of concrete materials such as colored chips, algebra tiles, or weights on a balance scale. ● Apply properties of real numbers and properties of equality to solve a one-step equation in one variable. Coefficients are limited	**6.PFA.3 The student will write and solve one-step linear equations in one variable, including contextual problems that require the solution of a one-step linear equation in one variable.** a) Identify and develop examples of the following algebraic vocabulary: equation, variable, expression, term, and coefficient. b) Represent and solve one-step linear equations in one variable, using a variety of concrete manipulatives and pictorial representations (e.g., colored chips, algebra tiles, weights on a balance scale). c) Apply properties of real numbers and properties of equality to solve a one-step equation in one variable. Coefficients are

2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Patterns, Functions, and Algebra**	2023 *Standards of Learning* Knowledge and Skills (KS) **Patterns, Functions, and Algebra (PFA)**
to integers and unit fractions. Numeric terms are limited to integers. ● Confirm solutions to one-step linear equations in one variable. ● Write verbal expressions and sentences as algebraic expressions and equations. ● Write algebraic expressions and equations as verbal expressions and sentences. ● Represent and solve a practical problem with a one-step linear equation in one variable.	limited to integers and unit fractions. Numeric terms are limited to integers. d) Confirm solutions to one-step linear equations in one variable using a variety of concrete manipulatives and pictorial representations (e.g., colored chips, algebra tiles, weights on a balance scale). e) Write a one-step linear equation in one variable to represent a verbal situation, including those in context. f) Create a verbal situation in context given a one-step linear equation in one variable.
6.14 The student will a) **represent a practical situation with a linear inequality in one variable; and** b) **solve one-step linear inequalities in one variable, involving addition or subtraction, and graph the solution on a number line.** ● Given a verbal description, represent a practical situation with a one-variable linear inequality. (a) ● Apply properties of real numbers and the addition or subtraction property of inequality to solve a one-step linear inequality in one variable, and graph the solution on a number line. Numeric terms being added or subtracted from the variable are limited to integers. (b) ● Given the graph of a linear inequality with integers, represent the inequality two different ways (e.g., x < -5 or -5 > x) using symbols. (b)	**6.PFA.4 The student will represent a contextual situation using a linear inequality in one variable with symbols and graphs on a number line.** a) Given the graph of a linear inequality in one variable on a number line, represent the inequality in two equivalent ways (e.g., $x < -5$ or $-5 > x$) using symbols. Symbols include $<, >, \leq, \geq$. b) Write a linear inequality in one variable to represent a given constraint or condition in context or given a graph on a number line. c) Given a linear inequality in one variable, create a corresponding contextual situation or create a number line graph. d) Use substitution or a number line graph to justify whether a given number in a specified set makes a linear inequality in one variable true. e) Identify a numerical value(s) that is part of the solution set of a given inequality in one variable.

2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Patterns, Functions, and Algebra**	2023 *Standards of Learning* Knowledge and Skills (KS) **Patterns, Functions, and Algebra (PFA)**
● Identify a numerical value(s) that is part of the solution set of a given inequality. (a, b)	

2023 Grade 6 Mathematics SOL – Summary of Changes

Grade 6 (2016 SOL to 2023 SOL Numbering)	Parameter Changes/Clarifications (2023 SOL)
6.1 → 6.PFA.1 6.2a-b → 6.NS.1 6.3a-c → 6.NS.2 6.4 → 6.NS.3 6.5a, b → 6.CE.1 6.5c → [Included in Grades 5 and 7] 6.6c → [Included in 6.CE.2 and Grade 7] 6.7a,b → 6.MG.1 6.7c → 6.MG.2 6.8a-b → 6.MG.3 6.9 → 6.MG.4 6.10a-c → 6.PS.1 6.11a-b → 6.PS.2 6.12a → 6.PFA.1 6.12b,c,d → 6.PFA.2 6.13 → 6.PFA.3 6.14a-b → 6.PFA.4 [Solving inequalities included in Grade 7]	6.NS.1 - Use multiple strategies & representations to express equivalency, and compare and order fractions, decimals, and percents 6.NS.3c - Justify if a number between 0 and 400 is a perfect square through modeling or mathematical reasoning 6.CE.1c - Investigate and explain the effect of multiplying or dividing a fraction, whole number, or mixed number by a number between zero and one 6.CE.1 and 6.CE.2 - Estimate, determine, and justify solutions 6.CE.2c - Simplify an expression that contains absolute value bars | | and an operation with two integers (e.g., –|5 − 8| or |-12|/8) and represent the result on a number line. 6.MG.1a,b,c,d -; identify and describe chord, diameter, radius, circumference, and area of a circle; describe relationships between diameter, radius, and circumference; remove $\frac{22}{7}$ as part of deriving pi; develop the formula for circumference using the relationship between diameter, radius, and pi 6.MG.2 - Include the development of the formula for triangles and parallelograms 6.PS.d,f - Include the use of technology to represent circle graphs; justify which graphical representation best represents the data 6. PFA.3e,f - Create a verbal situation in context given a one-step linear equation in one variable and write a one-step equation to represent a problem in context

Deletions from Grade 6 (2016 SOL)	Additions (2023 SOL)
6.5c - Solve multi-step practical problems involving addition, subtraction, multiplication, division of decimals [Included in 5.CE.3 and 7.CE.1] 6.6c – Order of Operations – [Included in 7.PFA.2] 6.7c – Area and Perimeter of Rectangles [Included in 4.MG.3 and 5.MG.2] 6.14b [EKS] - Solve one-step inequalities in one variable involving addition or subtraction and graph the solution on a number line [Included in 7.PFA.4]	6.MG.2 – Perimeter and area of parallelograms 6.PS.1 [KS] - Additional data analysis knowledge and skills representing the data cycle have been included 6.PS.2c- Identify outliers and determine their effect on mean, median, mode, or range

KEY: NS = Number and Number Sense; CE = Computation and Estimation; MG = Measurement and Geometry; PS = Probability and Statistics; PFA = Patterns, Functions, and Algebra; EKS = Essential Knowledge and Skills (2016); KS = Knowledge and Skills (2023); US = Understanding the Standard

Grade 7 Mathematics *Standards of Learning* - 2023 Overview of Revisions

This overview includes a summary of the content embedded in five content strands.

Number and Number Sense: *There are multiple representations of numbers and relationships among numbers that provide meaning and structure and allow for sense-making.*
- Describe the concept of exponents for powers of ten and compare and order numbers greater than zero written in scientific notation
- Compare and order rational numbers
- Recognize and describe the relationship between square roots and perfect squares

Computation and Estimation: *Estimation and the operations of addition, subtraction, multiplication, and division, allow us to model, represent, and solve different types of problems with rational numbers.*
- Solve multistep contextual problems with rational numbers
- Solve problems involving proportional relationships

Measurement and Geometry: *Analyzing and describing geometric objects, the relationships and structures among them, or the space that they occupy can be used to classify, quantify, measure, or count one or more attributes.*
- Solve problems involving volume and surface area of rectangular prisms and right cylinders
- Solve problems and justify relationships of similarity using proportional reasoning
- Compare and contrast quadrilaterals based on their properties and determine unknown side lengths and angle measures
- Apply dilations of polygons in the coordinate plane

Probability and Statistics: *The world can be investigated through posing questions and collecting, representing, analyzing, and interpreting data to describe and predict events and real-world phenomena.*
- Use statistical investigation to determine experimental and theoretical probability
- Apply the data cycle (formulate questions; collect or acquire data; organize and represent data; and analyze data and communicate results) with a focus on histograms

Patterns, Functions, and Algebra: *Proportional relationships can be described, and generalizations can be made using patterns, relations, and functions. Algebraic Equations and Inequalities can be used to represent and solve real world problems.*
- Analyze proportional relationships between two quantities using verbal descriptions, tables, equations in $y = mx$ form, and graphs
- Simplify and generate equivalent numerical and algebraic expressions, and evaluate algebraic expressions given replacement values
- Create and solve two-step linear equations in one variable
- Create and solve one and two-step linear inequalities in one variable

Comparison of Grade 7 Mathematics *Standards of Learning* – 2016 to 2023

2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Number and Number Sense** * On the state assessment, items measuring this objective are assessed without the use of a calculator.	2023 *Standards of Learning* Knowledge and Skills (KS) **Number and Number Sense (NS)** * On the state assessment, items measuring this knowledge and skill are assessed without the use of a calculator.
7.1 The student will **a) investigate and describe the concept of negative exponents for powers of ten;** **b) compare and order numbers greater than zero written in scientific notation;*** ● Recognize powers of 10 with negative exponents by examining patterns. (a) ● Represent a power of 10 with a negative exponent in fraction and decimal form. (a) ● Convert between numbers greater than 0 written in scientific notation and decimals. (b) ● Compare and order no more than four numbers greater than 0 written in scientific notation. Ordering may be in ascending or descending order. (b)	**7.NS.1 The student will investigate and describe the concept of exponents for powers of ten and compare and order numbers greater than zero written in scientific notation.** a) Investigate and describe powers of 10 with negative exponents by examining patterns. b) Represent a power of 10 with a negative exponent in fraction and decimal form. c) Convert between numbers greater than 0 written in scientific notation and decimals.* d) Compare and order no more than four numbers greater than 0 written in scientific notation. Ordering may be in ascending or descending order.*
7.1 The student will **c) compare and order rational numbers;*** ● Compare and order no more than four rational numbers expressed as integers, fractions (proper or improper), mixed numbers, decimals, and percents. Fractions and mixed numbers may be positive or negative. Decimals may be positive or negative and are limited to the thousandths place. Ordering may be in ascending or descending order. (c)	**7.NS.2 The student will reason and use multiple strategies to compare and order rational numbers.** a) Use multiple strategies (e.g., benchmarks, number line, equivalency) to compare (using symbols <, >, =) and order (a set of no more than four) rational numbers expressed as integers, fractions (proper or improper), mixed numbers, decimals, and percents. Fractions and mixed numbers may be positive or negative. Decimals may be positive or negative and are limited

2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Number and Number Sense** * On the state assessment, items measuring this objective are assessed without the use of a calculator.	2023 *Standards of Learning* Knowledge and Skills (KS) **Number and Number Sense (NS)** * On the state assessment, items measuring this knowledge and skill are assessed without the use of a calculator.
	to the thousandths place. Ordering may be in ascending or descending order. Justify solutions orally, in writing or with a model.*
7.1 The student will **d) determine square roots of perfect squares;* and** ● Identify the perfect squares from 0 to 400. (d) ● Determine the positive square root of a perfect square from 0 to 400. (d)	**7.NS.3 The student will recognize and describe the relationship between square roots and perfect squares.** a) Determine the positive square root of a perfect square from 0 to 400.* b) Describe the relationship between square roots and perfect squares.*
7.1 The student will **e) identify and describe the absolute value of rational numbers.** • Demonstrate absolute value using a number line. (e)	**[Included in Grade 6 and 7.PFA.2]**

2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Computation and Estimation**	2023 *Standards of Learning* Knowledge and Skills (KS) **Computation and Estimation (CE)**
7.2 The student will solve practical problems involving operations with rational numbers.	**7.CE.1 The student will estimate, solve, and justify solutions to multistep contextual problems involving operations with rational numbers.**

2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Computation and Estimation**	2023 *Standards of Learning* Knowledge and Skills (KS) **Computation and Estimation (CE)**
● Solve practical problems involving addition, subtraction, multiplication, and division with rational numbers expressed as integers, fractions (proper or improper), mixed numbers, decimals, and percents. Fractions may be positive or negative. Decimals may be positive or negative and are limited to the thousandths place.	a) Estimate, solve, and justify solutions to contextual problems involving addition, subtraction, multiplication, and division with rational numbers expressed as integers, fractions (proper or improper), mixed numbers, and decimals. Fractions may be positive or negative. Decimals may be positive or negative and are limited to the thousandths place.
7.3 The student will solve single-step and multistep practical problems, using proportional reasoning. ● Given a proportional relationship between two quantities, create and use a ratio table to determine missing values. ● Write and solve a proportion that represents a proportional relationship between two quantities to find a missing value. ● Apply proportional reasoning to convert units of measurement within and between the U.S. Customary System and the metric system when given the conversion factor. ● Apply proportional reasoning to solve practical problems, including scale drawings. Scale factors shall have denominators no greater than 12 and decimals no less than tenths. [Moved to 7.MG.2] ● Using 10% as a benchmark, compute 5%, 10%, 15%, or 20% of a given whole number. ● Using 10% as a benchmark, compute 5%, 10%, 15%, or 20% in a practical situation such as tips, tax, and discounts. [Included in Grade 8]	**7.CE.2 The student will solve problems, including those in context, involving proportional relationships.** a) Given a proportional relationship between two quantities, create and use a ratio table to determine missing values. b) Write and solve a proportion that represents a proportional relationship between two quantities to find a missing value, including problems in context. c) Apply proportional reasoning to solve problems in context, including converting units of measurement, when given the conversion factor. d) Estimate and determine the percentage of a given whole number, including but not limited to the use of benchmark percentages.

2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Computation and Estimation**	2023 *Standards of Learning* Knowledge and Skills (KS) **Computation and Estimation (CE)**
● Solve problems involving tips, tax, and discounts. Limit problems to only one percent computation per problem. [Included in Grade 8]	

2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Measurement and Geometry**	2023 *Standards of Learning* Knowledge and Skills (KS) **Measurement and Geometry (MG)**
7.4 The student will **a) describe and determine the volume and surface area of rectangular prisms and cylinders; and** **b) solve problems, including practical problems, involving the volume and surface area of rectangular prisms and cylinders.** ● Determine the surface area of rectangular prisms and cylinders using concrete objects, nets, diagrams, and formulas. (a) ● Determine the volume of rectangular prisms and cylinders using concrete objects, diagrams, and formulas. (a) ● Determine if a practical problem involving a rectangular prism or cylinder represents the application of volume or surface area. (b) ● Solve practical problems that require determining the surface area of rectangular prisms and cylinders. (b) ● Solve practical problems that require determining the volume of rectangular prisms and cylinders. (b).	**7.MG.1 The student will investigate and determine the volume formula for right cylinders and the surface area formulas for rectangular prisms and right cylinders and apply the formulas in context.** a) Develop the formulas for determining the volume of right cylinders and solve problems, including those in contextual situations, using concrete objects, diagrams, and formulas. b) Develop the formulas for determining the surface area of rectangular prisms and right cylinders and solve problems, including those in contextual situations, using concrete objects, two-dimensional diagrams, nets, and formulas. c) Determine if a problem in context, involving a rectangular prism or right cylinder, represents the application of volume or surface area. d) Describe how the volume of a rectangular prism is affected when one measured attribute is multiplied by a factor of $\frac{1}{4}, \frac{1}{3}, \frac{1}{2}$, 2, 3, or 4, including those in contextual situations. **[Moved from Grade 8]**

2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Measurement and Geometry**	2023 *Standards of Learning* Knowledge and Skills (KS) **Measurement and Geometry (MG)**
	e) Describe how the surface area of a rectangular prism is affected when one measured attribute is multiplied by a factor of $\frac{1}{2}$ or 2, including those in contextual situations. **[Moved from Grade 8]** **[Volume of rectangular prisms included in Grade 5]**
7.5 The student will solve problems, including practical problems, involving the relationship between corresponding sides and corresponding angles of similar quadrilaterals and triangle ● Identify corresponding sides and corresponding congruent angles of similar quadrilaterals and triangles. ● Given two similar quadrilaterals or triangles, write similarity statements using symbols. ● Write proportions to express the relationships between the lengths of corresponding sides of similar quadrilaterals and triangles. ● Solve a proportion to determine a missing side length of similar quadrilaterals or triangles. ● Given angle measures in a quadrilateral or triangle, determine unknown angle measures in a similar quadrilateral or triangle.	**7.MG.2 The student will solve problems and justify relationships of similarity using proportional reasoning.** a) Identify corresponding congruent angles of similar quadrilaterals and triangles, through the use of geometric markings. b) Identify corresponding sides of similar quadrilaterals and triangles. c) Given two similar quadrilaterals or triangles, write similarity statements using symbols. d) Write proportions to express the relationships between the lengths of corresponding sides of similar quadrilaterals and triangles. e) Recognize and justify if two quadrilaterals or triangles are similar using the ratios of corresponding side lengths. f) Solve a proportion to determine a missing side length of similar quadrilaterals or triangles. g) Given angle measures in a quadrilateral or triangle, determine unknown angle measures in a similar quadrilateral or triangle. h) Apply proportional reasoning to solve problems in context including scale drawings. Scale factors shall have denominators no greater than 12 and decimals no less than tenths

2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Measurement and Geometry**	2023 *Standards of Learning* Knowledge and Skills (KS) **Measurement and Geometry (MG)**

7.6 The student will
a) compare and contrast quadrilaterals based on their properties; and
b) determine unknown side lengths or angle measures of quadrilaterals.
 	
● Compare and contrast properties of the following quadrilaterals: parallelogram, rectangle, square, rhombus, and trapezoid. (a)
● Sort and classify quadrilaterals, as parallelograms, rectangles, trapezoids, rhombi, and/or squares based on their properties. (a)
● Given a diagram, determine an unknown angle measure in a quadrilateral, using properties of quadrilaterals. (b)
● Given a diagram, determine an unknown side length in a quadrilateral using properties of quadrilaterals. (b)
	7.MG.3 The student will compare and contrast quadrilaterals based on their properties and determine unknown side lengths and angle measures of quadrilaterals.
	
	a) Compare and contrast properties of the following quadrilaterals: parallelogram, rectangle, square, rhombus, and trapezoid:
	  i) parallel/perpendicular sides and diagonals;
	  ii) congruence of angle measures, side, and diagonal lengths; and
	  iii) lines of symmetry
	b) Sort and classify quadrilaterals, as parallelograms, rectangles, trapezoids, rhombi, and/or squares based on their properties:
	  i) parallel/perpendicular sides and diagonals;
	  ii) congruence of angle measures, side, and diagonal lengths; and
	  iii) lines of symmetry.
	c) Given a diagram, determine an unknown angle measure in a quadrilateral, using properties of quadrilaterals.
	d) Given a diagram, determine an unknown side length in a quadrilateral using properties of quadrilaterals.

7.7 The student will apply translations and reflections of right triangles or rectangles in the coordinate plane. ● Given a preimage in the coordinate plane, identify the coordinates of the image of a right triangle or rectangle that has been translated either vertically, horizontally, or a combination of a vertical and horizontal translation.	**[Included in Grade 8]**

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2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Measurement and Geometry**	2023 *Standards of Learning* Knowledge and Skills (KS) **Measurement and Geometry (MG)**
- Given a preimage in the coordinate plane, identify the coordinates of the image of a right triangle or a rectangle that has been reflected over the *x*- or *y*-axis. - Given a preimage in the coordinate plane, identify the coordinates of the image of a right triangle or rectangle that has been translated and reflected over the *x*- or *y*-axis or reflected over the *x*- or *y*-axis and then translated. - Sketch the image of a right triangle or rectangle that has been translated vertically, horizontally, or a combination of both. - Sketch the image of a right triangle or rectangle that has been reflected over the *x*- or *y*-axis. - Sketch the image of a right triangle or rectangle that has been translated and reflected over the *x*- or *y*-axis or reflected over the *x*- or *y*-axis and then translated.	
[Moved from Grade 8]	**7.MG.4 The student will apply dilations of polygons in the coordinate plane.** a) Given a preimage in the coordinate plane, identify the coordinates of the image of a polygon that has been dilated. Scale factors are limited to $\frac{1}{4}, \frac{1}{2}$, 2, 3, or 4. The center of the dilation will be the origin. b) Sketch the image of a dilation of a polygon limited to a scale factor of $\frac{1}{4}, \frac{1}{2}$, 2, 3, or 4. The center of the dilation will be the origin. c) Identify and describe dilations in context including, but not limited to, scale drawings and graphic design.

2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Probability and Statistics**	2023 *Standards of Learning* Knowledge and Skills (KS) **Probability and Statistics (PS)**
7.8 The student will a) **determine the theoretical and experimental probabilities of an event; and** b) **investigate and describe the difference between the experimental probability and theoretical probability of an event.** ● Determine the theoretical probability of an event. (a) ● Determine the experimental probability of an event. (a) ● Describe changes in the experimental probability as the number of trials increases. (b) ● Investigate and describe the difference between the probability of an event found through experiment or simulation versus the theoretical probability of that same event. (b)	**7.PS.1 The student will use statistical investigation to determine the probability of an event and investigate and describe the difference between the experimental and theoretical probability.** a) Determine the theoretical probability of an event. b) Given the results of a statistical investigation, determine the experimental probability of an event. c) Describe changes in the experimental probability as the number of trials increases. d) Investigate and describe the difference between the probability of an event found through experiment or simulation versus the theoretical probability of that same event.
7.9 The student, given data in a practical situation, will a) **represent data in a histogram;** b) **make observations and inferences about data represented in a histogram; and** c) **compare histograms with the same data represented in stem-and-leaf plots, line plots, and circle graphs.** ● Collect, organize, and represent data in a histogram. (a) ● Make observations and inferences about data represented in a histogram. (b) ● Compare data represented in histograms with the same data represented in line plots, circle graphs, and stem-and-leaf plots. (c)	**7.PS.2 The student will apply the data cycle (formulate questions; collect or acquire data; organize and represent data; and analyze data and communicate results) with a focus on histograms.** a) Formulate questions that require the collection or acquisition of data with a focus on histograms. b) Determine the data needed to answer a formulated question and collect the data (or acquire existing data) using various methods (e.g., observations, measurement, surveys, experiments). c) Determine how sample size and randomness will ensure that the data collected is a sample that is representative of a larger population.

2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Probability and Statistics**	2023 *Standards of Learning* Knowledge and Skills (KS) **Probability and Statistics (PS)**
	d) Organize and represent numerical data using histograms with and without the use of technology e) Investigate and explain how using different intervals could impact the representation of the data in a histogram. f) Compare data represented in histograms with the same data represented in other graphs, including but not limited to line plots (dot plots), circle graphs, and stem-and-leaf plots, and justify which graphical representation best represents the data. g) Analyze data represented in histograms by making observations and drawing conclusions. Determine how histograms reveal patterns in data that cannot be easily seen by looking at the corresponding given data set.

2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Patterns, Functions, and Algebra**	2023 *Standards of Learning* Knowledge and Skills (KS) **Patterns, Functions, and Algebra (PFA)** * On the state assessment, items measuring this knowledge and skill are assessed without the use of a calculator.
7.10 The student will a) **determine the slope, *m*, as a rate of change in a proportional relationship between two quantities and write an equation in the form *y = mx* to represent the relationship;** b) **graph a line representing a proportional relationship between two quantities given the slope and an ordered pair, or given the equation in *y = mx* form, where *m* represents the slope as rate of change;**	**7.PFA.1 The student will investigate and analyze proportional relationships between two quantities using verbal descriptions, tables, equations in y = mx form, and graphs, including problems in context.** a) Determine the slope, *m*, as the rate of change in a proportional relationship between two quantities given a table of values, graph, or contextual situation and write an equation in the form *y*

2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Patterns, Functions, and Algebra**	2023 *Standards of Learning* Knowledge and Skills (KS) **Patterns, Functions, and Algebra (PFA)** * On the state assessment, items measuring this knowledge and skill are assessed without the use of a calculator.
e) make connections between and among representations of a proportional or additive relationship between two quantities using verbal descriptions, tables, equations, and graphs. ● Determine the slope, m, as rate of change in a proportional relationship between two quantities given a table of values or a verbal description, including those represented in a practical situation, and write an equation in the form $y = mx$ to represent the relationship. Slope will be limited to positive values. (a) ● Graph a line representing a proportional relationship, between two quantities given an ordered pair on the line and the slope, m, as rate of change. Slope will be limited to positive values. (b) ● Graph a line representing a proportional relationship between two quantities given the equation of the line in the form $y = mx$, where m represents the slope as rate of change. Slope will be limited to positive values. (b) ● Make connections between and among representations of a proportional or additive relationship between two quantities using verbal descriptions, tables, equations, and graphs. (e)	$= mx$ to represent the direct variation relationship. Slope may include positive or negative values (slope will be limited to positive values in a contextual situation). b) Identify and describe a line with a slope that is positive, negative, or zero (0), given a graph. c) Graph a line representing a proportional relationship, between two quantities given an ordered pair on the line and the slope, m, as rate of change. Slope may include positive or negative values. d) Graph a line representing a proportional relationship between two quantities given the equation of the line in the form $y = mx$, where m represents the slope as rate of change. Slope may include positive or negative values. e) Make connections between and among representations of a proportional relationship between two quantities using problems in context, tables, equations, and graphs. Slope may include positive or negative values (slope will be limited to positive values in a contextual situation).
7.10 The student will **c) determine the *y*-intercept, *b*, in an additive relationship between two quantities and write an equation in the form $y = x + b$ to represent the relationship;** **d) graph a line representing an additive relationship between two quantities given the *y*-intercept and an ordered pair, or given the**	**[Included in Grade 8]**

2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Patterns, Functions, and Algebra**	2023 *Standards of Learning* Knowledge and Skills (KS) **Patterns, Functions, and Algebra (PFA)** * On the state assessment, items measuring this knowledge and skill are assessed without the use of a calculator.
equation in the form $y = x + b$, where b represents the y-intercept; and • Determine the y-intercept, b, in an additive relationship between two quantities given a table of values or a verbal description, including those represented in a practical situation, and write an equation in the form $y = x + b$, $b \neq 0$, to represent the relationship. (c) • Graph a line representing an additive relationship ($y = x + b$, $b \neq 0$) between two quantities, given an ordered pair on the line and the y-intercept (b). The y-intercept (b) is limited to integer values and slope is limited to 1. (d)	
7.11 The student will evaluate algebraic expressions for given replacement values of the variables. • Represent algebraic expressions using concrete materials and pictorial representations. Concrete materials may include colored chips or algebra tiles. • Use the order of operations and apply the properties of real numbers to evaluate expressions for given replacement values of the variables. Exponents are limited to 1, 2, 3, or 4 and bases are limited to positive integers. Expressions should not include braces { } but may include brackets [ ] and absolute value	|. Square roots are limited to perfect squares. Limit the number of replacements to no more than three per expression.	**7.PFA.2 The student will simplify numerical expressions, simplify and generate equivalent algebraic expressions in one variable, and evaluate algebraic expressions for given replacement values of the variables.** a) Use the order of operations and apply the properties of real numbers to simplify numerical expressions. Exponents are limited to 1, 2, 3, or 4 and bases are limited to positive integers. Expressions should not include braces { } but may include brackets [ ] and absolute value bars	|. Square roots are limited to perfect squares.* **[Moved from Grade 6]** b) Represent equivalent algebraic expressions in one variable using concrete manipulatives and pictorial representations (e.g., colored chips, algebra tiles).

2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Patterns, Functions, and Algebra**	2023 *Standards of Learning* Knowledge and Skills (KS) **Patterns, Functions, and Algebra (PFA)** * On the state assessment, items measuring this knowledge and skill are assessed without the use of a calculator.
	c) Simplify and generate equivalent algebraic expressions in one variable by applying the order of operations and properties of real numbers. Expressions may require combining like terms to simplify. Expressions will include only linear and numeric terms. Coefficients and numeric terms may be positive or negative rational numbers.* **[Moved from Grade 8]** d) Use the order of operations and apply the properties of real numbers to evaluate algebraic expressions for given replacement values of the variables. Exponents are limited to 1, 2, 3, or 4 and bases are limited to positive integers. Expressions should not include braces { } but may include brackets [ ] and absolute value bars | |. Square roots are limited to perfect squares. Limit the number of replacements to no more than three per expression. Replacement values may be positive or negative rational numbers.
7.12 The student will solve two-step linear equations in one variable, including practical problems that require the solution of a two-step linear equation in one variable. ● Represent and solve two-step linear equations in one variable using a variety of concrete materials and pictorial representations. ● Apply properties of real numbers and properties of equality to solve two-step linear equations in one variable. Coefficients and numeric terms will be rational. ● Confirm algebraic solutions to linear equations in one variable.	**7.PFA.3 The student will write and solve two-step linear equations in one variable, including problems in context, that require the solution of a two-step linear equation in one variable.** a) Represent and solve two-step linear equations in one variable using a variety of concrete materials and pictorial representations. b) Apply properties of real numbers and properties of equality to solve two-step linear equations in one variable. Coefficients and numeric terms will be rational. c) Confirm algebraic solutions to linear equations in one variable.

2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Patterns, Functions, and Algebra**	2023 *Standards of Learning* Knowledge and Skills (KS) **Patterns, Functions, and Algebra (PFA)** * On the state assessment, items measuring this knowledge and skill are assessed without the use of a calculator.
• Write verbal expressions and sentences as algebraic expressions and equations. • Write algebraic expressions and equations as verbal expressions and sentences. • Solve practical problems that require the solution of a two-step linear equation.	d) Write a two-step linear equation in one variable to represent a verbal situation, including those in context. e) Create a verbal situation in context given a two-step linear equation in one variable. f) Solve problems in context that require the solution of a two-step linear equation.
7.13 The student will solve one- and two-step linear inequalities in one variable, including practical problems, involving addition, subtraction, multiplication, and division, and graph the solution on a number line. • Apply properties of real numbers and the multiplication and division properties of inequality to solve one-step inequalities in one variable, and the addition, subtraction, multiplication, and division properties of inequality to solve two-step inequalities in one variable. Coefficients and numeric terms will be rational. • Represent solutions to inequalities algebraically and graphically using a number line. • Write verbal expressions and sentences as algebraic expressions and inequalities. • Write algebraic expressions and inequalities as verbal expressions and sentences. • Solve practical problems that require the solution of a one- or two-step inequality.	**7.PFA.4 The student will write and solve one- and two-step linear inequalities in one variable, including problems in context, that require the solution of a one- and two-step linear inequality in one variable.** a) Apply properties of real numbers and the addition, subtraction, multiplication, and division properties of inequality to solve one- and two-step inequalities in one variable. Coefficients and numeric terms will be rational. b) Investigate and explain how the solution set of a linear inequality is affected by multiplying or dividing both sides of the inequality statement by a rational number less than zero. c) Represent solutions to one- or two-step linear inequalities in one variable algebraically and graphically using a number line. d) Write one- or two-step linear inequalities in one variable to represent a verbal situation, including those in context. e) Create a verbal situation in context given a one or two-step linear inequality in one variable

2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Patterns, Functions, and Algebra**	2023 *Standards of Learning* Knowledge and Skills (KS) **Patterns, Functions, and Algebra (PFA)** * On the state assessment, items measuring this knowledge and skill are assessed without the use of a calculator.
● Identify a numerical value(s) that is part of the solution set of a given inequality.	f) Solve problems in context that require the solution of a one- or two-step inequality. g) Identify a numerical value(s) that is part of the solution set of as given one- or two-step linear inequality in one variable. h) Describe the differences and similarities between solving linear inequalities in one variable and linear equations in one variable.

2023 Grade 7 Mathematics SOL – Summary of Changes

Grade 7 (2016 SOL to 2023 SOL Numbering)	Parameter Changes/Clarifications (2023 SOL)
7.1a,b → 7.NS.1 7.1c → 7.NS.2 7.1d → 7.NS.3 7.1e → [Included in Grade 6 and 7.PFA.2} 7.2 → 7.CE.1 7.3 → 7.CE.2 7.4a-b → 7.MG.1 7.5 → 7.MG.2 7.6a-b → 7.MG.3 7.7 → [Included in Grade 8] [Dilations moved from Grade 8] → 7.MG.4 7.8a-b → 7.PS.1 7.9a-c → 7.PS.2 7.10a,b,e → 7.PFA.1 7.10c,d → [Moved to Grade 8] 7.11 → 7.PFA.2 7.12 → 7.PFA.3 7.13 → 7.PFA.4	7.NS.2d - Use multiple strategies to compare and order rational numbers 7.NS.3b- Describe the relationship between square roots and perfect squares 7.CE.1 -Estimate, solve, and justify solutions 7.CE.2d – Estimate and determine percentage of a whole number, including but not limited to benchmark percentages. 7.MG.1a,b - Develop the formula for volume of right cylinders and for surface area of rectangular prisms and right cylinders 7.MG.2e - Recognize and justify if two figures are similar using ratios of corresponding sides 7.PS.2d,e,f - Include the use of technology to represent histograms; explain how using different intervals impacts the representation of data in a histogram; justify which graphical representation best represents the data 7.PFA.1a - Include determining slope given a table of values, graph, or contextual situation and write an equation in the form $y = mx$ to represent the direct variation relationship 7.PFA.1c,d - Slope may include positive or negative values when writing an equation or graphing a line give a proportional relationship 7.PFA.3d,e - Create a verbal situation in context given a two-step linear equation in one variable and write a two-step equation to represent a problem in context 7.PFA.4b,d,e– Explain how the solution set of a linear inequality if affected by multiplying or dividing both sides of the inequality by a rational number less than zero; create a verbal situation in

context given a one or two-step linear inequality in one variable; write a one or two-step inequality to represent a problem in context

Deletions from Grade 7 (2016 SOL)	Additions (2023 SOL)
7.1e – Absolute Value [Included in Grade 6 and 7.PFA.2] 7.3 EKS - Sales tax, tip, and discount [Included in 8.CE.1] 7.4a EKS – Determining volume of rectangular prisms [Included in 5.MG.2] 7.7 – Translations and reflections of polygons [Included in 8.MG.3] 7.10 c, d, e- Additive relationships and *y*-intercept [Embedded in 8.PFA.3]	7.MG.1d,e - Change in attribute with volume and surface area of rectangular prisms [Moved from Grade 8] 7.MG.4 - Dilations [Moved from Grade 8] 7.PS.2 [KS] - Additional data analysis knowledge and skills representing the data cycle have been included 7.PFA.1b - Identify and describe a line with a positive, negative, or zero slope [Moved from Grade 8] 7.PFA.2a – Simplify Numerical Expressions without a calculator [moved from Grade 6] 7.PFA.2c – Simplify and generate equivalent algebraic expressions that may require combining like terms [Moved from Grade 8] 7.PFA.4h- Describe the differences and similarities between solving equations and inequalities

KEY: NS = Number and Number Sense; CE = Computation and Estimation; MG = Measurement and Geometry; PS = Probability and Statistics; PFA = Patterns, Functions, and Algebra; EKS = Essential Knowledge and Skills (2016); KS = Knowledge and Skills (2023); US = Understanding the Standard

Grade 8 Mathematics *Standards of Learning* - 2023 Overview of Revisions

This overview includes a summary of the content embedded in five content strands.

Number and Number Sense: *There are multiple representations of numbers and relationships among numbers that provide meaning and structure and allow for sense-making.*
- Compare and order real numbers, and determine the relationships between real numbers
- Investigate and describe the relationship between the subsets of the real number system

Computation and Estimation: *Estimation and the operations of addition, subtraction, multiplication, and division, allow us to model, represent, and solve different types of problems with rational numbers.*
- Estimate and apply proportional reasoning and computational procedures to solve contextual problems

Measurement and Geometry: *Analyzing and describing geometric objects, the relationships and structures among them, or the space that they occupy can be used to classify, quantify, measure, or count one or more attributes.*
- Use the relationships among pairs of angles that are vertical angles, adjacent angles, supplementary angles, and complementary angles to determine the measure of unknown angles
- Investigate and determine the surface area of square-based pyramids and the volume of cones and square-based pyramids
- Apply translations and reflections of polygons in the coordinate plane
- Apply the Pythagorean Theorem to solve problems involving right triangles
- Solve area and perimeter problems involving composite plane figures

Probability and Statistics: *The world can be investigated through posing questions and collecting, representing, analyzing, and interpreting data to describe and predict events and real-world phenomena.*
- Use statistical investigation to determine the probability of independent and dependent events
- Apply the data cycle (formulate questions; collect or acquire data; organize and represent data; and analyze data and communicate results) with a focus on boxplots and scatterplots

Patterns, Functions, and Algebra: *Proportional relationships can be described, and generalizations can be made using patterns, relations, and functions. Algebraic Equations and Inequalities can be used to represent and solve real world problems.*
- Represent, simplify, and generate equivalent algebraic expressions in one variable
- Determine whether a given relation is a function and determine the domain and range of a function
- Represent and solve problems using linear functions and analyze their key characteristics
- Create and solve multistep linear equations in one variable
- Create and solve multistep linear inequalities in one variable

Comparison of Grade 8 Mathematics *Standards of Learning* – 2016 to 2023

2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Number and Number Sense**	2023 *Standards of Learning* Knowledge and Skills (KS) **Number and Number Sense (NS)**
8.1 The student will compare and order real numbers. • Compare and order no more than five real numbers expressed as integers, fractions (proper or improper), decimals, mixed numbers, percents, numbers written in scientific notation, radicals, and π. Radicals may include both positive and negative square roots of values from 0 to 400. Ordering may be in ascending or descending order. • Use rational approximations (to the nearest hundredth) of irrational numbers to compare and order, locating values on a number line. Radicals may include both positive and negative square roots of values from 0 to 400 yielding an irrational number.	**8.NS.1 The student will compare and order real numbers and determine the relationships between real numbers.** a) Estimate and identify the two consecutive natural numbers between which the positive square root of a given number lies and justify which natural number is the better approximation. Numbers are limited to natural numbers from 1 to 400. b) Use rational approximations (to the nearest hundredth) of irrational numbers to compare, order, and locate values on a number line. Radicals may include both positive and negative square roots of values from 0 to 400 yielding an irrational number. c) Use multiple strategies (e.g., benchmarks, number line, equivalency) to compare and order no more than five real numbers expressed as integers, fractions (proper or improper), decimals, mixed numbers, percents, numbers written in scientific notation, radicals, and π. Radicals may include both positive and negative square roots of values from 0 to 400. Ordering may be in ascending or descending order. Justify solutions orally, in writing or with a model.

2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Number and Number Sense**	2023 *Standards of Learning* Knowledge and Skills (KS) **Number and Number Sense (NS)**
8.2 The student will describe the relationship between the subsets of the real number system. ● Describe and illustrate the relationships among the subsets of the real number system by using representations (graphic organizers, number lines, etc.). Subsets include rational numbers, irrational numbers, integers, whole numbers, and natural numbers. ● Classify a given number as a member of a particular subset or subsets of the real number system, and explain why. ● Describe each subset of the set of real numbers and include examples and non-examples. ● Recognize that the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.	**8.NS.2 The student will investigate and describe the relationship between the subsets of the real number system.** a) Describe and illustrate the relationships among the subsets of the real number system by using representations (e.g., graphic organizers, number lines). Subsets include rational numbers, irrational numbers, integers, whole numbers, and natural numbers. b) Classify and explain why a given number is a member of a particular subset or subsets of the real number system. c) Describe each subset of the set of real numbers and include examples and non-examples.
8.3 The student will **a) estimate and determine the two consecutive integers between which a square root lies; and** **b) determine both the positive and negative square roots of a given perfect square.** ● Estimate and identify the two consecutive integers between which the positive or negative square root of a given number lies. Numbers are limited to natural numbers from 1 to 400. (a) ● Determine the positive or negative square root of a given perfect square from 1 to 400. (b)	**[Included in 8.NS.1 and Grade 7]**

2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Computation and Estimation**	2023 *Standards of Learning* Knowledge and Skills (KS) **Computation and Estimation (CE)**
8.4 The student will solve practical problems involving consumer applications. ● Solve practical problems involving consumer applications by using proportional reasoning and computation procedures for rational numbers. ● Reconcile an account balance given a statement with five or fewer transactions. ● Compute a discount or markup and the resulting sale price for one discount or markup. ● Compute the sales tax or tip and resulting total. ● Compute the simple interest and new balance earned in an investment or on a loan given the principal amount, interest rate, and time period in years. ● Compute the percent increase or decrease found in a practical situation.	**8.CE.1 The student will estimate and apply proportional reasoning and computational procedures to solve contextual problems.** a) Estimate and solve contextual problems that require the computation of one discount or markup and the resulting sale price. b) Estimate and solve contextual problems that require the computation of the sales tax, tip and resulting total. c) Estimate and solve contextual problems that require the computation of the percent increase or decrease.

2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Measurement and Geometry**	2023 *Standards of Learning* Knowledge and Skills (KS) **Measurement and Geometry (MG)**
8.5 The student will use the relationships among pairs of angles that are vertical angles, adjacent angles, supplementary angles, and complementary angles to determine the measure of unknown angles.	**8.MG.1 The student will use the relationships among pairs of angles that are vertical angles, adjacent angles, supplementary angles, and complementary angles to determine the measure of unknown angles.**

2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Measurement and Geometry**	2023 *Standards of Learning* Knowledge and Skills (KS) **Measurement and Geometry (MG)**
• Identify and describe the relationship between pairs of angles that are vertical, adjacent, supplementary, and complementary. • Use the relationships among supplementary, complementary, vertical, and adjacent angles to solve problems, including practical problems, involving the measure of unknown angles.	a) Identify and describe the relationship between pairs of angles that are vertical, adjacent, supplementary, and complementary. b) Use the relationships among supplementary, complementary, vertical, and adjacent angles to solve problems, including those in context, involving the measure of unknown angles.
8.6 The student will **a) solve problems, including practical problems, involving volume and surface area of cones and square-based pyramids;** • Distinguish between situations that are applications of surface area and those that are applications of volume. (a) • Determine the surface area of cones and square-based pyramids by using concrete objects, nets, diagrams, and formulas. (a) • Determine the volume of cones and square-based pyramids, using concrete objects, diagrams, and formulas. (a) • Solve practical problems involving volume and surface area of cones and square-based pyramids. (a)	**8.MG.2 The student will investigate and determine the surface area of square-based pyramids and the volume of cones and square-based pyramids.** a) Determine the surface area of square-based pyramids by using concrete objects, nets, diagrams, and formulas. b) Determine the volume of cones and square-based pyramids, using concrete objects, diagrams, and formulas. c) Examine and explain the relationship between the volume of cones and cylinders, and the volume of rectangular prisms and square based pyramids. d) Solve problems in context involving volume of cones and square-based pyramids and the surface area of square-based pyramids.
8.6 The student will **b) describe how changing one measured attribute of a rectangular prism affects the volume and surface area.** • Describe how the volume of a rectangular prism is affected when one measured attribute is multiplied by a factor of $\frac{1}{4}, \frac{1}{3}, \frac{1}{2}$, 2, 3, or 4. (b)	**[Moved to Grade 7]**

2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Measurement and Geometry**	2023 *Standards of Learning* Knowledge and Skills (KS) **Measurement and Geometry (MG)**
• Describe how the surface area of a rectangular prism is affected when one measured attribute is multiplied by a factor of $\frac{1}{2}$ or 2. (b)	
8.7 The student will **a) given a polygon, apply transformations, to include translations, reflections, and dilations, in the coordinate plane; and** **b) identify practical applications of transformations.** • Given a preimage in the coordinate plane, identify the coordinate of the image of a polygon that has been translated vertically, horizontally, or a combination of both. (a) • Given a preimage in the coordinate plane, identify the coordinates of the image of a polygon that has been reflected over the *x*- or *y*-axis. (a) • Given a preimage in the coordinate plane, identify the coordinates of the image of a right triangle or a rectangle that has been dilated. Scale factors are limited to $\frac{1}{4}, \frac{1}{2}$, 2, 3, or 4. The center of the dilation will be the origin. (a) • Given a preimage in the coordinate plane, identify the coordinates of the image of a polygon that has been translated and reflected over the *x*-or *y*-axis, or reflected over the *x*- or *y*-axis and then translated. (a) • Sketch the image of a polygon that has been translated vertically, horizontally, or a combination of both. (a) • Sketch the image of a polygon that has been reflected over the *x*- or *y*-axis. (a)	**8.MG.3 The student will apply translations and reflections to polygons in the coordinate plane.** a) Given a preimage in the coordinate plane, identify the coordinates of the image of a polygon that has been translated vertically, horizontally, or a combination of both. b) Given a preimage in the coordinate plane, identify the coordinates of the image of a polygon that has been reflected over the x- or *y*-axis c) Given a preimage in the coordinate plane, identify the coordinates of the image of a polygon that has been translated and reflected over the x- or *y*-axis or reflected over the *x*- or *y*-axis and then translated. d) Sketch the image of a polygon that has been translated vertically, horizontally, or a combination of both. e) Sketch the image of a polygon that has been reflected over the *x*- or *y*-axis. f) Sketch the image of a polygon that has been translated and reflected over the *x*- or *y*-axis, or reflected over the *x*- or *y*-axis and then translated. g) Identify and describe transformations in context (e.g., tiling, fabric, wallpaper designs, art).

2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Measurement and Geometry**	2023 *Standards of Learning* Knowledge and Skills (KS) **Measurement and Geometry (MG)**
• Sketch the image of a dilation of a right triangle or a rectangle limited to a scale factor of $\frac{1}{4}, \frac{1}{2}$, 2, 3, or 4. The center of the dilation will be the origin. (a) • Sketch the image of a polygon that has been translated and reflected over the *x*- or *y*-axis, or reflected over the *x*- or *y*-axis and then translated. (a) • Identify the type of translation in a given example. (a, b) • Identify practical applications of transformations including, but not limited to, tiling, fabric, wallpaper designs, art, and scale drawings. (b)	
8.8 The student will construct a three-dimensional model, given the top or bottom, side, and front views. • Construct three-dimensional models, given the top or bottom, side, and front views. • Identify three-dimensional models given a two-dimensional perspective. • Identify the two-dimensional perspective from the top or bottom, side, and front view, given a three-dimensional model.	**[Deleted]**
8.9 The student will **a) verify the Pythagorean Theorem; and** **b) apply the Pythagorean Theorem.** • Verify the Pythagorean Theorem, using diagrams, concrete materials, and measurement. (a)	**8.MG.4 The student will apply the Pythagorean Theorem to solve problems involving right triangles, including those in context.** a) Verify the Pythagorean Theorem using diagrams, concrete materials, and measurement. b) Determine whether a triangle is a right triangle given the measures of its three sides.

2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Measurement and Geometry**	2023 *Standards of Learning* Knowledge and Skills (KS) **Measurement and Geometry (MG)**
● Determine whether a triangle is a right triangle given the measures of its three sides. (b) ● Determine the measure of a side of a right triangle, given the measures of the other two sides. (b) ● Solve practical problems involving right triangles by using the Pythagorean Theorem. (b)	c) Identify the parts of a right triangle (the hypotenuse and the legs) given figures in various orientations. d) Determine the measure of a side of a right triangle, given the measures of the other two sides. e) Apply the Pythagorean Theorem, and its converse, to solve problems involving right triangles in context.
8.10 The student will solve area and perimeter problems, including practical problems, involving composite plane figures. ● Subdivide a plane figure into triangles, rectangles, squares, trapezoids, parallelograms, and semicircles. Determine the area of subdivisions and combine to determine the area of the composite plane figure. ● Subdivide a plane figure into triangles, rectangles, squares, trapezoids, parallelograms, and semicircles. Use the attributes of the subdivisions to determine the perimeter of the composite plane figure. ● Apply perimeter, circumference, and area formulas to solve practical problems involving composite plane figures.	**8.MG.5 The student will solve area and perimeter problems involving composite plane figures, including those in context.** a) Subdivide a plane figure into triangles, rectangles, squares, trapezoids, parallelograms, circles, and semicircles. Determine the area of subdivisions and combine to determine the area of the composite plane figure. b) Subdivide a plane figure into triangles, rectangles, squares, trapezoids, parallelograms, and semicircles. Use the attributes of the subdivisions to determine the perimeter of the composite plane figure. c) Apply perimeter, circumference, and area formulas to solve contextual problems involving composite plane figures.

2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Probability and Statistics**	2023 *Standards of Learning* Knowledge and Skills (KS) **Probability and Statistics (PS)**
8.11 The student will a) **compare and contrast the probability of independent and dependent events; and** b) **determine probabilities for independent and dependent events.** ● Determine whether two events are independent or dependent. (a) ● Compare and contrast the probability of independent and dependent events. (a) ● Determine the probability of two independent events. (b) ● Determine the probability of two dependent events. (b)	**8.PS.1 The student will use statistical investigation to determine the probability of independent and dependent events, including those in context.** a) Determine whether two events are independent or dependent and explain how replacement impacts the probability. b) Compare and contrast the probability of independent and dependent events. c) Determine the probability of two independent events. d) Determine the probability of two dependent events.
8.12 The student will a) **represent numerical data in boxplots;** b) **make observations and inferences about data represented in boxplots; and** c) **compare and analyze two data sets using boxplots.** ● Collect and display a numeric data set of no more than 20 items, using boxplots. (a) ● Make observations and inferences about data represented in a boxplot. (b) ● Given a data set represented in a boxplot, identify, and describe the lower extreme (minimum), upper extreme (maximum), median, upper quartile, lower quartile, range, and interquartile range. (b) ● Compare and analyze two data sets represented in boxplots. (c)	**8.PS.2 The student will apply the data cycle (formulate questions; collect or acquire data; organize and represent data; and analyze data and communicate results) with a focus on boxplots.** a) Formulate questions that require the collection or acquisition of data with a focus on boxplots. b) Determine the data needed to answer a formulated question and collect the data (or acquire existing data) using various methods (e.g., observations, measurement, surveys, experiments). c) Determine how statistical bias might affect whether the data collected from the sample is representative of the larger population. d) Organize and represent a numeric data set of no more than 20 items, using boxplots, with and without the use of technology. e) Identify and describe the lower extreme (minimum), upper extreme (maximum), median, upper quartile, lower quartile,

2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Probability and Statistics**	2023 *Standards of Learning* Knowledge and Skills (KS) **Probability and Statistics (PS)**
	range, and interquartile range given a data set, represented by a boxplot. f) Describe how the presence of an extreme data point (outlier) affects the shape and spread of the data distribution of a boxplot. g) Analyze data represented in a boxplot by making observations and drawing conclusions. h) Compare and analyze two data sets represented in boxplots. i) Given a contextual situation, justify which graphical representation (e.g., pictographs, bar graphs, line graphs, line plots/dot plots, stem-and-leaf plots, circle graphs, histograms, and boxplots) best represents the data. j) Identify components of graphical displays that can be misleading.
8.13 The student will **a) represent data in scatterplots;** **b) make observations about data represented in scatterplots; and** **c) use a drawing to estimate the line of best fit for data represented in a scatterplot.** ● Collect, organize, and represent a data set of no more than 20 items using scatterplots. (a) ● Make observations about a set of data points in a scatterplot as having a positive linear relationship, a negative linear relationship, or no relationship. (b) ● Estimate the line of best fit with a drawing for data represented in a scatterplot. (c)	**8.PS.3 The student will apply the data cycle (formulate questions; collect or acquire data; organize and represent data; and analyze data and communicate results) with a focus on scatterplots.** a) Formulate questions that require the collection or acquisition of data with a focus on scatterplots. b) Determine the data needed to answer a formulated question and collect the data (or acquire existing data) of no more than 20 items using various methods (e.g., observations, measurement, surveys, experiments). c) Organize and represent numeric bivariate data using scatterplots with and without the use of technology.

2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Probability and Statistics**	2023 *Standards of Learning* Knowledge and Skills (KS) **Probability and Statistics (PS)**
	d) Make observations about a set of data points in a scatterplot as having a positive linear relationship, a negative linear relationship, or no relationship e) Analyze and justify the relationship of the quantitative bivariate data represented in scatterplots. f) Sketch the line of best fit for data represented in a scatterplot.

2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Patterns, Functions, and Algebra**	2023 *Standards of Learning* Knowledge and Skills (KS) **Patterns, Functions, and Algebra (PFA)**
8.14 The student will **a) evaluate an algebraic expression for given replacement values of the variables;** ● Use the order of operations and apply the properties of real numbers to evaluate algebraic expressions for the given replacement values of the variables. Exponents are limited to whole numbers and bases are limited to integers. Square roots are limited to perfect squares. Limit the number of replacements to no more than three per expression. (a)	**[Included in Grade 7]**
8.14 The student will **b) simplify algebraic expressions in one variable.**	**8.PFA.1 The student will represent, simplify, and generate equivalent algebraic expressions in one variable.**

2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Patterns, Functions, and Algebra**	2023 *Standards of Learning* Knowledge and Skills (KS) **Patterns, Functions, and Algebra (PFA)**
• Represent algebraic expressions using concrete materials and pictorial representations. Concrete materials may include colored chips or algebra tiles. (a) • Simplify algebraic expressions in one variable. Expressions may need to be expanded (using the distributive property) or require combining like terms to simplify. Expressions will include only linear and numeric terms. Coefficients and numeric terms may be rational. (b)	a) Represent algebraic expressions using concrete manipulatives or pictorial representations (e.g., colored chips, algebra tiles), including expressions that apply the distributive property. b) Simplify and generate equivalent algebraic expressions in one variable by applying the order of operations and properties of real numbers. Expressions may need to be expanded (using the distributive property) or require combining like terms to simplify. Expressions will include only linear and numeric terms. Coefficients and numeric terms may be rational.
8.15 The student will **a) determine whether a given relation is a function; and** **b) determine the domain and range of a function.** • Determine whether a relation, represented by a set of ordered pairs, a table, or a graph of discrete points is a function. Sets are limited to no more than 10 ordered pairs. (a) • Identify the domain and range of a function represented as a set of ordered pairs, a table, or a graph of discrete points. (b)	**8.PFA.2 The student will determine whether a given relation is a function and determine the domain and range of a function.** a) Determine whether a relation, represented by a set of ordered pairs, a table, or a graph of discrete points is a function. Sets are limited to no more than 10 ordered pairs. b) Identify the domain and range of a function represented as a set of ordered pairs, a table, or a graph of discrete points.
8.16 The student will **a) recognize and describe the graph of a linear function with a** **slope that is positive, negative, or zero;** • Recognize and describe a line with a slope that is positive, negative, or zero (0). (a)	**[Moved to Grade 7]**

2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Patterns, Functions, and Algebra**	2023 *Standards of Learning* Knowledge and Skills (KS) **Patterns, Functions, and Algebra (PFA)**
8.16 The student will **b) identify the slope and *y*-intercept of a linear function given a table of values, a graph, or an equation in *y* = *mx* + *b* form;** **c) determine the indepedent and dependent variable, given a practical situation modeled by a linear function;** **d) graph a linear function given the equation in *y* = *mx* + *b* form; and** **e) make connections between and among representations of a linear function using verbal descriptions, tables, equations, and graphs.** ● Given a table of values for a linear function, identify the slope and *y*-intercept. The table will include the coordinate of the *y*-intercept. (b) ● Given a linear function in the form *y* = *mx* + *b*, identify the slope and *y*-intercept. (b) ● Given the graph of a linear function, identify the slope and *y*-intercept. The value of the *y*-intercept will be limited to integers. The coordinates of the ordered pairs shown in the graph will be limited to integers. (b) ● Identify the dependent and independent variable, given a practical situation modeled by a linear function. (c) ● Given the equation of a linear function in the form *y* = *mx* + *b*, graph the function. The value of the *y*-intercept will be limited to integers. (d) ● Write the equation of a linear function in the form *y* = *mx* + *b* given values for the slope, *m,* and the *y*-intercept or given a practical situation in which the slope, *m*, and *y*-intercept are described verbally. (e)	**8.PFA.3 The student will represent and solve problems, including those in context, by using linear functions and analyzing their key characteristics (the value of the y-intercept (b) and the coordinates of the ordered pairs in graphs will be limited to integers).** a) Determine how adding a constant (*b*) to the equation of a proportional relationship *y* = *mx* will translate the line on a graph. b) Describe key characteristics of linear functions including slope (*m*), *y*-intercept (*b*), and independent and dependent variables. c) Graph a linear function given a table, equation, or a situation in context. d) Create a table of values for a linear function given a graph, equation in the form of *y* = *mx* + *b*, or context. e) Write an equation of a linear function in the form *y* = *mx* + *b*, given a graph, table, or a situation in context. f) Create a context for a linear function given a graph, table, or equation in the form *y* = *mx* + *b*.

2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Patterns, Functions, and Algebra**	2023 *Standards of Learning* Knowledge and Skills (KS) **Patterns, Functions, and Algebra (PFA)**
● Make connections between and among representations of a linear function using verbal descriptions, tables, equations, and graphs. (e).	
8.17 The student will solve multistep linear equations in one variable with the variable on one or both sides of the equation, including practical problems that require the solution of a multistep linear equation in one variable. ● Represent and solve multistep linear equations in one variable with the variable on one or both sides of the equation (up to four steps) using a variety of concrete materials and pictorial representations. ● Apply properties of real numbers and properties of equality to solve multistep linear equations in one variable (up to four steps). Coefficients and numeric terms will be rational. Equations may contain expressions that need to be expanded (using the distributive property) or require collecting like terms to solve. ● Write verbal expressions and sentences as algebraic expressions and equations. ● Write algebraic expressions and equations as verbal expressions and sentences. ● Solve practical problems that require the solution of a multistep linear equation. ● Confirm algebraic solutions to linear equations in one variable.	**8.PFA.4 The student will write and solve multistep linear equations in one variable, including problems in context that require the solution of a multistep linear equation in one variable.** a) Represent and solve multistep linear equations in one variable with the variable on one or both sides of the equation (up to four steps) using a variety of concrete materials and pictorial representations. b) Apply properties of real numbers and properties of equality to solve multistep linear equations in one variable (up to four steps). Coefficients and numeric terms will be rational. Equations may contain expressions that need to be expanded (using the distributive property) or require combining like terms to solve. c) Write a multistep linear equation in one variable to represent a verbal situation, including those in context. d) Create a verbal situation in context given a multistep linear equation in one variable. e) Solve problems in context that require the solution of a multistep linear equation. f) Interpret algebraic solutions in context to linear equations in one variable. g) Confirm algebraic solutions to linear equations in one variable.

2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Patterns, Functions, and Algebra**	2023 *Standards of Learning* Knowledge and Skills (KS) **Patterns, Functions, and Algebra (PFA)**
8.18 The student will solve multistep linear inequalities in one variable with the variable on one or both sides of the inequality symbol, including practical problems, and graph the solution on a number line. ● Apply properties of real numbers and properties of inequality to solve multistep linear inequalities (up to four steps) in one variable with the variable on one or both sides of the inequality. Coefficients and numeric terms will be rational. Inequalities may contain expressions that need to be expanded (using the distributive property) or require collecting like terms to solve. ● Graph solutions to multistep linear inequalities on a number line. ● Write verbal expressions and sentences as algebraic expressions and inequalities. ● Write algebraic expressions and inequalities as verbal expressions and sentences. ● Solve practical problems that require the solution of a multistep linear inequality in one variable. ● Identify a numerical value(s) that is part of the solution set of as given inequality.	**8.PFA.5 The student will create and solve multistep linear inequalities in one variable, including problems in context that require the solution of a multistep linear inequality in one variable.** a) Apply properties of real numbers and properties of inequality to solve multistep linear inequalities (up to four steps) in one variable with the variable on one or both sides of the inequality. Coefficients and numeric terms will be rational. Inequalities may contain expressions that need to be expanded (using the distributive property) or require collecting like terms to solve. b) Represent solutions to inequalities algebraically and graphically using a number line. c) Write multistep linear inequalities in one variable to represent a verbal situation, including those in context. d) Create a verbal situation in context given a multistep linear inequality in one variable. e) Solve problems in context that require the solution of a multistep linear inequality in one variable. f) Identify a numerical value(s) that is part of the solution set of as given inequality. g) Interpret algebraic solutions in context to linear inequalities in one variable.

2023 Grade 8 Mathematics SOL – Summary of Changes

Grade 8 (2016 SOL to 2023 SOL Numbering)	Parameter Changes/Clarifications (2023 SOL)
8.1 → 8.NS.1 8.2 → 8.NS.2 8.3a → 8.NS.1 8.3b → [Included in Grade 7] 8.4 → 8.CE.1 8.5 → 8.MG.1 8.6a → 8.MG.2 8.6b → [Moved to Grade 7] 8.7a-b → 8.MG.3 8.8 → [Deleted] 8.9a-b → 8.MG.4 8.10 → 8.MG.5 8.11a-b → 8.PS.1 8.12a-c → 8.PS.2 8.13a-c → 8.PS.3 8.14a → [Included in Grade 7] 8.14b → 8.PFA.1 8.15a-b → 8.PFA.2 8.16a → [Moved to Grade 7] 8.16b-e → 8.PFA.3 8.17 → 8.PFA.4 8.18 → 8.PFA.5	8.NS.1c - Use multiple strategies to compare and order real numbers 8.CE.1 - Include estimating with contextual problem solving 8.MG.2c - Examine and explain the relationship between the volume of related solids (cone/cylinder and square-based pyramid/rectangular prism) 8.MG.4c - Identify the parts of a right triangle (the hypotenuse and the legs) given figures in various positions 8.PS.1a - Explain how replacement impacts the probability of independent and dependent events 8.PS.2d,i - Include the use of technology to represent boxplots; justify which graphical representation best represents the data 8.PS.3c - Include use of technology to represent scatterplots 8.PFA.1a - Model the distributive property 8.PFA.3a,b - Determine how adding a constant (b) to the equation of a proportional relationship $y = mx$ will translate the line on a graph; Describe the key characteristics of linear functions (slope, y-intercept, independent and dependent variables) 8.PFA.4c,d - Create a verbal situation in context given multistep linear equation in one variable and write a multistep equation to represent a problem in context 8.PFA.5d,e - Create a verbal situation in context given a multistep linear inequality in one variable and write a multistep inequality to represent a problem in context

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Deletions from Grade 8 (2016 SOL)	Additions (2023 SOL)
8.2 [EKS]- Sum and products of rational and irrational numbers 8.3b – Determine positive or negative square roots of perfect squares [Included in Grade 7] 8.4 [EKS] - Reconcile account balance [Included in Economics and Personal Finance] 8.4 [EKS] - Compute simple interest [Included in Economics and Personal finance] 8.6a – Solve problems involving the surface area of a cone [Included in Geometry] 8.6b [EKS] – Describe how changing one attribute of a rectangular prism affects volume and surface area [Moved to Grade 7] 8.7a- Give a polygon, apply a dilation, in the coordinate plane [Moved to Grade 7] 8.8 - Constructing 3-D models (front, side, back view) 8.14a - Evaluating algebraic expressions given replacement values of the variables [Included in Grade 7] 8.16a - Recognize and describe a line with a slope that is positive, negative, or zero (0) [Moved to Grade 7]	8.PS.2 [KS] - Additional data analysis knowledge and skills representing the data cycle have been included with a focus on boxplots 8.PS.3 [KS] - Additional data analysis knowledge and skills representing the data cycle have been included, with a focus on scatterplots

KEY: NS = Number and Number Sense; CE = Computation and Estimation; MG = Measurement and Geometry; PS = Probability and Statistics; PFA = Patterns, Functions, and Algebra; EKS = Essential Knowledge and Skills (2016); KS = Knowledge and Skills (2023); US = Understanding the Standard

Algebra 1 *Standards of Learning* - 2023 Overview of Revisions

This overview includes a summary of the content embedded in four content strands.

Expressions and Operations
- Represent verbal quantitative situations algebraically and evaluate these expressions for given replacement values of the variables.
- Perform operations on and factor polynomial expressions in one variable.
- Derive and apply the laws of exponents.
- Simplify square roots of whole numbers and cube roots of integers.

Equations and Inequalities
- Represent, solve, and interpret the solution to multistep linear equations and inequalities in one variable and literal equations for a specified variable.
- Represent, solve, and interpret the solution to a system of two linear equations, a linear inequality in two variables, or a system of two linear inequalities in two variables.
- Represent, solve, and interpret the solution to a quadratic equation in one variable.

Functions
- Investigate, analyze, and compare linear functions algebraically and graphically
- Investigate, analyze, and compare characteristics of quadratic and exponential functions

Statistics
- Apply the data cycle (formulate questions; collect or acquire data; organize and represent data; and analyze data and communicate results) with a focus on representing bivariate data in scatterplots and determining the curve of best fit using linear and quadratic functions

Comparison of Algebra 1 Mathematics *Standards of Learning* – 2016 to 2023

2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Expressions and Operations**	2023 *Standards of Learning* Knowledge and Skills (KS) **Expressions and Operations (EO)**
A.1 The student will **a) represent verbal quantitative situations algebraically; and** **b) evaluate algebraic expressions for given replacement values of the variables.** ● Translate between verbal quantitative situations and algebraic expressions and equations. (a) ● Represent practical situations with algebraic expressions in a variety of representations (e.g., concrete, pictorial, symbolic, verbal). (a) ● Evaluate algebraic expressions, using the order of operations, which include absolute value, square roots, and cube roots for given replacement values to include rational numbers, without rationalizing the denominator. (b)	**A.EO.1 The student will represent verbal quantitative situations algebraically and evaluate these expressions for given replacement values of the variables.** a) Translate between verbal quantitative situations and algebraic expressions, including contextual situations. b) Evaluate algebraic expressions which include absolute value, square roots, and cube roots for given replacement values to include rational numbers, without rationalizing the denominator.
A.2 The student will perform operations on polynomials, including **a) applying the laws of exponents to perform operations on expressions;** ● Simplify monomial expressions and ratios of monomial expressions in which the exponents are integers, using the laws of exponents. (a)	**A.EO.3 The student will derive and apply the laws of exponents.** a) Derive the laws of exponents through explorations of patterns, to include products, quotients, and powers of bases. b) Simplify multivariable expressions and ratios of monomial expressions in which the exponents are integers, using the laws of exponents.
A.2 The student will perform operations on polynomials, including **b) adding, subtracting, multiplying, and dividing polynomials; and**	**A.EO.2 The student will perform operations on and factor polynomial expressions in one variable.**

2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Expressions and Operations**	2023 *Standards of Learning* Knowledge and Skills (KS) **Expressions and Operations (EO)**
- Model sums, differences, products, and quotients of polynomials with concrete objects and their related pictorial and symbolic representations. (b) - Determine sums and differences of polynomials. (b) - Determine products of polynomials. The factors should be limited to five or fewer terms (i.e., $(4x + 2)(3x + 5)$ represents four terms and $(x + 1)(2x^2 + x + 3)$ represents five terms). (b) - Determine the quotient of polynomials, using a monomial or binomial divisor, or a completely factored divisor. (b)	a) Determine sums and differences of polynomial expressions in one variable, using a variety of strategies, including concrete objects and their related pictorial and symbolic models. b) Determine the product of polynomial expressions in one variable, using a variety of strategies, including concrete objects and their related pictorial and symbolic models, the application of the distributive property, and the use of area models. The factors should be limited to five or fewer terms (e.g., $(4x + 2)(3x + 5)$ represents four terms and $(x + 1)(2x^2 + x + 3)$ represents five terms). d) Determine the quotient of polynomials, using a monomial or binomial divisor, or a completely factored divisor. e) Represent and demonstrate equality of quadratic expressions in different forms (e.g., concrete, verbal, symbolic, and graphical).
A.2 The student will perform operations on polynomials, including **c) factoring completely first- and second-degree binomials and trinomials in one variable.** - Factor completely first- and second-degree polynomials in one variable with integral coefficients. After factoring out the greatest common factor (GCF), leading coefficients should have no more than four factors. (c) - Factor and verify algebraic factorizations of polynomials with a graphing utility. (c)	**A.EO.2 The student will perform operations on and factor polynomial expressions in one variable.** c) Factor completely first- and second-degree polynomials in one variable with integral coefficients. After factoring out the greatest common factor (GCF), leading coefficients should have no more than four factors. e) Represent and demonstrate equality of quadratic expressions in different forms (e.g., concrete, verbal, symbolic, and graphical).

2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Expressions and Operations**	2023 *Standards of Learning* Knowledge and Skills (KS) **Expressions and Operations (EO)**
A.3 The student will simplify **a) square roots of whole numbers and monomial algebraic expressions;** **b) cube roots of integers; and** **c) numerical expressions containing square or cube roots.** ● Express the square root of a whole number in simplest form. (a) ● Express the principal square root of a monomial algebraic expression in simplest form where variables are assumed to have positive values. (a) ● Express the cube root of an integer in simplest form. (b) ● Simplify a numerical expression containing square or cube roots. (c) ● Add, subtract, and multiply two monomial radical expressions limited to a numerical radicand. (c)	**A.EO.4 The student will simplify and determine equivalent radical expressions involving square roots of whole numbers and cube roots of integers.** a) Simplify and determine equivalent radical expressions involving the square root of a whole number in simplest form. b) Simplify and determine equivalent radical expressions involving the cube root of an integer. c) Add, subtract, and multiply radicals, limited to numeric square and cube root expressions. d) Generate equivalent numerical expressions and justify their equivalency for radicals using rational exponents, limited to rational exponents of $\frac{1}{2}$ and $\frac{1}{3}$ (e.g., $\sqrt{5} = 5^{\frac{1}{2}}$; $\sqrt[3]{8} = 8^{\frac{1}{3}} = (2^3)^{\frac{1}{3}} = 2$).

2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Equations and Inequalities**	2023 *Standards of Learning* Knowledge and Skills (KS) **Equations and Inequalities (EI)**
A.4 The student will solve **a) multistep linear equations in one variable algebraically;** **c) literal equations for a specified variable;**	**A.EI.1 The student will represent, solve, explain, and interpret the solution to multistep linear equations and inequalities in one variable and literal equations for a specified variable.** a) Write a linear equation or inequality in one variable to represent a contextual situation.

2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Equations and Inequalities**	2023 *Standards of Learning* Knowledge and Skills (KS) **Equations and Inequalities (EI)**
• Determine whether a linear equation in one variable has one, an infinite number, or no solutions. (a) • Apply the properties of real numbers and properties of equality to simplify expressions and solve equations. (a, b) • Solve multistep linear equations in one variable algebraically. (a) • Solve quadratic equations in one variable algebraically. Solutions may be rational or irrational. (b) • Solve a literal equation for a specified variable. (c) • Given a system of two linear equations in two variables that has a unique solution, solve the system by substitution or elimination to identify the ordered pair which satisfies both equations. (d) • Given a system of two linear equations in two variables that has a unique solution, solve the system graphically by identifying the point of intersection. (d) • Solve and confirm algebraic solutions to a system of two linear equations using a graphing utility. (d) • Determine whether a system of two linear equations has one, an infinite number, or no solutions. (d) • Write a system of two linear equations that models a practical situation. (e) • Interpret and determine the reasonableness of the algebraic or graphical solution of a system of two linear equations that models a practical situation. (e) • Solve practical problems involving equations and systems of equations. (e)	b) Solve multistep linear equations in one variable, including those in contextual situations, by applying the properties of real numbers and/or properties of equality. d) Rearrange a formula or literal equation to solve for a specified variable by applying the properties of equality. e) Determine if a linear equation in one variable has one solution, no solution, or an infinite number of solutions. f) Verify possible solution(s) to multistep linear equations and inequalities in one variable algebraically, graphically, and with technology to justify the reasonableness of the answer(s). Explain the solution method and interpret solutions for problems given in context.
A.4 The student will solve **b) quadratic equations in one variable algebraically;**	**A.EI.3 The student will represent, solve, and interpret the solution to a quadratic equation in one variable.**

2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Equations and Inequalities**	2023 *Standards of Learning* Knowledge and Skills (KS) **Equations and Inequalities (EI)**
• Apply the properties of real numbers and properties of equality to simplify expressions and solve equations. (a, b) • Solve quadratic equations in one variable algebraically. Solutions may be rational or irrational. (b)	a) Solve a quadratic equation in one variable over the set of real numbers with rational or irrational solutions, including those that can be used to solve contextual problems. b) Determine and justify if a quadratic equation in one variable has no real solutions, one real solution, or two real solutions. c) Verify possible solution(s) to a quadratic equation in one variable algebraically, graphically, and with technology to justify the reasonableness of answer(s). Explain the solution method and interpret solutions for problems given in context.
A.4 The student will solve **d) systems of two linear equations in two variables algebraically and graphically; and** • Given a system of two linear equations in two variables that has a unique solution, solve the system by substitution or elimination to identify the ordered pair which satisfies both equations. (d) • Given a system of two linear equations in two variables that has a unique solution, solve the system graphically by identifying the point of intersection. (d) • Solve and confirm algebraic solutions to a system of two linear equations using a graphing utility. (d) • Determine whether a system of two linear equations has one, an infinite number, or no solutions. (d)	**A.EI.2 The student will represent, solve, explain, and interpret the solution to a system of two linear equations, a linear inequality in two variables, or a system of two linear inequalities in two variables.** b) Apply the properties of real numbers and/or properties of equality to solve a system of two linear equations in two variables, algebraically and graphically. c) Determine whether a system of two linear equations has one solution, no solution, or an infinite number of solutions.
A.4 The student will solve **e) practical problems involving equations and systems of equations**	**A.EI.2 The student will represent, solve, explain, and interpret the solution to a system of two linear equations, a linear inequality in two variables, or a system of two linear inequalities in two variables** a) Create a system of two linear equations in two variables to represent a contextual situation.

2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Equations and Inequalities**	2023 *Standards of Learning* Knowledge and Skills (KS) **Equations and Inequalities (EI)**
• Write a system of two linear equations that models a practical situation. (e) • Interpret and determine the reasonableness of the algebraic or graphical solution of a system of two linear equations that models a practical situation. (e) • Solve practical problems involving equations and systems of equations. (e)	h) Verify possible solution(s) to a system of two linear equations, a linear inequality in two variable, or a system of two linear inequalities algebraically, graphically, and with technology to justify the reasonableness of the answer(s). Explain the solution method and interpret solutions for problems given in context. **A.EI.1 The student will represent, solve, explain, and interpret the solution to multistep linear equations and inequalities in one variable and literal equations for a specified variable.** a) write a linear equation or inequality in one variable to represent a contextual situation;

A.5 The student will
a) solve multistep linear inequalities in one variable algebraically and represent the solution graphically;
b) represent the solution of linear inequalities in two variables graphically;
 	
• Solve multistep linear inequalities in one variable algebraically and represent the solution graphically. (a)
• Apply the properties of real numbers and properties of inequality to solve multistep linear inequalities in one variable algebraically. (a)
• Represent the solution of a linear inequality in two variables graphically. (b)
• Determine and verify algebraic solutions using a graphing utility. (a, b, c, d)
	A.EI.1 The student will represent, solve, explain, and interpret the solution to multistep linear equations and inequalities in one variable and literal equations for a specified variable.
	
	    c) Solve multistep linear inequalities in one variable algebraically and graph the solution set on a number line, including those in contextual situations, by applying the properties of real numbers and/or properties of inequality.
	
	A.EI.2 The student will represent, solve, explain, and interpret the solution to a system of two linear equations, a linear inequality in two variables, or a system of two linear inequalities in two variables.
	
	    d) Create a linear inequality in two variables to represent a contextual situation.

2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Equations and Inequalities**	2023 *Standards of Learning* Knowledge and Skills (KS) **Equations and Inequalities (EI)**
	e) Represent the solution of a linear inequality in two variables graphically on a coordinate plane.
A.5 The student will **c) solve multistep linear inequalities in one variable algebraically and represent the solution graphically;** • Solve practical problems involving linear inequalities. (c) • Determine whether a coordinate pair is a solution of a linear inequality or a system of linear inequalities. (c) • Determine and verify algebraic solutions using a graphing utility. (a, b, c, d)	**A.EI.1 The student will represent, solve, explain, and interpret the solution to multistep linear equations and inequalities in one variable and literal equations for a specified variable.** a) Write a linear equation or inequality in one variable to represent a contextual situation. c) Solve multistep linear inequalities in one variable algebraically and graph the solution set on a number line, including those in contextual situations, by applying the properties of real numbers and/or properties of inequality.
A.5 The student will **d) represent the solution to a system of inequalities graphically** • Represent the solution of a system of two linear inequalities graphically. (d) • Determine and verify algebraic solutions using a graphing utility. (a, b, c, d)	**A.EI.2 The student will represent, solve, explain, and interpret the solution to a system of two linear equations, a linear inequality in two variables, or a system of two linear inequalities in two variables.** f) Create a system of two linear inequalities in two variables to represent a contextual situation. g) Represent the solution set of a system of two linear inequalities in two variables, graphically on a coordinate plane. h) Verify possible solution(s) to a system of two linear equations, a linear inequality in two variable, or a system of two linear inequalities algebraically, graphically, and with technology to justify the reasonableness of the answer(s). Explain the solution method and interpret solutions for problems given in context.

2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Functions**	2023 *Standards of Learning* Knowledge and Skills (KS) **Functions (F)**
A.6 The student will **a) determine the slope of a line when given an equation of the line, the graph of the line, or two points on the line;** ● Determine the slope of the line, given the equation of a linear function. (a) ● Determine the slope of a line, given the coordinates of two points on the line. (a) ● Determine the slope of a line, given the graph of a line. (a) ● Recognize and describe a line with a slope or rate of change that is positive, negative, zero, or undefined. (a)	**A.F.1 The student will investigate, analyze, and compare linear functions algebraically and graphically, and model linear relationships.** a) Determine and identify the domain, range, zeros, slope, and intercepts of a linear function, presented algebraically or graphically, including the interpretation of these characteristics in contextual situations.

A.6 The student will
b) write the equation of a line when given the graph of the line, two points on the line, or the slope and a point on the line; and
 	
● Write the equation of a line when given the graph of a line. (b)
● Write the equation of a line when given two points on the line whose coordinates are integers. (b)
● Write the equation of a line when given the slope and a point on the line whose coordinates are integers. (b)
● Write the equation of a vertical line as $x = a$. (b)
● Write the equation of a horizontal line as $y = c$. (b)
● Write the equation of a line parallel or perpendicular to a given line through a given point. (b)
	A.F.1 The student will investigate, analyze, and compare linear functions algebraically and graphically.
	
	c) Write equivalent algebraic forms of linear functions, including slope-intercept form, standard form, and point-slope form, and analyze and interpret the information revealed by each form.
	d) Write the equation of a linear function to model a linear relationship between two quantities, including those that can represent contextual situations. Writing the equation of a linear function will include the following situations:
	  i) given the graph of a line;
	  ii) given two points on the line whose coordinates are integers;
	  iii) given the slope and a point on the line whose coordinates are integers;
	  iv) vertical lines as $x = a$; and
	  v) horizontal lines as $y = c$.

2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Functions**	2023 *Standards of Learning* Knowledge and Skills (KS) **Functions (F)**
	e) Write the equation of a line parallel or perpendicular to a given line through a given point.
A.6 The student will **c) graph linear equations in two variables.** - Graph a linear equation in two variables, including those that arise from a variety of practical situations. (c) - Use the parent function $y = x$ and describe transformations defined by changes in the slope or y-intercept. (c)	**A.F.1 The student will investigate, analyze, and compare linear functions algebraically and graphically.** b) Investigate and explain how transformations to the parent function $y = x$ affect the rate of change (slope) and the y-intercept of a linear function. f) Graph a linear function in two variables, with and without the use of technology, including those that can represent contextual situations. g) For any value, x, in the domain of f, determine $f(x)$, and determine x given any value $f(x)$ in the range of f, given an algebraic or graphical representation of a linear function. h) Compare and contrast the characteristics of linear functions represented algebraically, graphically, in tables, and in contextual situations.
A.7 The student will investigate and analyze linear and quadratic function families and their characteristics both algebraically and graphically, including **a) determining whether a relation is a function;** **b) domain and range;** **c) zeros;** **d) intercepts;** - Determine whether a relation, represented by a set of ordered pairs, a table, a mapping, or a graph is a function. (a)	**A.F.2 The student will investigate, analyze, and compare characteristics of functions, including quadratic and exponential functions, and model quadratic and exponential relationships.** a) Determine whether a relation, represented by a set of ordered pairs, a table, a mapping, or a graph is a function; for relations that are functions, determine the domain and range. b) Given an equation or graph, determine key characteristics of a quadratic function including x-intercepts (zeros), y-intercept, vertex (maximum or minimum), and domain and range (including when restricted by context); interpret key

2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Functions**	2023 *Standards of Learning* Knowledge and Skills (KS) **Functions (F)**
• Identify the domain, range, zeros, and intercepts of a function presented algebraically or graphically. (b, c, d) • Use the *x*-intercepts from the graphical representation of a quadratic function to determine and confirm its factors. (c, d) • Investigate and analyze characteristics and multiple representations of functions with a graphing utility. (a, b, c, d, e, f)	characteristics as related to contextual situations, where applicable. c) Graph a quadratic function, $f(x)$, in two variables using a variety of strategies, including transformations $f(x) + k$ and $kf(x)$, where k is limited to rational values. d) Make connections between the algebraic (standard and factored forms) and graphical representation of a quadratic function.
A.7 The student will investigate and analyze linear and quadratic function families and their characteristics both algebraically and graphically, including **e) values of a function for elements in its domain; and** **f) connections between and among multiple representations of functions using verbal descriptions, tables, equations, and graphs.** • For any value, *x*, in the domain of *f*, determine $f(x)$. (e) • Represent relations and functions using verbal descriptions, tables, equations, and graph. Given one representation, represent the relation in another form. (f) • Investigate and analyze characteristics and multiple representations of functions with a graphing utility. (a, b, c, d, e, f)	**A.F.2 The student will investigate, analyze, and compare characteristics of functions, including quadratic and exponential functions, and model quadratic and exponential relationships.** e) Given an equation or graph of an exponential function in the form $y = ab^x$ (where *b* is limited to a natural number), interpret key characteristics, including *y*-intercepts and domain and range; interpret key characteristics as related to contextual situations, where applicable. f) Graph an exponential function, $f(x)$, in two variables using a variety of strategies, including transformations $f(x) + k$ and $kf(x)$, where *k* is limited to rational values. g) For any value, *x*, in the domain of *f*, determine $f(x)$ of a quadratic or exponential function. Determine *x* given any value $f(x)$ in the range of *f* of a quadratic function. Explain the meaning of *x* and $f(x)$ in context. h) Compare and contrast the key characteristics of linear functions ($f(x) = x$), quadratic functions ($f(x) = x^2$), and exponential functions ($f(x) = b^x$) using tables and graphs.

2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Statistics**	2023 *Standards of Learning* Knowledge and Skills (KS) **Statistics (ST)**
A.8 The student, given a data set or practical situation, will analyze a relation to determine whether a direct or inverse variation exists, and represent a direct variation algebraically and graphically and an inverse variation algebraically. • Given a data set or practical situation, determine whether a direct variation exists. • Given a data set or practical situation, determine whether an inverse variation exists. • Given a data set or practical situation, write an equation for a direct variation. • Given a data set or practical situation, write an equation for an inverse variation. • Given a data set or practical situation, graph an equation representing a direct variation.	**[Direct variation is included in Grade 7]** **[Both direct and inverse variation are included in Algebra 2]**
A.9 The student will collect and analyze data, determine the equation of the curve of best fit in order to make predictions, and solve practical problems, using mathematical models of linear and quadratic functions. • Determine an equation of a curve of best fit, using a graphing utility, given a set of no more than twenty data points in a table, a graph, or a practical situation. • Make predictions, using data, scatterplots, or the equation of the curve of best fit. • Solve practical problems involving an equation of the curve of best fit. • Evaluate the reasonableness of a mathematical model of a practical situation.	**A.ST.1 The student will apply the data cycle (formulate questions; collect or acquire data; organize and represent data; and analyze data and communicate results) with a focus on representing bivariate data in scatterplots and determining the curve of best fit using linear and quadratic functions.** a) Formulate investigative questions that require the collection or acquisition of bivariate data. b) Determine what variables could be used to explain a given contextual problem or situation or answer investigative questions. c) Determine an appropriate method to collect a representative sample, which could include a simple random sample, to answer an investigative question.

2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Statistics**	2023 *Standards of Learning* Knowledge and Skills (KS) **Statistics (ST)**
	d) Given a table of ordered pairs or a scatterplot representing no more than 30 data points, use available technology to determine whether a linear or quadratic function would represent the relationship, and if so, determine the equation of the curve of best fit. e) Use linear and quadratic regression methods available through technology to write a linear or quadratic function that represents the data where appropriate and describe the strengths and weaknesses of the model. f) Use a linear model to predict outcomes and evaluate the strength and validity of these predictions, including through the use of technology. g) Investigate and explain the meaning of the rate of change (slope) and *y*-intercept (constant term) of a linear model in context. h) Analyze relationships between two quantitative variables revealed in a scatterplot. i) Make conclusions based on the analysis of a set of bivariate data and communicate the results.

2023 Algebra 1 Mathematics SOL – Summary of Changes

Algebra 1 (2016 SOL to 2023 SOL Numbering)	Parameter Changes/Clarifications (2023 SOL)
- A.1a,b → A.EO.1 - A.2b → A.EO.2 - A.2a → A.EO.3 - A.3a-c → A.EO.4 - A.4a,c,e → A.EI.1 - A.4d,e → A.EI.2 - A.4b → A.EI.3 - A.5a,c → A.EI.1 - A.5b,c,d → A.EI.2 - A.6a-c → A.F.1 - A.7a-h → A.F.2 - A.8 → [Included in Grade 7] - A.9 → A.ST.1	- A.EO.2 – Use area models to determine the product of polynomial expressions in one variable; use concrete, verbal, symbolic, and graphical forms to represent equality of quadratic expressions - A.EO.3 – Derive the laws of exponents specified - A.EI.1 – Justify answers, explain solution methods, and interpret solutions for problems in context when solving multistep linear equations and inequalities in one variable - A.EI.2 - Justify solutions to systems of two linear equations, a linear inequality with two variables, or a system of linear inequalities with technology; explain solution methods and interpret solution for problems in context involving systems of equations, linear inequalities, and systems of linear inequalities - A.EI.3 – Solve quadratic equations in one variable including those that can be used to solve contextual problems; justify solutions to quadratic equations with technology; explain solution methods and interpret solution for problems in context involving quadratic equations - A.ST.1 - Represent no more than 30 collected data points with a scatter plot using available technology; explain the meaning of the slope and *y*-intercept of a linear model; analyze relationships between two quantitative variables revealed in a scatterplot

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Deletions from Algebra 1 (2016 SOL)	**Additions to Algebra 1 (2023 SOL)**
- A.3a - Express the principal square root of a monomial algebraic expression in simplest form [Included in A2.EO.2] - A.8 – Analyze a relation to determine if a direct or inverse variation exists and represent a direct variation algebraically and graphically and an inverse variation algebraically [Direct variation included in 7.PFA.1; Direct and inverse variation included in A2.F.1d]	- A.EO.4 - Add, subtract, and multiply radicals includes numeric cube root expressions; generate equivalent numerical expressions for radicals using rational exponents, limited to rational exponents of $\frac{1}{2}$ and $\frac{1}{3}$ - A.EI.2 – Create a system of two linear inequalities in two variables to represent a contextual situation - A.EI.3 – Determine and justify if a quadratic equation has no real solutions, one real solution, or two real solutions - A.F.1 – Analyze and interpret information revealed by slope-intercept, standard, and point-slope forms of a linear function; compare and contrast characteristics of linear functions - A.F.2 - Identify the vertex (maximum and minimum) of a quadratic function; investigate, analyze, and compare functions, including quadratic and exponential functions; graph quadratic and exponential functions using transformations - A.ST.1 - Formulate and investigate questions about bivariate data using a data cycle; determine what variables could be used to explain a contextual problem or answer investigative questions; determine an appropriate method to collect a sample, including a simple random sample; describe strengths and weaknesses of a linear or quadratic model

KEY: EO = Expressions and Operations; EI = Equations and Inequalities; F = Functions; ST = Statistics; EKS = Essential Knowledge and Skills (2016); KS = Knowledge and Skills (2023); US = Understanding the Standard

Geometry *Standards of Learning* - 2023 Overview of Revisions

This overview includes a summary of the content embedded in four content strands.

Reasoning, Lines, and Transformations
- Translate, construct, and judge the validity of a logical argument and use and interpret Venn diagrams
- Analyze the relationships of parallel lines cut by a transversal
- Solve problems, including contextual problems, involving symmetry and transformation

Triangles
- Determine the relationships between the measures of angles and lengths of sides in triangles, including problems in context.
- Prove two triangles are congruent and solve contextual problems involving measured attributes of congruent triangles
- Given a triangle, use geometric constructions to create a congruent triangle
- Prove triangles are similar and solve contextual problems involving measured attributes of similar triangles
- Solve problems, including contextual problems, involving trigonometry in right triangles and applications of the Pythagorean Theorem

Polygons and Circles
- Prove and justify theorems and properties of quadrilaterals, and verify and use properties of quadrilaterals, including the relationships between the sides, angles, and diagonals, to solve problems, including those in context
- Use geometric constructions to verify properties of quadrilaterals
- Verify relationships and solve problems, including contextual problems, involving the number of sides and angles of convex polygons
- Solve problems, including those in context, by applying properties of circles
- Solve problems in the coordinate plane, including those in context, involving equations of circles

Two- and Three-Dimensional Figures
- Create models and solve problems, including those in context, involving surface area and volume of three-dimensional objects
- Determine the effects of changing one or more dimensions of a three-dimensional figure, including recognizing when two- and three-dimensional figures are similar

Comparison of Geometry Mathematics *Standards of Learning* – 2016 to 2023

2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Reasoning, Lines, and Transformations**	2023 *Standards of Learning* Knowledge and Skills (KS) **Reasoning, Lines and Transformations (RLT)**
G.1 The student will use deductive reasoning to construct and judge the validity of a logical argument consisting of a set of premises and a conclusion. This will include a) **identifying the converse, inverse, and contrapositive of a conditional statement;** b) **translating a short verbal argument into symbolic form; and** c) **determining the validity of a logical argument.** • Identify the converse, inverse, and contrapositive of a conditional statement. (a) • Translate verbal arguments into symbolic form using the symbols of formal logic. (b) • Determine the validity of a logical argument using valid forms of deductive reasoning. (c) • Determine that an argument is false using a counterexample. (c)	**G.RLT.1 The student will translate, construct, and judge the validity of a logical argument and use and interpret Venn diagrams.** a) Translate propositional statements and compound statements into symbolic form, including negations ($\sim p$, read "*not p*"), conjunctions ($p \land q$, read "*p and q*"), disjunctions ($p \lor q$, read "*p or q*"), conditionals ($p \rightarrow q$, read "*if p then q*"), and biconditionals ($p \leftrightarrow q$, read "*p if and only if q*"), including statements representing geometric relationships. b) Identify and determine the validity of the converse, inverse, and contrapositive of a conditional statement, and recognize the connection between a biconditional statement and a true conditional statement with a true converse, including statements representing geometric relationships. c) Use Venn diagrams to represent set relationships, including union, intersection, subset, and negation. d) Interpret Venn diagrams, including those representing contextual situations.

G.2 The student will use the relationships between angles formed by two lines intersected by a transversal to
a) **prove two or more lines are parallel; and**
b) **solve problems, including practical problems, involving angles formed when parallel lines are intersected by a transversal.**
	G.RLT.2 The student will analyze the relationships of parallel lines cut by a transversal.
	
	a) Prove and justify angle pair relationships formed by two parallel lines and a transversal, including
	  i) corresponding angles;
	  ii) alternate interior angles;

2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Reasoning, Lines, and Transformations**	2023 *Standards of Learning* Knowledge and Skills (KS) **Reasoning, Lines and Transformations (RLT)**
• Prove two or more lines are parallel given angle measurements expressed numerically or algebraically. (a) • Prove two lines are parallel using deductive proofs given relationships between and among angles. (a) • Solve problems by using the relationships between pairs of angles formed by the intersection of two parallel lines and a transversal including corresponding angles, alternate interior angles, alternate exterior angles, same-side (consecutive) interior angles, and same-side (consecutive) exterior angles. (b) • Solve problems, including practical problems, involving intersecting and parallel lines. (b)	iii) alternate exterior angles; iv) same-side (consecutive) interior angles; and v) same-side (consecutive) exterior angles. b) Prove two or more lines are parallel given angle measurements expressed numerically or algebraically. c) Solve problems by using the relationships between pairs of angles formed by the intersection of two parallel lines and a transversal.
G.3 The student will solve problems involving symmetry and transformation. This will include a) **investigating and using formulas for determining distance, midpoint, and slope;** b) **applying slope to verify and determine whether lines are parallel or perpendicular;** c) **investigating symmetry and determining whether a figure is symmetric with respect to a line or a point; and** d) **determining whether a figure has been translated, reflected, rotated, or dilated, using coordinate methods.** • Determine the coordinates of the midpoint or endpoint of a segment, using the midpoint formula. (a) • Use a formula to determine the slope of a line. (a) • Apply the distance formula to determine the length of a line segment when given the coordinates of the endpoints. (a)	**G.RLT.3 The student will solve problems, including contextual problems, involving symmetry and transformation.** a) Locate, count, and draw lines of symmetry given a figure, including figures in context. b) Determine whether a figure has point symmetry, line symmetry, both, or neither, including figures in context. c) Given an image or preimage, identify the transformation or combination of transformations that has/have occurred. Transformations include: i) translations; ii) reflections over any horizontal or vertical line or the lines $y = x$ or $y = -x$; iii) clockwise or counterclockwise rotations of 90°, 180°, 270°, or 360° on a coordinate grid where the center of rotation is limited to the origin; and

2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Reasoning, Lines, and Transformations**	2023 *Standards of Learning* Knowledge and Skills (KS) **Reasoning, Lines and Transformations (RLT)**

• Compare the slopes to determine whether two lines are parallel, perpendicular, or neither. (b)
• Determine whether a figure has point symmetry, line symmetry, both, or neither. (c)
• Given an image and preimage, identify the transformation or combination of transformations that has/have occurred. Transformations include:
   o a translation;
   o a reflection over any horizontal or vertical line or the lines $y = x$ or $y = -x;$
   o a clockwise or counterclockwise rotation of 90°, 180°, 270°, or 360° on a coordinate grid where the center of rotation is limited to the origin; and
   o dilations, from a fixed point on a coordinate grid.
	iv) dilations, from a fixed point on a coordinate grid.

G.4 The student will construct and justify the constructions of
a) a line segment congruent to a given line segment;
b) the perpendicular bisector of a line segment;
c) a perpendicular to a given line from a point not on the line;
d) a perpendicular to a given line at a given point on the line;
e) the bisector of a given angle;
f) an angle congruent to a given angle;
g) a line parallel to a given line through a point not on the line; and
h) an equilateral triangle, a square, and a regular hexagon inscribed in a circle.
 	
• Construct and justify the constructions of
   o a line segment congruent to a given line segment; (a)
   o the perpendicular bisector of a line segment; (b)
	[Included in G.TR.2 and G.PC.1]

2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Reasoning, Lines, and Transformations**	2023 *Standards of Learning* Knowledge and Skills (KS) **Reasoning, Lines and Transformations (RLT)**
o a perpendicular to a given line from a point not on the line; (c) o a perpendicular to a given line at a given point on the line; (d) o the bisector of a given angle; (e) o an angle congruent to a given angle; (f) o a line parallel to a given line through a point not on the given line; (g) and o an equilateral triangle, a square, and a regular hexagon inscribed in a circle. (h)	

2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Triangles**	2023 *Standards of Learning* Knowledge and Skills (KS) **Triangles (TR)**
G.5 The student, given information concerning the lengths of sides and/or measures of angles in triangles, will solve problems, including practical problems. This will include **a) ordering the sides by length, given angle measures;** **b) ordering the angles by degree measure, given side lengths;** **c) determining whether a triangle exists; and** **d) determining the range in which the length of the third side must lie.** ● Given information about the lengths of sides and/or measures of angles in triangles, solve problems, including practical problems. (a, b, c, d)	**G.TR.1 The student will determine the relationships between the measures of angles and lengths of sides in triangles, including problems in context.** a) Given the lengths of three segments, determine whether a triangle could be formed. b) Given the lengths of two sides of a triangle, determine the range in which the length of the third side must lie. c) Order the sides of a triangle by their lengths when given information about the measures of the angles.

2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Triangles**	2023 *Standards of Learning* Knowledge and Skills (KS) **Triangles (TR)**
• Order the sides of a triangle by their lengths when given information about the measures of the angles. (a) • Order the angles of a triangle by their measures when given information about the lengths of the sides. (b) • Given the lengths of three segments, determine whether a triangle could be formed. (c) • Given the lengths of two sides of a triangle, determine the range in which the length of the third side must lie. (d)	d) Order the angles of a triangle by their measures when given information about the lengths of the sides. e) Solve for interior and exterior angles of a triangle, when given two angles.
G.6 The student, given information in the form of a figure or statement, will prove two triangles are congruent. • Prove two triangles congruent given relationships among angles and sides of triangles expressed numerically or algebraically. • Prove two triangles congruent given representations in the coordinate plane and using coordinate methods (distance formula and slope formula). • Use direct proofs to prove two triangles congruent.	**G.TR.2 The student will, given information in the form of a figure or statement, prove two triangles are congruent using direct and indirect proofs, and solve problems involving measured attributes of congruent triangles.** a) Use definitions, postulates, and theorems (including Side-Side-Side (SSS); Side-Angle-Side (SAS); Angle-Side-Angle (ASA); Angle-Angle-Side (AAS); and Hypotenuse-Leg (HL)) to prove and justify two triangles are congruent. b) Use algebraic methods to prove that two triangles are congruent. c) Use coordinate methods, such as the slope formula and the distance formula, to prove two triangles are congruent. d) Given a triangle, use congruent segment, congruent angle, and/or perpendicular line constructions to create a congruent triangle (SSS, SAS, ASA, AAS, and HL).
G.7 The student, given information in the form of a figure or statement, will prove two triangles are similar.	**G.TR.3 The student will, given information in the form of a figure or statement, prove and justify two triangles are similar using**

2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Triangles**	2023 *Standards of Learning* Knowledge and Skills (KS) **Triangles (TR)**
● Prove two triangles similar given relationships among angles and sides of triangles expressed numerically or algebraically. ● Prove two triangles similar given representations in the coordinate plane and using coordinate methods (distance formula and slope formula). ● Use direct proofs to prove triangles similar.	**direct and indirect proofs, and solve problems, including those in context, involving measured attributes of similar triangles.** a) Use definitions, postulates, and theorems (including Side-Angle-Side (SAS); Side-Side-Side (SSS); and Angle-Angle (AA)) to prove and justify that triangles are similar. b) Use algebraic methods to prove that triangles are similar. c) Use coordinate methods, such as the slope formula and the distance formula, to prove two triangles are similar. d) Describe a sequence of transformations that can be used to verify similarity of triangles located in the same plane. e) Solve problems, including those in context, involving attributes of similar triangles.
G.8 The student will solve problems, including practical problems, involving right triangles. This will include applying **a) the Pythagorean Theorem and its converse;** **b) properties of special right triangles; and** **c) trigonometric ratios.** ● Solve problems, including practical problems, using right triangle trigonometry and properties of special right triangles. (a, b, c) ● Determine whether a triangle formed with three given lengths is a right triangle. (a) ● Solve for missing lengths in geometric figures, using properties of 45°-45°-90° triangles where rationalizing denominators may be necessary. (b)	**G.TR.4 The student will model and solve problems, including those in context, involving trigonometry in right triangles and applications of the Pythagorean Theorem.** a) Determine whether a triangle formed with three given lengths is a right triangle. b) Find and verify trigonometric ratios using right triangles. c) Model and solve problems, including those in context, involving right triangle trigonometry (sine, cosine, and tangent ratios). d) Solve problems using the properties of special right triangle. e) Solve for missing lengths in geometric figures, using properties of 45°-45°-90° triangles, where rationalizing denominators may be necessary.

2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Triangles**	2023 *Standards of Learning* Knowledge and Skills (KS) **Triangles (TR)**
• Solve for missing lengths in geometric figures, using properties of 30°-60°-90° triangles where rationalizing denominators may be necessary. (b). • Solve problems, including practical problems, involving right triangles with missing side lengths or angle measurements, using sine, cosine, and tangent ratios. (c)	f) Solve for missing lengths in geometric figures, using properties of 30°-60°-90° triangles, where rationalizing denominators may be necessary. g) Solve problems, including those in context, involving right triangles using the Pythagorean Theorem and its converse, including recognizing Pythagorean Triples.

2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Polygons and Circles**	2023 *Standards of Learning* Knowledge and Skills (KS) **Polygons and Circles (PC)**
G.9 The student will verify and use properties of quadrilaterals to solve problems, including practical problems. • Solve problems, including practical problems, using the properties specific to parallelograms, rectangles, rhombi, squares, isosceles trapezoids, and trapezoids. • Prove that quadrilaterals have specific properties, using coordinate and algebraic methods, such as the distance formula, slope, and midpoint formula. • Prove the properties of quadrilaterals, using direct proofs.	**G.PC.1 The student will prove and justify theorems and properties of quadrilaterals, and verify and use properties of quadrilaterals to solve problems, including the relationships between the sides, angles, and diagonals.** a) Solve problems, using the properties specific to parallelograms, rectangles, rhombi, squares, isosceles trapezoids, and trapezoids. b) Prove and justify that quadrilaterals have specific properties, using coordinate and algebraic methods, such as the slope formula, the distance formula, and the midpoint formula. c) Prove and justify theorems and properties of quadrilaterals using deductive reasoning. d) Use congruent segment, congruent angle, angle bisector, perpendicular line, and/or parallel line constructions to verify properties of quadrilaterals.

2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Polygons and Circles**	2023 *Standards of Learning* Knowledge and Skills (KS) **Polygons and Circles (PC)**
G.10 The student will solve problems, including practical problems, involving angles of convex polygons. This will include determining the **a) sum of the interior and/or exterior angles;** **b) measure of an interior and/or exterior angle; and** **c) number of sides of a regular polygon.** ● Solve problems, including practical problems, involving angles of convex polygons. (a, b, c) ● Determine the sum of the measures of the interior and exterior angles of a convex polygon. (a) ● Determine the measure of each interior and exterior angle of a regular polygon. (b) ● Determine angle measures of a regular polygon in a tessellation. (b) ● Determine the number of sides of a regular polygon, given the measures of interior or exterior angles of the polygon. (c)	**G.PC.2 The student will verify relationships and solve problems involving the number of sides and angles of convex polygons.** a) Solve problems involving the number of sides of a regular polygon given the measures of the interior and exterior angles of the polygon. b) Justify the relationship between the sum of the measures of the interior and exterior angles of a convex polygon and solve problems involving the sum of the measures of the angles. c) Justify the relationship between the measure of each interior and exterior angle of a regular polygon and solve problems involving the measures of the angles.
G.11 The student will solve problems, including practical problems, by applying properties of circles. This will include determining **a) angle measures formed by intersecting chords, secants, and/or tangents;** **b) lengths of segments formed by intersecting chords, secants, and/or tangents;** **c) arc length; and** **d) area of a sector.**	**G.PC.3 The student will solve problems, including those in context, by applying properties of circles.** a) Determine the proportional relationship between the arc length or area of a sector and other parts of a circle. b) Solve for arc measures and angles in a circle formed by central angles. c) Solve for arc measures and angles in a circle involving inscribed angles. d) Calculate the length of an arc of a circle.

2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Polygons and Circles**	2023 *Standards of Learning* Knowledge and Skills (KS) **Polygons and Circles (PC)**
- Solve problems, including practical problems, by applying properties of circles. (a, b, c, d) - Determine angle measures and arc measures associated with - two intersecting chords; - two intersecting secants; - an intersecting secant and tangent; - two intersecting tangents; and - central and inscribed angles. (a) - Determine segment lengths associated with: - two intersecting chords; - two intersecting secants; - an intersecting secant and tangent; and - two intersecting tangents. (b) - Calculate the length of an arc of a circle. (c) - Calculate the area of a sector. (d)	e) Calculate the area of a sector of a circle. f) Apply arc length or sector area to solve for an unknown measurement of the circle including the radius, diameter, arc measure, central angle, arc length, or sector area.
G.12 The student will solve problems involving equations of circles. - Given a graph or the equation of a circle in standard form, identify the coordinates of the center of the circle. - Given the coordinates of the endpoints of a diameter of a circle, determine the coordinates of the center of the circle. - Given a graph or the equation of a circle in standard form, identify the length of the radius or diameter of the circle. - Given the coordinates of the endpoints of the diameter of a circle, determine the length of the radius or diameter of the circle.	**G.PC.4 The student will solve problems in the coordinate plane involving equations of circles.** a) Derive the equation of a circle given the center and radius using the Pythagorean Theorem. b) Solve problems in the coordinate plane involving equations of circles: i) given a graph or the equation of a circle in standard form, identify the coordinates of the center of the circle; ii) given the coordinates of the endpoints of a diameter of a circle, determine the coordinates of the center of the circle.

2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Polygons and Circles**	2023 *Standards of Learning* Knowledge and Skills (KS) **Polygons and Circles (PC)**
- Given the coordinates of the center and the coordinates of a point on the circle, determine the length of the radius or diameter of the circle. - Given the coordinates of the center and length of the radius of a circle, identify the coordinates of a point(s) on the circle. - Determine the equation of a circle given: - a graph of a circle with a center with coordinates that are integers; - coordinates of the center and a point on the circle; - coordinates of the center and the length of the radius or diameter; or - coordinates of the endpoints of a diameter.	iii) given a graph or the equation of a circle in standard form, identify the length of the radius or diameter of the circle. iv) given the coordinates of the endpoints of the diameter of a circle, determine the length of the radius or diameter of the circle. v) given the coordinates of the center and the coordinates of a point on the circle, determine the length of the radius or diameter of the circle; and vi) given the coordinates of the center and length of the radius of a circle, identify the coordinates of a point(s) on the circle. c) Determine the equation of a circle given: i) a graph of a circle with a center with coordinates that are integers; ii) coordinates of the center and a point on the circle; iii) coordinates of the center and the length of the radius or diameter; and iv) coordinates of the endpoints of a diameter.

2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Two- and Three- Dimensional Figures**	2023 *Standards of Learning* Knowledge and Skills (KS) **Two- and Three-Dimensional Figures (DF)**
G.13 The student will use surface area and volume of three-dimensional objects to solve practical problems.	**G.DF.1 The student will create models and solve problems, including those in context, involving surface area and volume of**

2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Two- and Three- Dimensional Figures**	2023 *Standards of Learning* Knowledge and Skills (KS) **Two- and Three-Dimensional Figures (DF)**
• Determine the surface area of cylinders, prisms, pyramids, cones, hemispheres, and spheres, using the appropriate formulas. • Determine the volume of cylinders, prisms, pyramids, cones, hemispheres, and spheres, using the appropriate formulas. • Solve problems including practical problems, involving surface area and volume of cylinders, prisms, pyramids, cones, hemispheres, and spheres, as well as composite three-dimensional figures. • Solve problems, including practical problems, involving the lateral area of circular cylinders, prisms, and regular pyramids. • Given information about a three-dimensional figure such as length of a side, area of a face, or volume, determine missing information.	**rectangular and triangular prisms, cylinders, cones, pyramids, and spheres.** a) Identify the shape of a two-dimensional cross section of a three-dimensional figure. b) Create models and solve problems, including those in context, involving surface area of three-dimensional figures, as well as composite three-dimensional figures. c) Solve multistep problems, including those in context, involving volume of three-dimensional figures, as well as composite three-dimensional figures. d) Determine unknown measurements of three-dimensional figures using information such as length of a side, area of a face, or volume.
G.14 The student will apply the concepts of similarity to two- or three-dimensional geometric figures. This will include a) **comparing ratios between lengths, perimeters, areas, and volumes of similar figures;** b) **determining how changes in one or more dimensions of a figure affect area and/or volume of the figure;** c) **determining how changes in area and/or volume of a figure affect one or more dimensions of the figure; and** d) **solving problems, including practical problems, about similar geometric figures.** • Compare ratios between side lengths, perimeters, areas, and volumes, given two similar figures. (a)	**G.DF.2 The student will determine the effect of changing one or more dimensions of a three-dimensional geometric figure and describe the relationship between the original and changed figure.** a) Describe how changes in one or more dimensions of a figure affect other derived measures (perimeter, area, total surface area, and volume) of the figure. b) Describe how changes in surface area and/or volume of a figure affect the measures of one or more dimensions of the figure. c) Solve problems, including those in context, involving changing the dimensions or derived measures of a three-dimensional figure. d) Compare ratios between side lengths, perimeters, areas, and volumes of similar figures. e) Recognize when two- and three-dimensional figures are similar and

2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Two- and Three- Dimensional Figures**	2023 *Standards of Learning* Knowledge and Skills (KS) **Two- and Three-Dimensional Figures (DF)**
● Describe how changes in one or more dimensions affect other derived measures (perimeter, area, surface area, and volume) of a figure. (b) ● Describe how changes in one or more measures (perimeter, area, surface area, and volume) affect other measures of a figure. (c) ● Solve real-world problems involving measured attributes of similar figures. (d)	solve problems, including those in context, involving attributes of similar geometric figures.

2023 Geometry Mathematics SOL – Summary of Changes

Geometry (2016 SOL to 2023 SOL Numbering)	Parameter Changes/Clarifications (2023 SOL)
- G.1 → G.RLT.1 - G.2 → G.RLT.2 - G.3a,b → [Included in G.TR.2, G.TR.3, G.PC.1, and G.PC.4] - G.3c,d → G.RLT.3 - G.4a-g → [Included in G.TR.2 and G.PC.1] - G.4h → [Deleted] - G.5 → G.TR.1 - G.6 → G.TR.2 - G.7 → G.TR.3 - G.8 → G.TR.4 - G.9 → G.PC.1 - G.10 → G.PC.2 - G.11 → G.PC.3 - G.12 → G.PC.4 - G.13 → G.DF.1 - G.14 → G.DF.2	- G.RLT.1 – Translate logic statements includes statements representing geometric relationships - G.RLT.2 – Prove and justify angle pair relationships formed by two parallel lines and a transversal - G.RLT.3 – Include figures in context when determining whether a figure has point symmetry, line symmetry, both, or neither - G.TR.2 - Use Hypotenuse-Leg (HL) to prove triangles are congruent - G.TR.4 - Recognize Pythagorean Triples - G.TR.4 – Model and solve problems involving right triangle trigonometry - G.PC.1 – Prove and justify theorems and properties of quadrilaterals - G.PC.2 – Verify and justify angle and side relationships in convex polygons - G.PC.4 – Solve problems in the coordinate plane involving circle equations - G.DF.1 – Create models and solve problems involving the surface area of three-dimensional figures, as well as composite figures - G.DF.1 – Solve multistep problems, including those in context, involving volume of three-dimensional figures, as well as composite figures

Deletions from Geometry (2016 SOL)	Additions to Geometry (2023 SOL)
- G.4h – Construction and justify the constructions of an equilateral triangle, a square, and a regular hexagon inscribed in a circle [Deleted] - G.10b [EKS] - Determine angle measures of a regular polygon in a tessellation [Deleted] - G.11b [EKS] - Find lengths of segments and non-central angle measures in a circle formed by intersecting chords, secants, and/or tangents [Deleted]	- G.RLT.1 - Included recognizing the relationship between a biconditional statement and a true conditional statement with a true converse; added Venn diagrams to represent set relationships and interpret Venn diagrams, including those representing situations in context - G.RLT.3 – Locate, count, and draw lines of symmetry given a figure, including figures in context - G.TR.1 – Solve for interior and exterior angles of a triangle, when given two angles - G.TR.2 – Given a triangle, use constructions to create a congruent triangle - G.TR.3 – Describe a sequence of transformations that can be used to verify similarity of triangles located in the same plane; solve problems involving attributes of similar figures, including problems in context - G.TR.4 – Find and verify trigonometric ratios using right triangles - G.PC.1 - Use constructions to verify properties of quadrilaterals - G.PC.3 - Determine the proportional relationship between the arc length or area of a sector and other parts of a circle; apply arc length or sector area to solve for an unknown measurement - G.PC.4 – Derive the equation of a circle given the center and radius using the Pythagorean Theorem - G.DF.1 – Identify the shape of a two-dimensional cross section of a three-dimensional figure - G.DF.2 – Recognize when two- and three-dimensional figures are similar

KEY: RLT = Reasoning, Lines, and Transformations (2023); TR = Triangles (2023); PC = Polygons and Circles (2023); DF = Two-Dimensional and Three-Dimensional Figures (2023); EKS = Essential Knowledge and Skills (2016); KS = Knowledge and Skills (2023); US = Understanding the Standard

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Algebra, Functions, and Data Analysis *Standards of Learning* - 2023 Overview of Revisions

This overview includes a summary of the content embedded in two content strands.

Algebra and Functions

- Investigate, analyze, and compare linear, quadratic, and exponential function families, algebraically and graphically, using transformations
- Investigate and analyze characteristics of the graphs of linear, quadratic, exponential, and piecewise-defined functions
- Represent and interpret contextual situations with constraints that require optimization using linear programming techniques, including systems of linear equations or inequalities, solving graphically and when appropriate, algebraically

Data Analysis

- Apply the data cycle (formulate questions; collect or acquire data; organize and represent data; and analyze data and communicate results) with a focus on representing bivariate data in scatterplots and determining the curve of best fit using linear, quadratic, and exponential functions
- Apply the data cycle (formulate questions; collect or acquire data; organize and represent data; and analyze data and communicate results) with a focus on the design and implementation of an experiment and/or survey
- Calculate and interpret probabilities, including those arising from contextual situations
- Describe and apply the properties of normal distribution, including those that arise from contextual situations

Comparison of AFDA Mathematics *Standards of Learning* – 2016 to 2023

2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Algebra and Functions**	2023 *Standards of Learning* Knowledge and Skills (KS) **Algebra and Functions (AF)**
AFDA.1 The student will investigate and analyze linear, quadratic, exponential, and logarithmic function families, and their characteristics. Key concepts include **a) domain and range;** **b) intervals on which a function is increasing or decreasing;** **c) absolute maxima and minima;** **d) zeros;** **e) intercepts;** **f) values of a function for elements in its domain;** **g) connections between and among multiple representations of functions using verbal descriptions, tables, equations, and graphs;** **h) end behavior; and** **i) vertical and horizontal asymptotes.** • Identify the domain, range, zeros, and intercepts of a function presented algebraically or graphically. Domains may be limited by problem context or in graphical representations. (a, d, e) • Identify intervals on which the function is increasing or decreasing. (b) • Identify the location and value of the absolute maximum and absolute minimum of a function over the domain of the function graphically or by using a graphing utility. (c) • For any x value in the domain of f, determine f(x). (f)	**AFDA.AF.2 The student will investigate and analyze characteristics of the graphs of linear, quadratic, exponential, and piecewise-defined functions.** a) Determine the domain and range of a function given a graphical representation, including those limited by contexts. b) Identify intervals on a graph for which a function is increasing, decreasing, or constant. c) Given a graph, identify the location and value of the absolute maximum and absolute minimum of a function over the domain of a function. d) Given a graph, determine the zeros and intercepts of a function. e) Describe and recognize the connection between points on the graph and the value of a function. f) Describe the end behavior of a function given its graph. g) Identify horizontal and/or vertical asymptotes from the graph of a function, if they exist. h) Describe and relate the characteristics of the graphs of linear, quadratic, exponential, and piecewise-defined functions, including those in contextual situations.

2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Algebra and Functions**	2023 *Standards of Learning* Knowledge and Skills (KS) **Algebra and Functions (AF)**
• Represent relations and functions using verbal descriptions, tables, equations, and graphs. Given one representation, represent the relation in another form. (g) • Detect patterns in data and represent arithmetic and geometric patterns algebraically. (g) • Describe the end behavior of a function. (h) • Determine the equations of the horizontal asymptote of an exponential function and the vertical asymptote of a logarithmic function. (i) • Investigate and analyze characteristics and multiple representations of functions with a graphing utility. (a, b, c, d, e, f, g, h, i)	
AFDA.2 The student will use knowledge of transformations to write an equation, given the graph of a linear, quadratic, exponential, and logarithmic function. • Write an equation of a line when given the graph of a line. • Recognize graphs of parent functions for linear, quadratic, exponential and logarithmic functions. • Write the equation of a linear, quadratic, exponential, or logarithmic function in vertex form, given the graph of the parent function and transformation information. • Describe the transformation from the parent function given the equation written in vertex form or the graph of the function. • Given the equation of a function, recognize the parent function and transformation to graph the given function.	**AFDA.AF.1 The student will investigate, analyze, and compare linear, quadratic, and exponential function families, algebraically and graphically, using transformations.** a) Identify graphs and equations of parent functions for linear, quadratic, and exponential function families. b) Describe the transformation from the parent function given the equation or the graph of the function. c) Determine and analyze whether a linear, quadratic, or exponential function best models a given representation, including those in context. d) Write the equation of a linear, quadratic, or exponential function, given a graph, using transformations of the parent function.

2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Algebra and Functions**	2023 *Standards of Learning* Knowledge and Skills (KS) **Algebra and Functions (AF)**
● Recognize the vertex of a parabola given a quadratic equation in vertex form or graphed. ● Describe the parent function represented by a scatterplot.	e) Use a graphical or algebraic representation of a function to solve problems within a context, graphically and algebraically, when appropriate. f) Graph a function given the equation of a function, using transformations of the parent function. Use technology to verify transformations of functions. g) Compare and contrast linear, quadratic, and exponential functions using multiple representations (e.g., graphs, tables, equations, verbal descriptions).
AFDA.3 The student will collect and analyze data, determine the equation of the curve of best fit in order to make predictions, and solve practical problems using models of linear, quadratic, and exponential functions. ● Determine an equation for the curve of best fit, given a set of no more than 20 data points in a table, on a graph, or practical situation. ● Make predictions, using data, scatterplots, or the equation of the curve of best fit. ● Solve practical problems involving an equation of the curve of best fit. ● Evaluate the reasonableness of a mathematical model of a practical situation.	**[Moved to AFDA.DA.1]**
AFDA.4 The student will use multiple representations of functions for analysis, interpretation, and prediction.	**[Moved to AFDA.AF.1]**

2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Algebra and Functions**	2023 *Standards of Learning* Knowledge and Skills (KS) **Algebra and Functions (AF)**
• Given an equation, graph a linear, quadratic, exponential or logarithmic function. • Make predictions given a table of values, a graph, or an algebraic formula. • Describe relationships between data represented in a table, in a scatterplot, and as elements of a function. • Determine the appropriate representation of data derived from • real-world situations. • Analyze and interpret the data in context of the practical situation. • Use a graphing utility to graph, analyze, interpret, and make predictions.	
AFDA.5 The student will determine optimal values in problem situations by identifying constraints and using linear programming techniques. • Model practical problems with systems of linear inequalities. • Solve systems of no more than four linear inequalities with pencil and paper and using a graphing utility. • Solve systems of no more than four equations algebraically and graphically. • Identify the feasible region of a system of linear inequalities. • Identify the coordinates of the vertices of a feasible region. • Determine and describe the maximum or minimum value for the function defined over a feasible region.	**AFDA.AF.3 The student will represent and interpret contextual situations with constraints that require optimization using linear programming techniques, including systems of linear equations or inequalities, solving graphically and when appropriate, algebraically.** a) Represent and interpret contextual problems requiring optimization with systems of linear equations or inequalities. b) Solve systems of no more than four equations or inequalities graphically and when appropriate, algebraically. c) Identify the feasible region of a system of linear inequalities. d) Identify the coordinates of the vertices of a feasible region. e) Determine and describe the maximum or minimum value for the function defined over a feasible region.

2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Algebra and Functions**	2023 *Standards of Learning* Knowledge and Skills (KS) **Algebra and Functions (AF)**
	f) Interpret the validity of possible solution(s) algebraically, graphically, using technology, and in context and justify the reasonableness of the answer(s) or the solution method in context.

2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Data Analysis**	2023 *Standards of Learning* Knowledge and Skills (KS) **Data Analysis (DA)**
[Moved from AFDA.3]	**AFDA.DA.1 The student will apply the data cycle (formulate questions; collect or acquire data; organize and represent data; and analyze data and communicate results) with a focus on representing bivariate data in scatterplots and determining the curve of best fit using linear, quadratic, and exponential functions.** a) Formulate investigative questions that require the collection or acquisition of bivariate data, where exactly two of the variables are quantitative. b) Collect or acquire bivariate data from a representative sample to answer an investigative question. c) Represent bivariate data with a scatterplot using technology and describe how the variables are related in terms of the given context. d) Make predictions, decisions, and critical judgments using data, scatterplots, or the equation(s) of the mathematical model.

2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Data Analysis**	2023 *Standards of Learning* Knowledge and Skills (KS) **Data Analysis (DA)**
[Moved from AFDA.8]	**AFDA.DA.2 The student will apply the data cycle (formulate questions; collect or acquire data; organize and represent data; and analyze data and communicate results) with a focus on the design and implementation of an experiment and/or observational study.** a) Formulate questions that can be addressed with data and assess the type of data relevant to the question (e.g., quantitative versus categorical). b) Investigate, describe, and determine best sampling techniques, such as simple random sampling, stratified sampling, and cluster sampling. c) Plan and conduct an experiment and/or observational study. The experimental design should address control, randomization, and minimization of experimental error. d) Collect or acquire data to answer a statistical question. e) Recognize that data may contain errors, have missing values, or may be biased, and make decisions about how to account for these issues. f) Identify biased sampling methods. g) Given a plan for an observational study, identify possible sources of bias, and describe ways to reduce bias. h) Select, create, and use appropriate visual representations of data to brainstorm solutions. i) Use appropriate statistical methods to analyze data. j) Communicate the description of an experiment and/or observational study, the resulting data, analysis, and the validity of the conclusions.

2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Data Analysis**	2023 *Standards of Learning* Knowledge and Skills (KS) **Data Analysis (DA)**
AFDA.6 The student will calculate probabilities. Key concepts include **a) conditional probability;** **b) dependent and independent events;** **c) mutually exclusive events;** **d) counting techniques (permutations and combinations); and** **e) Law of Large Numbers.** • Analyze, interpret, and make predictions based on theoretical probability within practical context. (a, b, c, e) • Determine conditional probabilities for dependent, independent, and mutually exclusive events. (a, b, c) • Represent and calculate probabilities using Venn diagrams and probability trees. (a) • Define and give contextual examples of complementary, dependent, independent, and mutually exclusive events. (b, c) • Given two or more events in a problem setting, determine whether the events are complementary, dependent, independent, and/or mutually exclusive. (b, c) • Compare and contrast permutations and combinations, including those occurring in practical situations. (d) • Calculate the number of permutations of n objects taken r at a time, without repetition. (d) • Calculate the number of combinations of n objects taken r at a time, without repetition. (d)	**AFDA.DA.3 The student will calculate and interpret probabilities, including those in contextual situations.** a) Analyze, interpret, and make predictions based on theoretical probability. b) Calculate conditional probabilities for dependent, independent, and mutually exclusive events. c) Represent and calculate probabilities using Venn diagrams, probability trees, organized lists, two-way tables, simulations, or other probability models. d) Interpret probabilities from simulations or experiments to make informed decisions and justify the rationale. e) Define and give contextual examples of complementary, dependent, independent, and mutually exclusive events. f) Given two or more events in a problem setting, determine whether the events are complementary, dependent, independent, and/or mutually exclusive. g) Compare and contrast permutations and combinations, including those in contextual situations. h) Calculate the number of permutations of n objects taken r at a time, without repetition. i) Calculate the number of combinations of n objects taken r at a time, without repetition.

2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Data Analysis**	2023 *Standards of Learning* Knowledge and Skills (KS) **Data Analysis (DA)**
AFDA.7 The student will **a) identify and describe properties of a normal distribution;** **b) interpret and compare z-scores for normally distributed data; and** **c) apply properties of normal distributions to determine probabilities associated with areas under the standard normal curve.** • Identify the properties of a normal distribution. (a) • Describe how the standard deviation and the mean affect the graph of the normal distribution. (a) • Given standard deviation and mean, calculate, and interpret the • z-score for a data point. (b) • Compare two sets of normally distributed data using a standard normal distribution and z-scores, given mean and standard deviation. (b) • Represent probability as area under the curve of a standard normal distribution. (c) • Use a graphing utility or a table of Standard Normal Probabilities to determine probabilities associated with areas under the standard normal curve. (c) • Use a graphing utility to investigate, represent, and determine relationships between a normally distributed data set and its descriptive statistics. (a, b, c)	**AFDA.DA.4 The student will describe and apply the properties of normal distribution, including those in contextual situations.** a) Identify and describe the properties of a normal distribution. b) Determine when the normal distribution is a reasonable representation of the data. c) Describe how the mean and the standard deviation affect the graph of the normal distribution. d) Calculate and interpret the z-score for a data point, given the mean and the standard deviation. e) Compare two sets of normally distributed data using a standard normal distribution and z-scores, given the mean and the standard deviation. f) Represent probability as the area under the curve of a standard normal distribution. g) Determine probabilities associated with areas under the standard normal curve, using technology or a table of Standard Normal Probabilities. h) Investigate, represent, and determine relationships between a normally distributed data set and its descriptive statistics.
AFDA.8 The student will design and conduct an experiment/survey. Key concepts include **a) sample size;**	**[Moved to AFDA.DA.2]**

2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Data Analysis**	2023 *Standards of Learning* Knowledge and Skills (KS) **Data Analysis (DA)**
b) sampling technique; **c) controlling sources of bias and experimental error;** **d) data collection; and** **e) data analysis and reporting.** ● Investigate and describe sampling techniques, such as simple random sampling, stratified sampling, and cluster sampling. (a, b) ● Determine which sampling technique is best, given a particular context. (b) ● Identify biased sampling methods. (c) ● Given a plan for a survey, identify possible sources of bias, and describe ways to reduce bias. (c) ● Plan and conduct an experiment or survey. The experimental design should address control, randomization, and minimization of experimental error. (a, b, c, d) ● Compare and contrast controlled experiments, observational studies, and the conclusions one may draw from each. (e) ● Write a report describing the experiment/survey and the resulting data and analysis. (e)	

2023 Algebra, Functions, and Data Analysis (AFDA) Mathematics SOL – Summary of Changes

AFDA (2016 SOL to 2023 SOL Numbering)	Parameter Changes/Clarifications (2023 SOL)
- AFDA.1a-i → AFDA.AF.2 - AFDA.2 → AFDA.AF.1 - AFDA.3 → AFDA.DA.1 - AFDA.4 → AFDA.AF.1 - AFDA.5 → AFDA.AF.3 - AFDA.6a-e → AFDA.DA.3 - AFDA.7a-c → AFDA.DA.4 - AFDA.8a-e → AFDA.DA.2	- AFDA.DA.1 - Removed the limitation of "no more than 20 data points to determine the curve of best fit" to allow for the analysis of larger data sets - AFDA.DA.1 - Includes the use of a data cycle to formulate and investigate questions about bivariate data with scatterplots and using mathematical models - AFDA.DA.2 - Includes the use of a data cycle to formulate and investigate questions and design surveys and experiments

Deletions from AFDA (2016 SOL)	Additions to AFDA (2023 SOL)
- AFDA.2 - Investigate and analyze logarithmic function families and their characteristics [Included in Algebra 2] - AFDA.1 - Use knowledge of transformations to write an equation, given the graph of a logarithmic function [Included in Algebra 2]	- AFDA.AF.2 - Investigate and analyze the characteristics of piecewise-defined functions - AFDA.DA.3 - Interpret probabilities from simulations or experiments to make informed decisions and justify the rationale

KEY: AF = Algebra and Functions; DA = Data Analysis; EKS = Essential Knowledge and Skills (2016); KS = Knowledge and Skills (2023); US = Understanding the Standard

Algebra 2 Mathematics *Standards of Learning* - 2023 Overview of Revisions

This overview includes a summary of the content embedded in four content strands.

Expressions and Operations
- Perform operations on and simplify rational expressions
- Perform operations on and simplify radical expressions
- Perform operations on and factor polynomial expressions in one and two variables
- Perform operations on complex numbers

Equations and Inequalities
- Represent, solve, and interpret the solution to absolute value equations and inequalities in one variable
- Represent, solve, and interpret the solution to quadratic equations in one variable over the set of complex numbers and solve quadratic, inequalities in one variable
- Represent, solve, and interpret the solution to an equation containing rational algebraic expressions
- Represent, solve, and interpret the solution to an equation containing a radical expression
- Solve systems of equations containing a quadratic expression
- Represent, solve, and interpret the solution to a polynomial equation

Functions
- Investigate, analyze, and compare square root, cube root, rational, exponential, and logarithmic function families, algebraically and graphically, using transformations
- Investigate and analyze characteristics of square root, cube root, rational, polynomial, exponential, logarithmic, and piecewise-defined functions algebraically and graphically

Statistics
- Apply the data cycle (formulate questions; collect or acquire data; organize and represent data; and analyze data and communicate results) with a focus on univariate quantitative data represented by a smooth curve, including a normal curve
- Apply the data cycle (formulate questions; collect or acquire data; organize and represent data; and analyze data and communicate results) with a focus on representing bivariate data in scatterplots and determining the curve of best fit using linear, quadratic, exponential, or a combination of these functions
- Compute and distinguish between permutations and combinations

Comparison of Algebra 2 Mathematics *Standards of Learning* – 2016 to 2023

2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Expressions and Operations**	2023 *Standards of Learning* Knowledge and Skills (KS) **Expressions and Operations (EO)**
AII.1 The student will **a) add, subtract, multiply, divide, and simplify rational algebraic expressions;** ● Add, subtract, multiply, and divide rational algebraic expressions. (a) ● Simplify a rational algebraic expression with monomial or binomial factors. Algebraic expressions should be limited to linear and quadratic expressions. (a) ● Recognize a complex algebraic fraction, and simplify it as a quotient or product of simple algebraic fractions. (a)	**A2.EO.1 The student will perform operations on and simplify rational expressions.** a) Add, subtract, multiply, or divide rational algebraic expressions, simplifying the result. b) Justify and determine equivalent rational algebraic expressions with monomial and binomial factors. Algebraic expressions should be limited to linear and quadratic expressions. c) Recognize a complex algebraic fraction and simplify it as a product or quotient of simple algebraic fractions. d) Represent and demonstrate equivalence of rational expressions written in different forms.
AII.1 The student will **b) add, subtract, multiply, divide, and simplify radical expressions containing rational numbers and variables, and expressions containing rational exponents; and** ● Simplify radical expressions containing positive rational numbers and variables. (b) ● Convert between radical expressions and expressions containing rational exponents. (b) ● Add and subtract radical expressions. (b) ● Multiply and divide radical expressions. Simplification may include rationalizing denominators. (b)	**A2.EO.2 The student will perform operations on and simplify radical expressions.** a) Simplify and determine equivalent radical expressions that include numeric and algebraic radicands. b) Add, subtract, multiply, and divide radical expressions that include numeric and algebraic radicands, simplifying the result. Simplification may include rationalizing the denominator. c) Convert between radical expressions and expressions containing rational exponents.

2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Expressions and Operations**	2023 *Standards of Learning* Knowledge and Skills (KS) **Expressions and Operations (EO)**
AII.1 The student will **c) factor polynomials completely in one or two variables.** ● Factor polynomials in one or two variables with no more than four terms completely over the set of integers. Factors of the polynomial should be constant, linear, or quadratic. (c) ● Verify polynomial identities including the difference of squares, sum and difference of cubes, and perfect square trinomials. (c)	**A2.EO.3 The student will perform operations on polynomial expressions and factor polynomial expressions in one and two variables.** a) Determine sums, differences, and products of polynomials in one and two variables. b) Factor polynomials completely in one and two variables with no more than four terms over the set of integers. c) Determine the quotient of polynomials in one and two variables, using monomial, binomial, and factorable trinomial divisors. d) Represent and demonstrate equality of polynomial expressions written in different forms and verify polynomial identities including the difference of squares, sum and difference of cubes, and perfect square trinomials.
AII.2 The student will perform operations on complex numbers and express the results in simplest form using patterns of the powers of *i*. ● Recognize that the square root of –1 is represented as *i*. ● Simplify radical expressions containing negative rational numbers and express in $a + bi$ form. ● Simplify powers of *i*. ● Add, subtract, and multiply complex numbers.	**A2.EO.4 The student will perform operations on complex numbers.** a) Explain the meaning of *i*. b) Identify equivalent radical expressions containing negative rational numbers and expressions in $a + bi$ form. c) Apply properties to add, subtract, and multiply complex numbers.

2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Equations and Inequalities**	2023 *Standards of Learning* Knowledge and Skills (KS) **Equations and Inequalities (EI)**
AII.3 The student will solve **a) absolute value linear equations and inequalities;** ● Solve absolute value linear equations or inequalities in one variable algebraically. (a) ● Represent solutions to absolute value linear inequalities in one variable graphically. (a) ● Solve equations and verify algebraic solutions using a graphing utility. (a, b, c, d)	**A2.EI.1 The student will represent, solve, and interpret the solution to absolute value equations and inequalities in one variable.** a) Create an absolute value equation in one variable to model a contextual situation. b) Solve an absolute value equation in one variable algebraically and verify the solution graphically. c) Create an absolute value inequality in one variable to model a contextual situation. d) Solve an absolute value inequality in one variable and represent the solution set using set notation, interval notation, and using a number line. e) Verify possible solution(s) to absolute value equations and inequalities in one variable algebraically, graphically, and with technology to justify the reasonableness of answer(s). Explain the solution method and interpret solutions for problems given in context.
AII.3 The student will solve **b) quadratic equations over the set of complex numbers;** ● Solve a quadratic equation over the set of complex numbers algebraically. (b) ● Calculate the discriminant of a quadratic equation to determine the number and type of solutions. (b) ● Solve equations and verify algebraic solutions using a graphing utility. (a, b, c, d)	**A2.EI.2 The student will represent, solve, and interpret the solution to quadratic equations in one variable over the set of complex numbers and solve quadratic inequalities in one variable.** a) Create a quadratic equation or inequality in one variable to model a contextual situation. b) Solve a quadratic equation in one variable over the set of complex numbers algebraically. c) Determine the solution to a quadratic inequality in one variable over the set of real numbers algebraically.

2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Equations and Inequalities**	2023 *Standards of Learning* Knowledge and Skills (KS) **Equations and Inequalities (EI)**
	d) Verify possible solution(s) to quadratic equations or inequalities in one variable algebraically, graphically, and with technology to justify the reasonableness of answer(s). Explain the solution method and interpret solutions for problems given in context.
AII.3 The student will solve **c) equations containing rational algebraic expressions;** ● Solve rational equations with real solutions containing factorable algebraic expressions algebraically and graphically. Algebraic expressions should be limited to linear and quadratic expressions. ● Solve equations and verify algebraic solutions using a graphing utility.	**A2.EI.4 The student will represent, solve, and interpret the solution to an equation containing rational algebraic expressions.** a) Create an equation containing a rational expression to model a contextual situation. b) Solve rational equations with real solutions containing factorable algebraic expressions algebraically and graphically. Algebraic expressions should be limited to linear and quadratic expressions. c) Verify possible solution(s) to rational equations algebraically, graphically, and with technology to justify the reasonableness of answer(s). Explain the solution method and interpret solutions for problems given in context. d) Justify why a possible solution to an equation containing a rational expression might be extraneous.
AII.3d The student will solve **d) equations containing radical expressions.** ● Solve an equation containing no more than one radical expression algebraically and graphically. (d) ● Solve equations and verify algebraic solutions using a graphing utility. (d)	**A2.EI.5 The student will represent, solve, and interpret the solution to an equation containing a radical expression.** a) Solve an equation containing no more than one radical expression algebraically and graphically. b) Verify possible solution(s) to radical equations algebraically, graphically, and with technology, to justify the reasonableness

2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Equations and Inequalities**	2023 *Standards of Learning* Knowledge and Skills (KS) **Equations and Inequalities (EI)**
● Solve equations and verify algebraic solutions using a graphing utility. (a, b, c, d)	of answer(s). Explain the solution method and interpret solutions for problems given in context. c) Justify why a possible solution to an equation with a square root might be extraneous.
AII.4 The student will solve systems of linear-quadratic and quadratic-quadratic equations, algebraically and graphically. ● Determine the number of solutions to a linear-quadratic and quadratic-quadratic system of equations in two variables. ● Solve a linear-quadratic system of two equations in two variables algebraically and graphically. ● Solve a quadratic-quadratic system of two equations in two variables algebraically and graphically. ● Solve systems of equations and verify solutions of systems of equations with a graphing utility.	**A2.EI.3 The student will solve a system of equations in two variables containing a quadratic expression.** a) Create a linear-quadratic or quadratic-quadratic system of equations to model a contextual situation. b) Determine the number of solutions to a linear-quadratic and quadratic-quadratic system of equations in two variables. c) Solve a linear-quadratic and quadratic-quadratic system of equations algebraically and graphically, including situations in context. d) Verify possible solution(s) to linear-quadratic or quadratic-quadratic system of equations algebraically, graphically, and with technology to justify the reasonableness of answer(s). Explain the solution method and interpret solutions for problems given in context.
AII.8 The student will investigate and describe the relationships among solutions of an equation, zeros of a function, *x*-intercepts of a graph, and factors of a polynomial expression. ● Define a polynomial function in factored form, given its zeros. ● Determine a factored form of a polynomial expression from the *x*-intercepts of the graph of its corresponding function.	**A2.EI.6 The student will represent, solve, and interpret the solution to a polynomial equation.** a) Determine a factored form of a polynomial equation, of degree three or higher, given its zeros or the *x*-intercepts of the graph of its related function.

2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Equations and Inequalities**	2023 *Standards of Learning* Knowledge and Skills (KS) **Equations and Inequalities (EI)**
• For a function, identify zeros of multiplicity greater than 1 and describe the effect of those zeros on the graph of the function. • Given a polynomial equation, determine the number and type of solutions.	b) Determine the number and type of solutions (real or imaginary) of a polynomial equation of degree three or higher. c) Solve a polynomial equation over the set of complex numbers. d) Verify possible solution(s) to polynomial equations of degree three or higher algebraically, graphically, and with technology, to justify the reasonableness of answer(s). Explain the solution method and interpret solutions in context.

2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Functions**	2023 *Standards of Learning* Knowledge and Skills (EK) **Functions (F)**
AII.5 The student will investigate and apply the properties of arithmetic and geometric sequences and series to solve practical problems, including writing the first *n* terms, determining the nth term, and evaluating summation formulas. Notation will include $\sum$ and a_n. • Distinguish between a sequence and a series. • Generalize patterns in a sequence using explicit and recursive formulas. • Use and interpret the notations $\sum$, n, n^{th} term, and a_n. • Given the formula, determine a_n (the nth term) for an arithmetic or a geometric sequence. • Given formulas, write the first *n* terms and determine the sum, S_n, of the first *n* terms of an arithmetic or geometric series.	**[Moved to Mathematical Analysis]**

2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Functions**	2023 *Standards of Learning* Knowledge and Skills (EK) **Functions (F)**
- Given the formula, determine the sum of a convergent infinite series. - Model practical situations using sequences and series.	
AII.6 For absolute value, square root, cube root, rational, polynomial, exponential, and logarithmic functions, the student will **a) recognize the general shape of function families; and** **b) use knowledge of transformations to convert between equations and the corresponding graphs of functions.** - Recognize the general shape of function families. (a) - Recognize graphs of parent functions. (a) - Identify the graph of a function from the equation. (b) - Write the equation of a function given the graph. (b) - Graph a transformation of a parent function, given the equation. (b) - Identify the transformation(s) of a function. Transformations of exponential and logarithmic functions, given a graph, should be limited to a single transformation. (b) - Investigate and verify transformations of functions using a graphing utility. (a, b)	**A2.F.1 The student will investigate, analyze, and compare square root, cube root, rational, exponential, and logarithmic function families, algebraically and graphically, using transformations.** a) Distinguish between the graphs of parent functions for square root, cube root, rational, exponential, and logarithmic function families. b) Write the equation of a square root, cube root, rational, exponential, and logarithmic function, given a graph, using transformations of the parent function, including $f(x) + k$; $f(kx)$; $f(x + k)$; and $kf(x)$, where k is limited to rational values. Transformations of exponential and logarithmic functions, given a graph, should be limited to a single transformation. c) Graph a square root, cube root, rational, exponential, and logarithmic function, given the equation, using transformations of the parent function including $f(x) + k$; $f(kx)$; $f(x + k)$; and $kf(x)$, where k is limited to rational values. Use technology to verify transformations of the functions.
AII.7 The student will investigate and analyze linear, quadratic, absolute value, square root, cube root, rational, polynomial, exponential, and logarithmic function families algebraically and graphically. Key concepts include **a) domain, range, and continuity;**	**A2.F.2 The student will investigate and analyze characteristics of square root, cube root, rational, polynomial, exponential, logarithmic, and piecewise-defined functions algebraically and graphically.**

2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Functions**	2023 *Standards of Learning* Knowledge and Skills (EK) **Functions (F)**
b) intervals in which a function is increasing or decreasing; **c) extrema;** **d) zeros;** **e) intercepts;** • Identify the domain, range, zeros, and intercepts of a function presented algebraically or graphically, including graphs with discontinuities. (a, d, e) • Describe a function as continuous or discontinuous. (a) • Given the graph of a function, identify intervals on which the function (linear, quadratic, absolute value, square root, cube root, polynomial, exponential, and logarithmic) is increasing or decreasing. (b) • Identify the location and value of absolute maxima and absolute minima of a function over the domain of the function graphically or by using a graphing utility. (c) • Identify the location and value of relative maxima or relative minima of a function over some interval of the domain graphically or by using a graphing utility. (c)	a) Determine and identify the domain, range, zeros, and intercepts of a function presented algebraically or graphically, including graphs with discontinuities. b) Compare and contrast the characteristics of square root, cube root, rational, polynomial, exponential, logarithmic, and piecewise-defined functions. c) Determine the intervals on which the graph of a function is increasing, decreasing, or constant. d) Determine the location and value of absolute (global) maxima and absolute (global) minima of a function. e) Determine the location and value of relative (local) maxima or relative (local) minima of a function.
AII.7 The student will investigate and analyze linear, quadratic, absolute value, square root, cube root, rational, polynomial, exponential, and logarithmic function families algebraically and graphically. Key concepts include **f) values of a function for elements in its domain;** **h) end behavior;** **i) vertical and horizontal asymptotes;** • For any x value in the domain of f, determine f(x). (f)	**A2.F.2 The student will investigate and analyze characteristics of square root, cube root, rational, polynomial, exponential, logarithmic, and piecewise-defined functions algebraically and graphically.** f) For any value, x, in the domain of f, determine $f(x)$ using a graph or equation. Explain the meaning of x and $f(x)$ in context, where applicable. g) Describe the end behavior of a function given a graph.

2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Functions**	2023 *Standards of Learning* Knowledge and Skills (EK) **Functions (F)**
• Describe the end behavior of a function. (h) • Determine the equations of vertical and horizontal asymptotes of functions (rational, exponential, and logarithmic). (i)	h) Determine the equations of any vertical and horizontal asymptotes of a function using a graph or equation (rational, exponential, and logarithmic).
AII.7 The student will investigate and analyze linear, quadratic, absolute value, square root, cube root, rational, polynomial, exponential, and logarithmic function families algebraically and graphically. Key concepts include **j) inverse of a function; and** **k) composition of functions, algebraically and graphically.** • Determine the inverse of a function (linear, quadratic, cubic, square root, and cube root). (j) • Graph the inverse of a function as a reflection over the line *y* = *x*. (j) • Determine the composition of two functions algebraically and graphically. (k)	**A2.F.2 The student will investigate and analyze characteristics of square root, cube root, rational, polynomial, exponential, logarithmic, and piecewise-defined functions algebraically and graphically.** i) Determine the inverse of a function algebraically and graphically, given the equation of a linear or quadratic function (linear, quadratic, and square root). Justify and explain why two functions are inverses of each other. j) Graph the inverse of a function as a reflection over the line *y* = *x*. k) Determine the composition of two functions algebraically and graphically.
AII.7 The student will investigate and analyze linear, quadratic, absolute value, square root, cube root, rational, polynomial, exponential, and logarithmic function families algebraically and graphically. Key concepts include **g) connections between and among multiple representations of functions using verbal descriptions, tables, equations, and graphs;**	**A2.F.1 The student will investigate, analyze, and compare square root, cube root, rational, exponential, and logarithmic function families, algebraically and graphically, using transformations.** e) Compare and contrast the graphs, tables, and equations of square root, cube root, rational, exponential, and logarithmic functions.

2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Functions**	2023 *Standards of Learning* Knowledge and Skills (EK) **Functions (F)**
• Represent relations and functions using verbal descriptions, tables, equations, and graphs. Given one representation, represent the relation in another form. (g) • Investigate and analyze characteristics and multiple representations of functions with a graphing utility. (a, b, c, d, e, f, g, h, i, j, k)	
AII.8 The student will investigate and describe the relationships among solutions of an equation, zeros of a function, *x*-intercepts of a graph, and factors of a polynomial expression. • Define a polynomial function in factored form, given its zeros. • Determine a factored form of a polynomial expression from the *x*-intercepts of the graph of its corresponding function. • For a function, identify zeros of multiplicity greater than 1 and describe the effect of those zeros on the graph of the function. • Given a polynomial equation, determine the number and type of solutions.	**[Included in A2.EI.6]**

2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Statistics**	2023 *Standards of Learning* Knowledge and Skills (EK) **Statistics (ST)**
AII.9 The student will collect and analyze data, determine the equation of the curve of best fit in order to make predictions, and	**A2.ST.2 The student will apply the data cycle (formulate questions; collect or acquire data; organize and represent data; and analyze data and communicate results) with a focus on representing bivariate data in scatterplots and determining the curve of best fit**

2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Statistics**	2023 *Standards of Learning* Knowledge and Skills (EK) **Statistics (ST)**
solve practical problems, using mathematical models of quadratic and exponential functions. ● Determine an equation of the curve of best fit, using a graphing utility, given a set of no more than 20 data points in a table, graph, or practical situation. ● Make predictions, using data, scatterplots, or the equation of the curve of best fit. ● Solve practical problems involving an equation of the curve of best fit. ● Evaluate the reasonableness of a mathematical model of a practical situation.	**using linear, quadratic, exponential, or a combination of these functions.** a) Formulate investigative questions that require the collection or acquisition of bivariate data and investigate questions using a data cycle. b) Collect or acquire bivariate data through research, or using surveys, observations, scientific experiments, polls, or questionnaires. c) Represent bivariate data with a scatterplot using technology. d) Determine whether the relationship between two quantitative variables is best approximated by a linear, quadratic, exponential, or a combination of these functions. e) Determine the equation(s) of the function(s) that best models the relationship between two variables using technology. Curves of best fit may include a combination of linear, quadratic, or exponential (piecewise-defined) functions. f) Use the correlation coefficient to designate the goodness of fit of a linear function using technology. g) Make predictions, decisions, and critical judgments using data, scatterplots, or the equation(s) of the mathematical model. h) Evaluate the reasonableness of a mathematical model of a contextual situation.
AII.10 The student will represent and solve problems, including practical problems, involving inverse variation, joint variation, and a combination of direct and inverse variations.	**A2.F.1 The student will investigate, analyze, and compare square root, cube root, rational, exponential, and logarithmic function families, algebraically and graphically, using transformations.**

2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Statistics**	2023 *Standards of Learning* Knowledge and Skills (EK) **Statistics (ST)**
- Given a data set or practical situation, write the equation for an inverse variation. - Given a data set or practical situation, write the equation for a joint variation. - Solve problems, including practical problems, involving inverse variation, joint variation, and a combination of direct and inverse variations.	d) Determine when two variables are directly proportional, inversely proportional, or neither, given a table of values. Write an equation and create a graph to represent a direct or inverse variation, including situations in context.
AII.11 The student will **b) interpret and compare z-scores for normally distributed data; and** **c) apply properties of normal distribution to determine probabilities associated with areas under the standard normal curve.** - Solve problems involving the relationship of the mean, standard deviation, and *z*-score of a normally distributed data set. (b) - Compare two sets of normally distributed data using a standard normal distribution and *z*-scores, given the mean and standard deviation. (b) - Represent probability as area under the curve of a standard normal distribution. (c) - Use the graphing utility or a table of Standard Normal Probabilities to determine probabilities associated with areas under the standard normal curve. (c)	**A2.ST.1 The student will apply the data cycle (formulate questions; collect or acquire data; organize and represent data; and analyze data and communicate results) with a focus on univariate quantitative data represented by a smooth curve, including a normal curve.** a) Formulate investigative questions that require the collection or acquisition of a large set of univariate quantitative data or summary statistics of a large set of univariate quantitative data and investigate questions using a data cycle. b) Collect or acquire univariate data through research, or using surveys, observations, scientific experiments, polls, or questionnaires.

2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Statistics**	2023 *Standards of Learning* Knowledge and Skills (EK) **Statistics (ST)**
• Use a graphing utility to investigate, represent, and determine relationships between a normally distributed data set and its descriptive statistics. (a, b, c)	
AII.11 The student will **a) identify and describe properties of a normal distribution;** **b) interpret and compare z-scores for normally distributed data; and** • Identify the properties of a normal distribution. (a) • Describe how the standard deviation and the mean affect the graph of the normal distribution. (a) • Solve problems involving the relationship of the mean, standard deviation, and *z*-score of a normally distributed data set. (b) • Compare two sets of normally distributed data using a standard normal distribution and *z*-scores, given the mean and standard deviation. (b) • Use a graphing utility to investigate, represent, and determine relationships between a normally distributed data set and its descriptive statistics. (a, b, c)	**A2.ST.1 The student will apply the data cycle (formulate questions; collect or acquire data; organize and represent data; and analyze data and communicate results) with a focus on univariate quantitative data represented by a smooth curve, including a normal curve.** c) Examine the shape of a data set (skewed versus symmetric) that can be represented by a histogram, and sketch a smooth curve to model the distribution. d) Identify the properties of a normal distribution. e) Describe and interpret a data distribution represented by a smooth curve by analyzing measures of center, measures of spread, and shape of the curve. f) Calculate and interpret the *z*-score for a value in a data set. g) Compare two data points from two different distributions using *z*-scores. h) Determine the solution to problems involving the relationship of the mean, standard deviation, and *z*-score of a data set represented by a smooth or normal curve. j) Compare multiple data distributions using measures of center, measures of spread, and shape of the distributions.
AII.11 The student will	**A2.ST.1 The student will apply the data cycle (formulate questions; collect or acquire data; organize and represent data; and analyze**

2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Statistics**	2023 *Standards of Learning* Knowledge and Skills (EK) **Statistics (ST)**
c) apply properties of normal distribution to determine probabilities associated with areas under the standard normal curve. ● Represent probability as area under the curve of a standard normal distribution. (c) ● Use the graphing utility or a table of Standard Normal Probabilities to determine probabilities associated with areas under the standard normal curve. (c) ● Use a graphing utility to investigate, represent, and determine relationships between a normally distributed data set and its descriptive statistics. (a, b, c)	**data and communicate results) with a focus on univariate quantitative data represented by a smooth curve, including a normal curve.** i) Apply the Empirical Rule to answer investigative questions. j) Compare multiple data distributions using measures of center, measures of spread, and shape of the distributions.
AII.12 The student will compute and distinguish between permutations and combinations. ● Compare and contrast permutations and combinations. ● Calculate the number of permutations of *n* objects taken *r* at a time. ● Calculate the number of combinations of *n* objects taken *r* at a time. ● Use permutations and combinations as counting techniques to solve practical problems. ● Calculate and verify permutations and combinations using a graphing utility.	**A2.ST.3 The student will compute and distinguish between permutations and combinations.** a) Compare and contrast permutations and combinations to count the number of ways that events can occur. b) Calculate the number of permutations of *n* objects taken *r* at a time. c) Calculate the number of combinations of *n* objects taken *r* at a time. d) Use permutations and combinations as counting techniques to solve practical problems. e) Calculate and verify permutations and combinations using technology.

2023 Algebra 2 Mathematics SOL – Summary of Changes

Algebra II (2016 SOL to 2023 SOL Numbering)	Parameter Changes/Clarifications (2023 SOL)
- AII.1a → A2.EO.1 - AII.1b → A2.EO.2 - AII.1c → A2.EO.3 - AII.2 → A2.EO.4 - AII.3a → A2.EI.1 - AII.3b → A2.EI.2 - AII.3c → A2.EI.4 - AII.3d → A2.EI.5 - AII.4 → A2.EI.3 - AII.5 → [Moved to Mathematical Analysis] - AII.6a,b → A2.F.1 - AII.7a-f, h-k → A2.F.2 - AII.7g → A2.F.1 - AII.8 → A2.EI.6 - AII.9 → A2.ST.2 - AII.10 → A2.F.1 - AII.11 → A2.ST.1 - AII.12 → A2.ST.3	- A2.EO.1 – Justify and determine equivalent rational algebraic expressions with monomial and binomial factors; represent and demonstrate equality of rational expressions written in different forms - A2.EO.2 – Simplify and determine equivalent radical expressions that include numeric and algebraic radicands - A2.EO.3 – Represent and demonstrate equality of polynomial expressions written in different forms - A2.EO.4 – Identify equivalent radical expressions containing negative rational numbers and expressions in $a + bi$ form - A2.EI.1 – Represent the solution set of an absolute value inequality using set notation, interval notation, and using a number line. - A2.EI.3 – Create a linear-quadratic or quadratic-quadratic system of equations to model a contextual situation - A2.EI.4 – Create an equation containing a rational expression to model a contextual situation; justify why a possible solution to an equation containing a rational expression might be extraneous - A2.EI.5 - Justify why a possible solution to an equation with a square root might be extraneous. - A2.F.1 - Transformation notation is specifically addressed and the constant, k, includes both integer and rational values. - A2.F.2 - Explain the meaning of x and $f(x)$ in context, where applicable; justify and explain why two functions are inverses of each other - A2.ST.1- Interpret the z-score; apply the Empirical Rule to answer investigative questions - A2.ST.2 – Curves of best fit may include a combination of linear, quadratic, or exponential (piecewise-defined) functions

Deletions from Algebra II (2016 SOL)	Additions to Algebra 2 (2023 SOL)
- AII.2 [EKS] – Simplify powers of *i* [Deleted] - AII.5 – Investigate and apply properties of arithmetic and geometric series; evaluate formulas in summation notation [Included in MA.FR.3] - AII.10 [EKS] – Solve problems involving joint variation and a combination of direct and inverse variation [Deleted] - AII.11 – [EKS] Determine probabilities associated with areas under the standard normal curve [Included in AFDA.DA.4 and PS.P.3]	- A2.EO.3 – Determine sums, differences, and products of polynomials in one and two variables; determine the quotient of polynomials in one and two variables, using monomial, binomial, and factorable trinomial divisors - A2.EI.1 – Create an absolute value equation and inequality in one variable to model a contextual situation - A2.EI.2 – Create a quadratic equation or inequality in one variable to model a contextual situation; determine the solution to a quadratic inequality in one variable over the set of real numbers algebraically - A2.EI.6 – Solve a polynomial equation over the set of complex numbers; verify solutions to polynomial equations of degree three or higher, algebraically, graphically, and with technology to justify answers; explain the solution method and interpret solutions in context - A2.F.2 - Investigate and analyze piecewise-defined functions - A2.ST.1 and A2.ST.2 – Includes data cycle process of collecting and acquiring data and formulating investigative questions that require the collection of the data - A2.ST.1 - Examine the shape of a data set that can be represented by a histogram and sketch a smooth curve to model the distribution; analyze measures of center and spread of a univariate data set represented by a smooth curve - A2.ST.2 - Use the correlation coefficient to designate goodness of fit of a linear function using technology

KEY: EO = Expressions and Operations; EI = Equations and Inequalities; F = Functions; ST = Statistics; EKS = Essential Knowledge and Skills (2016); KS = Knowledge and Skills (2023); US = Understanding the Standard

Trigonometry *Standards of Learning* - 2023 Overview of Revisions

This overview includes a summary of the content embedded in four content strands.

Triangle Trigonometry
- Determine the sine, cosine, tangent, cotangent, secant, and cosecant of the acute angles in a right triangle and use these ratios to solve for missing sides and angle measures, including application in contextual problems
- Find the area of any triangle and solve for the lengths of the sides and measures of the angles in a non-right triangle using the Law of Sines and the Law of Cosines

Circular Trigonometry
- Determine the degree and radian measure of angles; sketch angles in standard position on a coordinate plane; and determine the sine, cosine, tangent, cosecant, secant, and cotangent of an angle, given a point on the terminal side of an angle in standard position or the value of a trigonometric function of the angle
- Develop and apply the properties of the unit circle in degrees and radians

Graphs of Trigonometric Functions
- Graph and analyze trigonometric functions and apply trigonometric functions to represent periodic phenomena
- Graph the six inverse trigonometric functions

Identities and Equations
- Evaluate expressions involving the six trigonometric functions and the inverse sine, cosine, and tangent functions
- Use basic trigonometric identity substitutions to simplify and verify trigonometric identities
- Solve trigonometric equations and inequalities

Comparison of Trigonometry Mathematics *Standards of Learning* – 2016 to 2023

2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Triangular and Circular Trigonometric Functions**	2023 *Standards of Learning* Knowledge and Skills (KS) **Triangle Trigonometry (TT)**
T.1 The student, given a point on the terminal side of an angle in standard position, or the value of the trigonometric function of the angle, will determine the sine, cosine, tangent, cotangent, secant, and cosecant of the angle. ● Define the six triangular trigonometric functions of an angle in a right triangle. ● Draw a reference right triangle when given a point on the terminal side of the angle in standard position. ● Draw a reference right triangle when given the value of a trigonometric function of the angle. ● Determine the value of any trigonometric function when given a point on the terminal side of an angle in standard position. ● Given one trigonometric function value, determine the other five trigonometric function values.	**T.TT.1 The student will determine the sine, cosine, tangent, cotangent, secant, and cosecant of the acute angles in a right triangle and use these ratios to solve for missing sides and angle measures, including application in contextual problems.** a) Define and represent the six triangular trigonometric ratios (sine, cosine, tangent, cosecant, secant, and cotangent) of an angle in a right triangle. b) Describe the relationships between side lengths in special right triangles (30°-60°-90° and 45°-45°-90°). c) Use the trigonometric functions, the Pythagorean Theorem, the Law of Sines, and the Law of Cosines to solve contextual problems. d) Represent and solve contextual problems involving right triangles, including problems involving angles of elevation and depression.
[Moved from T.8]	**T.TT.2 The student will find the area of any triangle and solve for the lengths of the sides and measures of the angles in a non-right triangle using the Law of Sines and the Law of Cosines.** a) Apply the Law of Sines, and the Law of Cosines, as appropriate, to find missing sides and angles in non-right triangles. b) Recognize the ambiguous case when applying the Law of Sines and the potential for two triangle solutions in some situations.

2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Triangular and Circular Trigonometric Functions**	2023 *Standards of Learning* Knowledge and Skills (KS) **Triangle Trigonometry (TT)**
	c) Solve problems that integrate the use of the Law of Sines and the Law of Cosines and the triangle area formula (Area = $\frac{1}{2}absinC$, where a and b are triangle sides and C is the included angle) to find the area of any triangle, including those in contextual problems.

2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Graphs of Trigonometric Functions**	2023 *Standards of Learning* Knowledge and Skills (KS) **Circular Trigonometry (CT)**
[Moved from T.1 and T.9a]	**T.CT.1 The student will determine the degree and radian measure of angles; sketch angles in standard position on a coordinate plane; and determine the sine, cosine, tangent, cosecant, secant, and cotangent of an angle, given a point on the terminal side of an angle in standard position or the value of a trigonometric function of the angle.**
	a) Define a radian as a unit of angle measure and determine the relationship between the radian measure of an angle and the length of the intercepted arc in a circle.
	b) Determine the degree and radian measure of angles to include both negative and positive rotations in the coordinate plane.
	c) Find both positive and negative coterminal angles for a given angle.
	d) Identify the quadrant or axis in/on which the terminal side of an angle lies.

2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Graphs of Trigonometric Functions**	2023 *Standards of Learning* Knowledge and Skills (KS) **Circular Trigonometry (CT)**
	e) Draw a reference right triangle when given a point on the terminal side of an angle in standard position. f) Draw a reference right triangle when given the value of a trigonometric function of an angle (sine, cosine, tangent, cosecant, secant, and cotangent). g) Determine the value of any trigonometric function (sine, cosine, tangent, cosecant, secant, and cotangent) when given a point on the terminal side of an angle in standard position. h) Given one trigonometric function value, determine the other five trigonometric function values. i) Calculate the length of an arc of a circle in radians. j) Calculate the area of a sector of a circle.
T.2 The student will develop and apply the properties of the unit circle in degrees and radians. ● Define the six circular trigonometric functions of an angle in standard position. ● Apply the properties of the unit circle to determine trigonometric function values of special angles and their related angles in both degrees and radians without using a graphing utility. ● Apply the properties of the unit circle to convert between special angles expressed in radians and degrees, without using a graphing utility.	**T.CT.2 The student will develop and apply the properties of the unit circle in degrees and radians.** a) Convert between radian and degree measure of special angles of the unit circle without the use of technology. b) Define the six circular trigonometric functions of an angle in standard position on the unit circle. c) Apply knowledge of right triangle trigonometry, special right triangles, and the properties of the unit circle to determine trigonometric functions values of special angles (0°, 30°, 45°, 60°, and 90°) and their related angles in degree and radians without the use of technology.

2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Graphs of Trigonometric Functions**	2023 *Standards of Learning* Knowledge and Skills (KS) **Graphs of Trigonometric Functions (GT)**
T.3 The student, given one of the six trigonometric functions in standard form, will **a) state the domain and the range of the function;** **b) determine the amplitude, period, phase shift, vertical shift, and asymptotes;** **c) sketch the graph of the function by using transformations for at least a two-period interval; and** **d) investigate the effect of changing the parameters in a trigonometric function on the graph of the function.** ● State the domain and the range of a trigonometric function written in standard form. (a) ● Determine the amplitude, period, phase shift, vertical shift, and asymptotes of a trigonometric function from the equation of the function and from the graph of the function. (b) ● Describe the effect of changing A, B, C, or D in the standard form of a trigonometric equation. (d) ● Sketch the graph of a function written in standard form by using transformations for at least a two-period interval, including both positive and negative values for the domain. (c)	**T.GT.1 The student will graph and analyze trigonometric functions and apply trigonometric functions to represent periodic phenomena.** a) Sketch the graph of the six parent trigonometric functions (sine, cosine, tangent, cosecant, secant, and cotangent) for at least a two-period interval. b) Determine the domain and range, amplitude, period, and asymptote locations for a trigonometric function, given a graph or an equation. c) Describe the effects of changing the parameters (A, B, C, or D in the standard form of a trigonometric equation) on the graph of the function using graphing technology. d) Sketch the graph of a transformed sine, cosine, and tangent function written in standard form by using transformations for at least a two-period interval, including both positive and negative values for the domain. f) Apply trigonometric functions and their graphs to represent periodic phenomena.
T.4 The student will graph the six inverse trigonometric functions. ● Determine the domain and range of the inverse trigonometric functions. ● Use the restrictions on the domains of the inverse trigonometric functions in determining the values of the inverse trigonometric functions.	**T.GT.2 The student will graph the six inverse trigonometric functions.** a) Determine the domain and range of the inverse trigonometric functions. b) Use the restrictions on the domain of an inverse trigonometric function to determine a value of the inverse trigonometric function.

2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Graphs of Trigonometric Functions**	2023 *Standards of Learning* Knowledge and Skills (KS) **Graphs of Trigonometric Functions (GT)**
● Graph inverse trigonometric functions.	c) Graph inverse trigonometric functions.

2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Equations and Identities**	2023 *Standards of Learning* Knowledge and Skills (KS) **Identities and Equations (IE)**
[Moved from T.7]	**T.IE.1 The student will evaluate expressions involving the six trigonometric functions and the inverse sine, cosine, and tangent functions.** a) Determine the values of trigonometric functions, with and without graphing technology. b) Determine angle measures by using the inverse trigonometric functions, with and without a graphing technology. c) Evaluate composite functions that involve trigonometric functions and inverse trigonometric functions.

T.5 The student will verify basic trigonometric identities and make substitutions, using the basic identities.
 	
● Use trigonometric identities to make algebraic substitutions to simplify and verify trigonometric identities. The basic trigonometric identities include
  o reciprocal identities;
  o Pythagorean identities;
  o sum and difference identities;
  o double-angle identities; and
	T.IE.2 The student will use basic trigonometric identity substitutions to simplify and verify trigonometric identities.
	
	a) Use trigonometric identities to make algebraic substitutions to simplify and verify trigonometric identities. The basic trigonometric identities include
	  i) reciprocal identities;
	  ii) Pythagorean identities;
	  iii) sum and difference identities;

2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Equations and Identities**	2023 *Standards of Learning* Knowledge and Skills (KS) **Identities and Equations (IE)**
o half-angle identities.	iv) double-angle identities; and v) half-angle identities. b) Apply the sum, difference, and half-angle identities to evaluate trigonometric function values of angles that are not integer multiples of the special angles to solve problems, including contextual situations.
T.6 The student will solve trigonometric equations and inequalities. ● Solve trigonometric equations with and without restricted domains algebraically and graphically. ● Solve trigonometric inequalities algebraically and graphically. ● Verify algebraic solutions, using a graphing utility.	**T.IE.3 The student will solve trigonometric equations and inequalities.** a) Solve trigonometric equations with and without restricted domains algebraically and graphically. b) Solve trigonometric inequalities algebraically and graphically. c) Verify and justify algebraic solutions to trigonometric equations and inequalities, using graphing technology.
T.7 The student will determine the value of any trigonometric function and inverse trigonometric function. ● Use a graphing utility to determine the trigonometric function values of any angle in either degrees or radians. ● Define inverse trigonometric functions. ● Determine angle measures by using the inverse trigonometric functions when the trigonometric function values are given.	**[Moved to T.IE.1]**

2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Applications of Trig Functions**	2023 *Standards of Learning* Knowledge and Skills (KS) **Applications of Trig Functions (AT)**
T.8 The student will create and solve practical problems involving triangles. ● Create and solve practical problems involving triangles. ● Use the trigonometric functions, Pythagorean Theorem, Law of Sines, and Law of Cosines to solve practical problems. ● Use the trigonometric functions to model practical situations. ● Identify a solution technique associated with triangles that could be used with a given problem. ● Apply the sum and difference identities for sine, cosine, and tangent to solve problems.	**[Moved to T.TT.2]**
T.9 The student will solve problems, including practical problems, involving **a) arc length and area of sectors in circles using radians and degrees; and** **b) linear and angular velocity.** ● Convert between any angle expressed in radians and degrees without the use of technology. (a) ● Derive the relationship between the radian measure of an angle and the length of the intercepted arc. (a) ● Calculate the length of an arc in radians. (a) ● Calculate the area of sectors in circles. (a) ● Solve practical problems involving linear and angular velocity. (b)	**[T.9a moved to T.CT.1]** **[T.9b Included in Physics PH.2]**

2023 Trigonometry Mathematics SOL – Summary of Changes

Trigonometry (2016 SOL to 2023 SOL Numbering)	Parameter Changes/Clarifications (2023 SOL)
- T.1 → T.TT.1, T.CT.1 - T.2 → T.CT.2 - T.3a-d → T.GT.1 - T.4 → T.GT.2 - T.5 → T.IE.2 - T.6 → T.IE.3 - T.7 → T.IE.1 - T.8 → T.TT.2 - T.9a → T.CT.1 - T.9b → [Included in Physics]	T.TT.2 [KS] - Recognize the ambiguous case when applying the Law of Sines

Deletions from Trigonometry (2016 SOL)	Additions to Trigonometry (2023 SOL)
T.9b - Linear and angular velocity [Included in Physics]	- T.TT.1 [KS] - Relationships between side lengths in special right triangles - T.TT.1 [KS] - Model and solve problems, including contextual problems, involving angles of elevation and depression - T.TT.2 [KS] - Solve problems that integrate the triangle area formula (Area $= \frac{1}{2}ab\sin C$, where a and b are triangle sides and C is the included angle) - T.IE.1 [KS] - Evaluate composite functions that involve trigonometric functions and inverse trigonometric functions

KEY: TT = Triangle Trigonometry; CT = Circular Trigonometry; GT = Graphs of Trigonometric Functions; IE = Identities and Equations; AT = Applications of Trig Functions; EKS = Essential Knowledge and Skills (2016); KS = Knowledge and Skills (2023); US = Understanding the Standard

Computer Mathematics *Standards of Learning* - 2023 Overview of Revisions

This overview includes a summary of the content embedded in four content strands.

Data Representation and Storage
- Represent data and convert data between different number systems
- Differentiate between variable data types based upon their characteristics
- Represent data using appropriate data structures

Components of Programming
- Design a step-by-step plan to perform a task or solve a problem, including those arising from mathematical and interdisciplinary contexts
- Construct Boolean expressions and implement conditional statements
- Perform interaction with loops
- Write and implement the output and input phases of a computer program
- Implement library functions to process data
- Write and implement user-defined functions
- Implement pre-defined algorithms, including search routines and sort routines

Applications of Programming
- Write and implement programs using sequencing, selection, and iteration to perform a specific task or solve a problem, including those arising from mathematical and interdisciplinary contexts
- Create documentation using written comments to annotate the intended purpose of the components of a user-created program
- Verify how programs access and process variables
- Translate a mathematical expression or statement into computer code
- Trace existing code to interpret the intended purpose

Evaluation of Programming
- Test a program to match a sample output, using a set of data
- Debug a program using appropriate techniques
- Compare and contrast the efficiency of computer programs

Comparison of Computer Mathematics *Standards of Learning* – 2016 to 2023

2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Problem Solving**	2023 *Standards of Learning* Knowledge and Skills (KS) **Data Representation and Storage (DRS)**

COM.1 The student will design and apply computer programs to solve practical problems in mathematics arising from business and applications in mathematics.
 	
● Design and implement computer programs to solve practical problems.
● Analyze and interpret graphs, charts, and tables in the design and implementation of a computer program.
● Design and implement computer programs to
   o solve practical problems arising from business; and
   o solve mathematical problems, using formulas, equations, and functions.
	[Moved to CM.CP.1 and CM.AP.1]

[Moved from COM.6 and COM.15]	**CM.DRS.1 The student will represent data and convert data between different number systems.** a) Represent data in different number systems, including binary, decimal, and hexadecimal. b) Convert data between number systems (e.g., binary to decimal, decimal to hexadecimal).

2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Problem Solving**	2023 *Standards of Learning* Knowledge and Skills (KS) **Data Representation and Storage (DRS)**

[Moved from COM.14]	**CM.DRS.2 The student will differentiate between variable data types based upon their characteristics.**
	
	a) Describe the characteristics of different variable data types, including
	   i) Boolean;
	   ii) character;
	   iii) integer;
	   iv) decimal (double/float); and
	   v) string.
	b) Differentiate between variable data types to determine the data type needed based upon intended use (e.g., character versus string, integer versus double/float).

[Moved from COM.16]	**CM.DRS.3 The student will represent data using appropriate data structures.**
	
	a) Given a specific task or problem, determine the appropriate data structure (e.g., lists, arrays, objects) to represent data.
	b) Perform tasks related to lists or arrays (one-dimensional or two-dimensional), including
	   i) declare a list or array (one-dimensional or two-dimensional);
	   ii) choose an appropriate data type for a list or an array; and
	   iii) fill the list or array with data.
	c) Access and manipulate a particular element of a list or an array.
	d) Implement predefined objects to consolidate related information of different data types.

2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Program Design**	2023 *Standards of Learning* Knowledge and Skills (KS) **Components of Programming (CP)**
COM.2 The student will design, write, document, test, and debug a computer program. • Describe a computer program. • Design, write, document, test, and debug a complete computer program. • List and describe the processes involved in writing a computer program. • Describe the function of an algorithm. • Provide required documentation for a program. • Determine what a given output statement will print. • Debug a program.	**[Moved to CM.CP.1, CM.AP.1, CM.AP.2, CM.EP.1, and CM.EP.2]**
COM.3 The student will write program specifications that define the constraints of a given problem. • Write program specifications that define the constraints of a given problem. • Describe the preconditions, postconditions, and desired input and output of a given problem. • Determine whether or not a given problem is solvable using a computer program.	**[Moved to CM.CP.1]**
COM.4 The student will design an algorithm to solve a given problem. • Design a step-by-step algorithm to solve a problem.	**[Moved to CM.CP.1]**

2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Program Design**	2023 *Standards of Learning* Knowledge and Skills (KS) **Components of Programming (CP)**

COM.5 The student will divide a given problem into modules by task and implement the solution.
 	
● Divide a problem into modules by task.
● Write task-oriented modules, which may include
    ○ a user-defined function;
    ○ subroutines; or
    ○ procedures.
● Determine the need for a subroutine or user-defined function.
● Determine the difference between and the need for internal and external subroutines and functions.
● Implement the solution of the problem.
	CM.CP.1 The student will design a step-by-step plan to perform a task or solve a problem, including those arising from mathematical or interdisciplinary contexts.
	
	a) Design a step-by-step plan to perform a task or solve a problem using a flowchart or pseudocode that outlines the subtasks needed.
	b) Define the variables needed to perform a task or solve a problem.
	c) Define the constraints of a task or problem (e.g., pre-conditions, post-conditions) to determine the desired input and output.

COM.6 The student will translate mathematical expressions into programming expressions by declaring variables, writing assignment statements, and using the order of operations. ● Translate mathematical expressions into a programming expression. ● Declare appropriately named variables to store values used in computations. ● Write variable assignment statements. ● Use the order of operations to simplify expressions. ● Construct and evaluate expressions that include multiple arithmetic operations.	**[Moved to CM.DRS.1; CM.AP.4]**

2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Program Design**	2023 *Standards of Learning* Knowledge and Skills (KS) **Components of Programming (CP)**
COM.7 The student will select and call library functions to process data, as appropriate. ● Use library functions in designing programs to process data. ● Use library functions that are arithmetic or string operations. ● Invoke a value-returning library function.	**[Moved to CM.CP.7]**
COM.8 The student will implement conditional statements that include "if/then" statements, "if/then/else" statements, case statements, and Boolean logic. ● Construct a simple logical (Boolean) expression to evaluate a given condition. ● Construct an "if/then" statement to perform a specific task. ● Construct an "if/then/else" statement to perform a specific task. ● Construct a case statement to perform a specific task. ● Use conditional statements to incorporate decision making into programs.	**CM.CP.2 The student will construct Boolean expressions and implement conditional statements.** a) Write and implement Boolean expressions using logical and relational operators (e.g., !, &&, ||, ==, <, >, >=, <=, !=). b) Write and implement "if" conditional statements. c) Write and implement "if/else" conditional statements. d) Write and implement compound conditional statements (e.g., nested conditionals, chained conditional statements). e) Determine which parts of an algorithm are executed based on a condition being true or false.
COM.9 The student will implement pre-defined algorithms, including sort routines, search routines, and simple animation routines. ● Implement pre-defined algorithms into a program. ● Implement a sort routine on a one-dimensional array. ● Implement a sequential search routine on a one-dimensional array. ● Implement a binary search routine on a one-dimensional array. ● Implement a simple animation routine.	**[Moved to CM.CP.8]**

2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Program Design**	2023 *Standards of Learning* Knowledge and Skills (KS) **Components of Programming (CP)**
[Moved from COM.13]	**CM.CP.3 The student will perform iteration with loops.** a) Write and implement "while" and "for" loops. b) Differentiate between loops that run a fixed number of times and loops that run an indefinite number of times (e.g., stopping dependent on variable conditions). c) Identify conditions that cause infinite loops. d) Determine the outcome of code segments that include loops.

2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Program Implementation**	2023 *Standards of Learning* Knowledge and Skills (KS) **Components of Programming (CP)**

COM.10 The student will design and implement the input phase of a program, which will include designing screen layout, getting information into the program by way of user interaction and/or file input, and validating input.
 	
● Design a screen layout to facilitate input.
● Design program information input by user interaction and file input.
● Validate data, using a variety of methods.
● Construct input statements to read values into a program.
● Determine the contents of variables that have been assigned values by input statements.
	CM.CP.4 The student will write and implement the output phase of a computer program.
	
	a) Write and implement the output phase of a computer program, which may include:
	   i) formatting output in text-based environments;
	   ii) displaying output through a graphical user interface; and
	   iii) sending output to a physical device (e.g., speakers, robots, LED lights).
	b) Write output to a file.

2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Program Implementation**	2023 *Standards of Learning* Knowledge and Skills (KS) **Components of Programming (CP)**
COM.11 The student will design and implement the output phase of a computer program, which will include designing output layout, accessing available output devices, using output statements, and labeling results. ● Design an output layout. ● Access available output devices. ● Use output statements. ● Label results.	**CM.CP.5 The student will write and implement the input phase of a computer program.** a) Write and implement input statements to store user given values into a program. b) Validate input data using exception coding (e.g., using a "while" loop to control valid input by a user). c) Determine what output a program will produce given a specific input.
[New Expectation]	**CM.CP.6 The student will implement library functions.** a) Implement library functions to process data. b) Implement library functions to perform mathematical operations (e.g., random, absolute value, square root, power). c) Implement void library functions and return library functions. d) Implement overloaded library functions.

COM.12 The student will design and implement computer graphics to enhance output.
 	
● Design and implement computer graphics using various techniques such as
   o plotting points or shapes;
   o determining and setting window or screen dimensions;
   o determining and setting screen or background colors; and
   o using box commands.
● Describe the role of graphics in the computer environment.
	[Embedded in CM.CP.4; CM.CP.5]

2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Program Implementation**	2023 *Standards of Learning* Knowledge and Skills (KS) **Components of Programming (CP)**
[Moved from COM.7]	**CM.CP.7 The student will write and implement user-defined functions.** a) Write and implement a user-defined function to complete a task or sub-task. b) Write and implement void functions and return functions. c) Write and implement functions that accept parameters.
[Moved from COM.9]	**CM.CP.8 The student will implement pre-defined algorithms, including search routines and sort routines.** a) Differentiate between types of search routines. b) Differentiate between types of sort routines. c) Implement pre-defined algorithms. d) Implement a search routine on a one-dimensional list or an array, including sequential search and binary search. e) Implement a sort routine on a one-dimensional list or an array (e.g., selection sort, insertion sort, merge sort).
COM.13 The student will implement various mechanisms for performing iteration with an algorithm. ● Determine when an iterative algorithm is needed in a computer program. ● Incorporate single entry point, single exit point, preconditions, and postconditions into iterative algorithms.	**[Moved to CM.CP.3]**
COM.14 The student will select and implement appropriate data structures, including arrays (one- and/or two-dimensional) and objects.	**[Moved to CM.DRS.2; CM.DRS.3]**

2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Program Implementation**	2023 *Standards of Learning* Knowledge and Skills (KS) **Components of Programming (CP)**

• Implement a one-dimensional or two-dimensional array for a given problem:
   o choose an appropriate data type for an array;
   o assign a value to an array element;
   o fill an array with data, and process the data in the array;
   o access a particular element of a two-dimensional array;
   o process a two-dimensional array by rows and by columns; and
   o retrieve data from an array.
• Use data files in computer programs, both as a source of input data and as a way to save data for the next program execution.
• Implement objects to consolidate related information of different data types.	

2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Data Manipulation**	2023 *Standards of Learning* Knowledge and Skills (KS) **Applications of Programming (AP)**

COM.15 The student will define and use appropriate variable data types that include integer, real (fixed and scientific notation), character, string, Boolean, and object.
 	
• Define variables using data types, including
   o integer;
   o real (fixed and scientific notation);
   o character;
   o string;
   o Boolean; and
	[Moved to CM.DRS.2]

2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Data Manipulation**	2023 *Standards of Learning* Knowledge and Skills (KS) **Applications of Programming (AP)**
o object. ● Use standard naming conventions to create variable names.	
COM.16 The student will describe the way the computer stores, accesses, and processes variables, including the following topics: the use of variables versus constants, parameter passing, scope of variables, and local versus global variables. ● Determine when the use of a variable is appropriate. ● Describe how a computer stores, accesses, and processes variables. ● Incorporate parameter passing into programs. ● Differentiate between local and global variables and describe their appropriate use. ● Compare and contrast variables and constants. ● Describe the basic interplay between hardware and software in program execution.	[Moved to CM.AP.3]
[Moved from COM.1, COM.4]	**CM.AP.1 The student will write and implement programs using sequencing, selection, and iteration to perform a specific task or solve a problem, including those arising from mathematical and interdisciplinary contexts.** a) Determine what components of programming are needed to implement a step-by-step plan to perform a specific task or solve a problem.

2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Data Manipulation**	2023 *Standards of Learning* Knowledge and Skills (KS) **Applications of Programming (AP)**

	b) Write a computer program that includes sequencing, selection (conditionals), and iteration (loops).
	c) Write and implement computer programs to solve mathematical problems using
	   i) formulas and equations;
	   ii) functions;
	   iii) probability and statistics; and
	   iv) data-analysis.

[Moved from COM.2]	**CM.AP.2 The student will create documentation using written comments to annotate the intended purpose of the components of a user-created program.**
	
	a) Create documentation using written comments to:
	   i) describe the overall purpose of a program;
	   ii) align a previously created step-by-step plan to a written program;
	   iii) describe pre-conditions and post-conditions; and
	   iv) improve the readability of a program.

[New Expectation]	**CM.AP.3 The student will verify how programs access and process variables.** a) Verify that the variable types are aligned to the purpose of the algorithm. b) Verify that global variables are set to constant values before run time. c) Differentiate between the scopes of variables (e.g., global scope versus local scope) and verify the intended use.

2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Data Manipulation**	2023 *Standards of Learning* Knowledge and Skills (KS) **Applications of Programming (AP)**
[Moved from COM.6]	**CM.AP.4 The student will translate a mathematical expression or statement into computer code.** a) Declare, initialize, and assign variables to represent mathematical expressions or statements. b) Implement order of operations, including logical and relational operators. c) Translate a mathematical expression or statement into a programming statement(s).

[New Expectation]	**CM.AP.5 The student will trace existing code to interpret the intended purpose.**
	
	a) Trace existing code of an algorithm to
	  i) identify values at each stage of an algorithm; and
	  ii) predict return values of functions given specific arguments.
	b) Use tracing to describe the intended purpose of existing code for an algorithm.

2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Program Testing**	2023 *Standards of Learning* Knowledge and Skills (KS) **Evaluation of Programming (EP)**
COM.17 The student will test a program using an appropriate set of data. The test data should include boundary cases and test all branches of a program. ● Test a program, using an appropriate and complete set of data. ● Demonstrate that a set of data tests all branches of a program.	**CM.EP.1 The student will test a program to match a sample output, using a set of data.** a) Produce a given output by entering a data set. b) Test a program including boundary cases and inaccurate data types to verify the intended outcomes.

2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Program Testing**	2023 *Standards of Learning* Knowledge and Skills (KS) **Evaluation of Programming (EP)**

COM.18 The student will debug a program using appropriate techniques (e.g., appropriately placed controlled breaks, the printing of intermediate results, and other debugging tools available in the programming environment), and identify the difference among syntax errors, runtime errors, and logic errors.
 	
• Debug a program, using controlled breaks, the printing of intermediate results, and other debugging tools.
• Identify the differences among syntax errors, runtime errors, and logic errors.
	CM.EP.2 The student will identify errors and debug a program using various techniques.
	
	a) Differentiate among syntax errors, runtime errors, and logic errors.
	b) Debug a program using various techniques:
	   i) interpret syntax and runtime error messages;
	   ii) place controlled breaks;
	   iii) output intermediate results;
	   iv) disable a section of code by converting it into a comment;
	   v) trace code to identify logic errors; and
	   vi) use debugging tools available in the programming environment.

[New Expectation]	**CM.EP.3 The student will compare and contrast the efficiency of computer programs.**
	
	a) Compare and contrast the efficiency of computer programs in terms of
	   i) complexity of algorithms with the same intended outcomes;
	   ii) memory space used; and
	   iii) run time.

2023 Computer Mathematics SOL – Summary of Changes

Computer Mathematics (2016 SOL to 2023 SOL Numbering)	Parameter Changes/Clarifications (2023 SOL)
- COM.1 → CM.CP.1; CM.AP.1 - COM.2 → CM.CP.1; CM.AP.1; CM.AP.2; CM.EP.1; CM.EP.2 - COM.3 → CM.CP.1; CM.AP.1 - COM.4 → CM.CP.1 - COM.5 → CM.CP.1 - COM.6 → CM.DRS.1; CM.AP.4 - COM.7 → CM.CP.7 - COM.8 → CM.CP.2 - COM.9 → CM.CP.8 - COM.10 → CM.CP.4 - COM.11 → CM.CP.5 - New Expectation → CM.CP.6 - COM.12 → CM.CP.4; CM.CP.5 - COM.13 → CM.CP.3 - COM.14 → CM.DRS.2, CM.DRS.3 - COM.15 → CM.DRS.2 - COM.16 → CM.AP.3 - COM.17 → CM.EP.1 - COM.18 → CM.EP.2 - New Expectation → CM.EP.3	- CM.AP.2 - Improve the readability of a program - CM.EP.2 - Trace code to identify logic errors - CM.CP.3 - Perform iteration with loops

Deletions from Computer Mathematics (2016 SOL)	Additions to Computer Mathematics (2023 SOL)
● COM.12 - The student will design and implement computer graphics to enhance output (not explicitly included but may be embedded in CM.CP.4 and CM.CP.5)	● CM.CP.6 - Implement library functions to process data ● CM.AP.5 - Trace existing code to interpret the intended purpose

KEY: DRS = Data Representation and Storage (2023); CP = Components of Programming (2023); AP = Applications of Programming (2023); EP = Evaluation of Programming (2023); EKS = Essential Knowledge and Skills (2016); KS = Knowledge and Skills (2023); US = Understanding the Standard

Probability and Statistics *Standards of Learning* - 2023 Overview of Revision

This overview includes a summary of the content embedded in four content strands.

Data in Context
- Use a statistical cycle to formulate questions, describe types of data and data sources, and constraints within the context of a problem[†]
- Compare and contrast data collection methods to plan and conduct an observational study[†]
- Utilize the principles of experimental design to plan and conduct a well-designed experiment[†]

Descriptive Statistics
- Represent and analyze data visualizations of univariate quantitative data, including dotplots, stemplots, boxplots, cumulative frequency graphs, and histograms, to identify and describe patterns and departures from patterns, using central tendency, spread, clusters, gaps, and outliers, within the context of a problem[†]
- Represent and analyze numerical characteristics of univariate quantitative data sets to describe patterns and departures from patterns within the context of a problem[†]
- Represent, compare, and analyze distributions of two or more univariate quantitative data sets, numerically and graphically[†]
- Represent and analyze categorical data, using two-way tables and other graphical displays, to describe patterns and relationships
- Represent and analyze quantitative bivariate data with scatterplots to identify and describe the relationship between two variables
- Create and interpret a linear model using the least squares regression method to assess the relationship between two quantitative variables

Probability
- Organize information and apply probability rules to compute probabilities of events within the context of a problem[†]
- Represent and interpret situations using discrete random distributions, including binomial distributions
- Represent and interpret situations using normal distributions[†]

Inferential Statistics
- Apply properties of sampling distributions and inference procedures to make decisions about population proportions
- Apply properties of sampling distributions and inference procedures to make decisions about populations

[†] **Content intended for a one-semester course only.**

Comparison of Probability and Statistics *Standards of Learning* – 2016 to 2023

2016 *Standards of Learning* Essential Knowledge and Skills (EKS)	2023 *Standards of Learning* Knowledge and Skills (KS) **Data in Context (DC)**

[New Expectation]	**PS.DC.1† The student will use a statistical cycle to formulate questions, describe types of data, data sources, and constraints within the context of a problem.**
	
	  a) Define the stages of the statistical cycle and how each stage relates to the others.
	  b) Formulate questions and conclusions based on context.
	  c) Understand the type of data relevant to the question at hand (e.g., quantitative versus categorical).
	  d) Compare and contrast population and sample, and parameter and statistic.
	  e) Identify and explain constraints of the statistical approach.

Moved from PS.8† and PS.9†	**PS.DC.2† The student will compare and contrast data collection methods to plan and conduct an observational study.**
	
	  a) Investigate and describe sampling techniques (e.g., simple random sampling, stratified sampling, systematic sampling, cluster sampling).
	  b) Determine which sampling technique is best, given a particular context.
	  c) Investigate and explain biased influences inherent within sampling methods and various forms of response bias.
	  d) Use the statistical cycle to plan and conduct an observational study to answer a question or address a problem.

2016 *Standards of Learning* Essential Knowledge and Skills (EKS)	2023 *Standards of Learning* Knowledge and Skills (KS) **Data in Context (DC)**

Moved from PS.10[†]	**PS.DC.3**[†] **The student will utilize the principles of experimental design to plan and conduct a well-designed experiment.**
	
	a) Describe the principles of experimental design, including:
	   i) treatment/control groups;
	   ii) blinding/placebo effects;
	   iii) experimental units/subjects; and
	   iv) blocking/matched pairs and completely randomized designs.
	b) Evaluate the principles of experimental design to address comparison, randomization, replication, and control within the context of the problem.
	c) Compare and contrast controlled experiments and observational studies and the conclusions that may be drawn from each.
	d) Use the statistical cycle to plan and conduct a well-designed experiment to answer a question or address a problem.
	e) Select a data collection method appropriate for a given context.

2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Descriptive Statistics**	2023 *Standards of Learning* Knowledge and Skills (KS) **Descriptive Statistics (DS)**
PS.1[†] **The student will analyze graphical displays of univariate data, including dotplots, stemplots, boxplots, cumulative frequency graphs, and histograms, to identify and describe patterns and**	**PS.DS.1**[†] **The student will represent and analyze data visualizations of univariate quantitative data, including dot plots, stemplots, boxplots, cumulative frequency graphs, and histograms, to identify and describe patterns and departures from patterns,**

2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Descriptive Statistics**	2023 *Standards of Learning* Knowledge and Skills (KS) **Descriptive Statistics (DS)**

departures from patterns, using central tendency, spread, clusters, gaps, and outliers.
 	
● Create and interpret graphical displays of data, including dotplots, stemplots, boxplots, cumulative frequency graphs, and histograms, using appropriate technology.
● Examine graphs of data for clusters and gaps, and relate those phenomena to the data in context.
● Examine graphs of data for outliers, and explain the outlier(s) within the context of the data.
● Examine graphs of data and identify the central tendency of the data as well as the spread.
● Explain the central tendency and the spread of the data within the context of the data.
	using central tendency, spread, clusters, gaps, and outliers, within the context of a problem.
	
	a) Create and interpret graphical displays of data, including dot plots, stemplots, boxplots, cumulative frequency graphs, and histograms, using appropriate technology.
	b) Examine the graphs within the context of the problem by analyzing:
	  i) shape;
	  ii) measures of center;
	  iii) spread; and
	  iv) unusual features of the data (e.g., outliers, clusters, gaps).

PS.2[†] The student will analyze numerical characteristics of univariate data sets to describe patterns and departures from patterns, using mean, median, mode, variance, standard deviation, interquartile range, range, and outliers. ● Interpret mean, median, mode, range, interquartile range, variance, and standard deviation of a univariate data set in terms of the problem's context. ● Identify possible outliers, using an algorithm. ● Explain the influence of outliers on a univariate data set. ● Explain ways in which standard deviation addresses dispersion by examining the formula for standard deviation.	**PS.DS.2[†] The student will represent and analyze numerical characteristics of univariate quantitative data sets to describe patterns and departures from patterns within the context of a problem.** a) Interpret measures of central tendency: mean, median, and mode. b) Interpret measures of spread: range, interquartile range, variance, and standard deviation. c) Identify possible outliers, using an algorithm. d) Investigate and explain the influence of outliers on a univariate data set.

2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Descriptive Statistics**	2023 *Standards of Learning* Knowledge and Skills (KS) **Descriptive Statistics (DS)**
	e) Investigate and explain ways in which standard deviation addresses variability by examining the formula for standard deviation.

PS.3† The student will compare distributions of two or more univariate data sets, numerically and graphically, analyzing center and spread (within group and between group variations), clusters and gaps, shapes, outliers, or other unusual features.
 	
● Compare and contrast two or more univariate data sets, numerically and graphically, by analyzing measures of center and spread within a contextual framework.
● Describe any unusual features of the data, such as clusters, gaps, or outliers, within the context of the data.
● Analyze skewness in conjunction with measures of center and spread in a contextual framework.
	PS.DS.3† The student will represent, compare, and analyze distributions of two or more univariate quantitative data sets, numerically and graphically.
	
	a) Create graphical displays of data, including back-to-back stemplots, parallel dot plots, parallel boxplots, and histograms, using appropriate technology.
	b) Compare and contrast two or more univariate data sets, numerically and graphically, within the context of a problem by analyzing:
	    i) shape;
	    ii) measures of center;
	    iii) measures of spread; and
	    iv) unusual features of the data (e.g., clusters, gaps, outliers).

Moved from PS.7†	**PS.DS.4 The student will represent and analyze categorical data, using two-way tables and other graphical displays, to describe patterns and relationships.** a) Create and interpret graphical displays of univariate categorical data, including bar graphs within the context of the problem, using appropriate technology. b) Create and interpret graphical displays comparing distributions of two or more univariate categorical data sets including segmented and side-by-side bar graphs within the context of the problem, using appropriate technology.

2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Descriptive Statistics**	2023 *Standards of Learning* Knowledge and Skills (KS) **Descriptive Statistics (DS)**
	c) Generate and interpret a two-way table as a summary of the information obtained from two categorical variables.
	d) Calculate and interpret marginal, relative, and conditional frequencies to analyze data in a two-way table within the context of a problem.

PS.4[†] The student will analyze scatterplots to identify and describe the relationship between two variables, using shape; strength of relationship; clusters; positive, negative, or no association; outliers; and influential points.
 	
● Examine scatterplots of data, and describe skewness, and correlation within the context of the data.
● Describe and explain any unusual features of the data, such as clusters, gaps, or outliers, within the context of the data.
● Identify influential data points (observations that have a great effect on a line of best fit because of extreme *x*-values) and describe the effect of the influential points.
	PS.DS.5 The student will represent and analyze quantitative bivariate data with scatterplots to identify and describe the relationship between two variables.
	
	a) Create scatterplots, using appropriate technology.
	b) Examine and interpret scatterplots in the context of the problem by analyzing:
	  i) the form of relationship for linear and nonlinear trends;
	  ii) the direction of the relationship for positive, negative, or no association;
	  iii) the strength of the relationship such as strong, moderate, or weak; and
	  iv) the presence of unusual features within the data (e.g., clusters, gaps, influential points, outliers).

PS.5 The student will determine and interpret linear correlation, use the method of least squares regression to model the linear relationship between two variables, and use the residual plot to assess linearity. ● Calculate a correlation coefficient, *r*. ● Explain how the correlation coefficient, *r*, measures association by looking at its formula.	**PS.DS.6 The student will create and interpret a linear model using the least squares regression method to assess the relationship between two quantitative variables.** a) Create the least squares regression model using technology to interpret the contextual meaning of the slope and *y*-intercept. b) Using technology, calculate and interpret the correlation coefficient, *r*, within the context of a problem.

2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Descriptive Statistics**	2023 *Standards of Learning* Knowledge and Skills (KS) **Descriptive Statistics (DS)**
• Interpret the coefficient of determination, r^2, in a contextual framework. • Use regression lines to make predictions, and identify the limitations of the predictions. • Use residual plots to determine whether a linear model is satisfactory for describing the relationship between two variables. • Describe the errors inherent in extrapolation beyond the range of the data. • Use least squares regression to determine the equation of the line of best fit for a set of data. • Interpret the slope and *y*-intercept of the least squares regression line in a contextual framework. • Explain how least squares regression generates the equation of the line of best fit by examining the formulas used in computation.	c) Using technology, calculate and interpret the coefficient of determination, r^2, within the context of a problem. d) Use regression lines to make predictions, and identify the limitations of the predictions, such as extrapolation. e) Calculate and interpret a residual to understand the error of a prediction. f) Using technology, calculate and interpret the standard deviation of the residuals, *s*.
PS.6 The student will make logarithmic and power transformations to achieve linearity. • Apply a logarithmic transformation to data. • Explain how a logarithmic transformation works to achieve a linear relationship between variables. • Apply a power transformation to data. • Explain how a power transformation works to achieve a linear relationship between variables.	**[Included in AP Statistics]**
PS.7[†] The student, using two-way tables and other graphical displays, will analyze categorical data to describe patterns and departures from patterns and to determine marginal frequency and relative frequencies, including conditional frequencies.	**[Moved to PS.DS.4]**

2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Descriptive Statistics**	2023 *Standards of Learning* Knowledge and Skills (KS) **Descriptive Statistics (DS)**
● Produce a two-way table as a summary of the information obtained from two categorical variables. ● Create and interpret graphical displays of categorical data including bar charts. ● Calculate marginal, relative, and conditional frequencies in a two-way table. ● Use marginal, relative, and conditional frequencies to analyze data in two-way tables within the context of the data.	

2016 *Standards of Learning* Essential Knowledge and Skills **Data Collection**	2023 *Standards of Learning* Knowledge and Skills
PS.8[†] The student will describe the methods of data collection in a census, sample survey, experiment, and observational study and identify an appropriate method of solution for a given problem setting. ● Compare and contrast controlled experiments and observational studies and the conclusions one can draw from each. ● Compare and contrast population and sample, and parameter and statistic. ● Identify biased sampling methods. ● Describe simple random sampling. ● Select a data collection method appropriate for a given context.	**[Moved to PS.DC.2[†]]**

2016 *Standards of Learning* Essential Knowledge and Skills **Data Collection**	2023 *Standards of Learning* Knowledge and Skills
PS.9† The student will plan and conduct a survey. The plan will address sampling techniques and methods to reduce bias. ● Distinguish between a population and a sample. ● Investigate and describe sampling techniques, such as simple random sampling, stratified sampling, and cluster sampling. ● Determine which sampling technique is best, given a particular context. ● Plan a survey to answer a question or address an issue. ● Given a plan for a survey, identify possible sources of bias, and describe ways to reduce bias. ● Design a survey instrument. ● Conduct a survey.	**[Moved to PS.DC.2†]**
PS.10† The student will plan and conduct a well-designed experiment. The plan will address control, randomization, replication, blinding, and measurement of experimental error. ● Plan and conduct a well-designed experiment. The experimental design should address control, randomization, replication, blinding and minimization of experimental error. ● Identify treatments, levels, factors, control groups, and experimental units in an experimental design. ● Identify sources of bias and confounding, including the placebo effect. ● Identify a situation when a block design, including matched pairs, would reduce the effects of confounding variables.	**[Moved to PS.DC.3†]**

2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Probability**	2023 *Standards of Learning* Knowledge and Skills (KS) **Probability (P)**
PS.11[†] The student will identify and describe two or more events as complementary, dependent, independent, and/or mutually exclusive. ● Define and give contextual examples of complementary, dependent, independent, and mutually exclusive events. ● Given two or more events in a problem setting, determine whether the events are complementary, dependent, independent, and/or mutually exclusive.	**PS.P.1[†] The student will organize information and apply probability rules to compute probabilities of events within the context of a problem.** a) Given two or more events, determine whether the events are complementary, dependent, independent, and/or mutually exclusive, and compute the probability of those events. b) Represent and calculate probabilities using Venn diagrams, tree diagrams, and two-way tables. c) Apply the addition rule, the multiplication rule, and complementary rule to calculate probabilities. d) Calculate conditional probabilities to determine the association or independence of two events.
PS.12[†] The student will determine probabilities (relative frequency and theoretical), including conditional probabilities for events that are either dependent or independent, by applying the Law of Large Numbers concept, the addition rule, and the multiplication rule. ● Calculate relative frequency and expected frequency. ● Determine conditional probabilities for dependent, independent, and mutually exclusive events.	**[Moved to PS.P.1[†]]**
PS.13 The student will develop, interpret, and apply the binomial and geometric probability distributions for discrete random variables, including computing the mean and standard deviation for the binomial and geometric variables. ● Develop the binomial and geometric probability distributions within a practical context.	**PS.P.2 The student will represent and interpret situations using discrete random distributions, including binomial distributions.** a) Identify discrete random variables and create a table to represent valid discrete probability distributions within the context of a problem.

2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Probability**	2023 *Standards of Learning* Knowledge and Skills (KS) **Probability (P)**
• Calculate the mean and standard deviation for the binomial and geometric variables. • Use the binomial and geometric distributions to calculate probabilities associated with experiments for which there are only two possible outcomes.	b) Calculate and interpret the mean (expected value) and standard deviation for a discrete random variable within the context of a problem. c) Determine if a discrete random variable satisfies the conditions for a binomial distribution. d) Design and conduct a simulation of a binomial distribution. e) Calculate and interpret probabilities from a binomial distribution within the context of a problem. f) Calculate the mean and standard deviation for binomial distributions. g) Describe the center, shape, and spread of a discrete random variable within the context of a problem.
PS.14 The student will simulate probability distributions, including binomial and geometric. • Design and conduct a simulation of a binomial distribution. • Design and conduct a simulation of a geometric distribution. • Calculate probabilities resulting from simulations of binomial and geometric distributions.	**[Moved to PS.P.2]**
PS.15 The student will identify random variables as independent or dependent and determine the mean and standard deviations for random variables and sums and differences of independent random variables. • Compare and contrast independent and dependent random variables. • Determine the mean (expected value) and standard deviation for a random variable and linear transformation of a random variable.	**[Moved to PS.P.2]**

2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Probability**	2023 *Standards of Learning* Knowledge and Skills (KS) **Probability (P)**
● Determine the mean (expected value) for sums and differences of random variables. ● Determine the standard deviation for sums and differences of independent random variables.	
PS.16[†] The student will identify properties of a normal distribution and apply the normal distribution to determine probabilities. ● Identify the properties of a normal distribution. ● Describe how the standard deviation and the mean affect the graph of the normal distribution. ● Calculate and interpret the *z*-score of a given data value from a normal distribution. ● Determine the probability of a given event, using the normal distribution. ● Use a graphing utility and a table of Standard Normal Probabilities to determine probabilities.	**PS.P.3[†] The student will represent and interpret situations using normal distributions.** a) Compare and contrast discrete and continuous distributions. b) Represent probability as the area under the curve of a normal distribution using the Empirical Rule and graphing technology. c) Describe the center, shape, and spread of normal distributions within the context of a problem. d) Compare and contrast two or more sets of normally distributed data using *z*-scores, percentiles, or probabilities within the context of a problem. e) Standardize a data value from a normal distribution and interpret the *z*-score within the context of a problem. f) Calculate and interpret probabilities of a normal distribution using technology within the context of a problem.

2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Inferential Statistics**	2023 *Standards of Learning* Knowledge and Skills (KS) **Inferential Statistics (IS)**

PS.17 The student, given data from a large sample, will determine and interpret appropriate point estimates and confidence intervals for parameters. The parameters will include proportion and mean, difference between two proportions, difference between two means (independent and paired), and slope of a least-squares regression line.
 	
● Construct confidence intervals to estimate a population parameter, such as a proportion or the difference between two proportions; a mean or the difference between two means; or slope of a least-squares regression line.
● Select a value for the confidence level of a confidence interval.
● Interpret confidence intervals and confidence levels in the context of the data.
● Explain the importance of random sampling for confidence intervals.
● Explain how changes in confidence level and sample size effect width of the confidence interval and margin of error.
● Calculate point estimates for parameters and discuss the limitations of point estimates.
	PS.IS.1 The student will apply properties of sampling distributions and inference procedures to make decisions about population proportions.
	
	a) Describe the shape, center, and spread of the sampling distribution of a proportion within the context of a problem.
	b) Given a problem, construct a one sample z confidence interval:
	   i) identify the basic conditions for inference: random sample, independence, and normality;
	   ii) calculate a confidence interval using technology; and
	   iii) interpret the interval within the context of the problem.
	c) Explain how changes in confidence level and sample size affect width of the confidence interval and margin of error.
	d) Calculate and interpret a point estimate and margin of error of a confidence interval for a proportion within the context of the problem.
	e) Explain how and why the hypothesis testing procedure allows one to reach a statistical decision.
	f) Given a problem, apply the one sample z hypothesis testing procedures:
	   i) construct appropriate null and alternate hypotheses;
	   ii) identify the basic conditions for inference: random sample; independence, and normality;
	   iii) calculate and interpret the p-value using technology;
	   iv) determine and justify whether to reject the null hypothesis; and
	   v) interpret the results within the context of the problem.

2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Inferential Statistics**	2023 *Standards of Learning* Knowledge and Skills (KS) **Inferential Statistics (IS)**
	g) Use the statistical cycle to plan and conduct a statistical study about a proportion to answer a question or address a problem with inference.

PS.18 The student will apply and interpret the logic of an appropriate hypothesis-testing procedure. Tests will include large sample test for proportion, mean, difference between two proportions, difference between two means (independent and paired); chi-squared tests for goodness of fit, homogeneity of proportions, and independence; and slope of a least-squares regression line.
 	
• Use the chi-squared test for goodness of fit to decide whether the population being analyzed fits a particular distribution pattern.
• Use hypothesis-testing procedures to determine whether or not to reject the null hypothesis. The null hypothesis may address proportion, mean, difference between two proportions or two means, goodness of fit, homogeneity of proportions, independence, and the slope of a least-squares regression line.
• Compare and contrast Type I and Type II errors.
• Explain how and why the hypothesis-testing procedure allows one to reach a statistical decision
	PS.IS.2 The student will apply properties of sampling distributions and inference procedures to make decisions about populations.
	
	a) Describe the shape, center, and spread of the sampling distribution of a mean within the context of a problem.
	b) Calculate and interpret a point estimate and a margin of error for a confidence interval of a mean within the context of a problem.
	c) Describe the use of the Central Limit Theorem in satisfying the assumptions and conditions for inference about a mean.
	d) Identify the properties of a t distribution.
	e) Given a problem, construct a one sample t confidence interval:
	   i) identify the basic conditions for inference: random sample, independence, and approximate normality;
	   ii) calculate a confidence interval using technology; and
	   iii) interpret the interval within the context of the problem.
	f) Given a problem, apply the one sample t hypothesis testing procedures:
	   i) construct appropriate null and alternate hypotheses;
	   ii) identify the basic conditions for inference: random sample, independence, and approximate normality;
	   iii) calculate and interpret the p value using technology;
	   iv) determine and justify whether to reject the null hypothesis; and
	   v) interpret the results within the context of the problem.

2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Inferential Statistics**	2023 *Standards of Learning* Knowledge and Skills (KS) **Inferential Statistics (IS)**
PS.19 The student will identify the meaning of sampling distribution with reference to random variable, sampling statistic, and parameter and explain the Central Limit Theorem. This will include sampling distribution of a sample proportion, a sample mean, a difference between two sample proportions, and a difference between two sample means. ● Describe the use of the Central Limit Theorem for drawing inferences about a population parameter based on a sample statistic. ● Describe the effect of sample size on the sampling distribution and on related probabilities. ● Use the normal approximation to calculate probabilities of sample statistics falling within a given interval. ● Identify and describe the characteristics of a sampling distribution of a sample proportion, mean, difference between two sample proportions, or difference between two sample means.	**[Moved to PS.IS.1, PS.IS.2, or included in AP Statistics]**
PS.20 The student will identify properties of a *t*-distribution and apply *t*-distributions to single-sample and two-sample (independent and matched pairs) *t*-procedures. ● Identify the properties of a *t*-distribution. ● Compare and contrast a *t*-distribution and a normal distribution. ● Use a *t*-test for single-sample and two-sample data.	**[Moved to PS.IS.2, or included in AP Statistics]**

2023 Probability and Statistics Mathematics SOL – Summary of Changes

Probability and Statistics (2016 SOL to 2023 SOL Numbering)	Parameter Changes/Clarifications (2023 SOL)
[New Expectation] → PS.DC.1[†] - PS.1[†] → PS.DS.1[†] - PS.2[†] → PS.DS.2[†] - PS.3[†] → PS.DS.3 - PS.4[†] → PS.DS.5 - PS.5 → PS.DS.6 - PS.6 → [Deleted] - PS.7[†] → PS.DS.4 - PS.8[†] → PS.DC.2[†] - PS.9[†] → PS.DC.2[†] - PS.10[†] → PS.DC.3[†] - PS.11[†] → PS.P.1[†] - PS.12[†] → PS.P.1[†] - PS.13 → PS.P.2 - PS.14 → PS.P.2 - PS.15 → PS.P.2 - PS.16[†] → PS.P.3 - PS.17 → PS.IS.1 - PS.18 → PS.IS.2 - PS.19 → PS.IS.1, PS.IS.2 - PS.20 → PS.IS.2	- PS.IS.2 [KS] - Hypothesis testing limited to one-sample tests - PS.DS.6 [KS] - Generating the equation of the line of best fit by using technology versus examining the formulas - PS.P.2 [KS] - Limited discrete random variables to binomial distributions

Deletions from Probability and Statistics (2016 SOL)	Additions to Probability and Statistics (2023 SOL)
• PS.5 [EKS] – Use residual plots to determine whether a linear model is satisfactory for describing the relationship between two variables [Included in AP Statistics] • PS.6 – Logarithmic and power transformation to achieve linearity [Included in AP Statistics] • PS.10 [EKS] – Factors and levels of experimental design [Included in AP Statistics] • PS.13 [EKS] – Geometric distributions [Included in AP Statistics] • PS.15 [EKS] – Compare and contrast independent and dependent random variables; determine the mean (expected value) for sums and differences of random variables; determine the standard deviation for sums and differences of independent random variables [Included in AP Statistics] • PS.17 [EKS] – Given sample data, determine and interpret point estimates and confidence intervals for the parameters for the difference between two proportions, difference between two means (independent and paired), and slope of a least-squares regression line [Included in AP Statistics] • PS.18 [EKS] – Hypothesis testing for the difference between two proportions or two means, goodness of fit, homogeneity of proportions, independence, and the slope of a least-squares regression line; compare and contrast Type I and Type II errors [Included in AP Statistics] • PS.19 [EKS] – Identify and describe the characteristics of a sampling distribution of a difference between two sample proportions and a difference between two sample means [Included in AP Statistics] • PS.20 [EKS] – Use a *t*-test for two-sample data [Included in AP Statistics]	• PS.DC.1[†]; PS.DC.2[†]; PS.DC.3[†]; and PS.IS.1 [KS] - Describe and use the statistical cycle to answer questions, solve problems, and communicate within the context of the problem • PS.DC.2 [KS] – Investigate and describe the systematic sampling technique • PS.DS.6 [KS] – Use technology to calculate and interpret the standard deviation of the residual

KEY: DC = Data in Context; DS = Descriptive Statistics; P = Probability; IS = Inferential Statistics; EKS =Essential Knowledge and Skills (2016); KS = Knowledge and Skills (2023); US = Understanding the Standard

Discrete Mathematics *Standards of Learning* - 2023 Overview of Revisions

This overview includes a summary of the content embedded in four content strands.

Logical Reasoning
- Use reasoning to develop and apply logical arguments[†]
- Apply logic and proof techniques in the construction of a sound argument[†]
- Apply Boolean algebra to represent and analyze the function of logical gates and circuits[†]
- Use mathematical induction to prove formulas and mathematical statements

Set and Number Theory
- Identify and use the properties of sets and set operations[†]
- Apply the formulas of combinatorics[†]
- Use Pascal's Triangle to analyze numerical patterns and relationships

Graph Theory
- Represent problems using vertex-edge graphs. The concepts of degree, connectedness, paths, planarity, and directed graphs will be analyzed.[†]
- Solve problems through analysis and application of circuits, cycles, Euler paths, Euler circuits, Hamilton paths, and Hamilton circuits. Optimal solutions will be determined using existing algorithms and student-created algorithms.[†]
- Apply graphs to conflict-resolution problems, such as map coloring, scheduling, matching, and optimization[†]
- Recognize and apply algorithms to solve configuration, conflict-resolution, and sorting problems
- Use algorithms to schedule tasks to determine a minimum project time

Computational Methods
- Describe and apply sorting and searching algorithms used in processing and communicating information[†]
- Use recursive processes[†]
- Identify and apply cryptographic methods
- Analyze the limitations of algorithms and their contextual relationships in computing.

[†] **Content intended for a one-semester course only.**

Comparison of Discrete Mathematics *Standards of Learning* – 2016 to 2023

The following standards outline the content of a one-year course in Discrete Mathematics.
If a one-semester course is desired, the standards with a dagger (†) would apply.

2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Graphs**	2023 *Standards of Learning* Knowledge and Skills (KS) **Logical Reasoning (LR)**
[Moved from DM.9†]	**DM.LR.1† The student will use reasoning to develop and apply logical arguments.** a) Use Venn diagrams to codify and solve logic problems. b) Express logical statements in symbolic form. c) Represent a conditional statement as its converse, inverse, and contrapositive. d) Describe how symbolic logic can be used to map the processes of computer applications. e) Construct a truth table to display all possible input combinations and their outputs. f) Identify the rules of inference and model basic logical statements including De Morgan's Law. g) Apply logical reasoning to model contextual situations and make decisions.
[New Expectation]	**DM.LR.2† The student will apply logic and proof techniques in the construction of a sound argument.** a) Apply informal logical reasoning to contextual problems (e.g., predicting the behavior of software, solving puzzles). b) Outline the basic structure of a proof technique (e.g., direct proof, proof by contradiction, induction).

2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Graphs**	2023 *Standards of Learning* Knowledge and Skills (KS) **Logical Reasoning (LR)**
	c) Deduce the best type of proof for a given problem. d) Use the rules of inference to construct direct proofs and proofs by contradiction. e) Construct induction proofs involving summations and inequalities. f) Use a truth table to prove the logical equivalence of statements.
[Moved from DM.9†; New Expectations]	**DM.LR.3† The student will apply Boolean algebra to represent and analyze the function of logical gates and circuits.** a) Explain basic properties of Boolean algebra: duality, complements, and standard forms. b) Represent verbal statements as Boolean expressions. c) Apply Boolean algebra to prove identities and simplify expressions. d) Generate truth tables that encode the truth and falsity of two or more statements. e) Explain the operation of discrete logic gates. f) Describe the relationship between Boolean algebra and electronic circuits. g) Analyze a combinational network using Boolean expressions. h) Design simple combinational networks that use NAND (AND followed by NOT), NOR (OR followed by NOT), and XOR (exclusive-OR) gates.
[Moved from Mathematical Analysis]	**DM.LR.4 The student will use mathematical induction to prove formulas and mathematical statements.**

2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Graphs**	2023 *Standards of Learning* Knowledge and Skills (KS) **Logical Reasoning (LR)**
	a) Compare and contrast inductive and deductive reasoning. b) Explain the relationship between weak and strong induction. c) Construct induction proofs involving a divisibility argument. d) Prove the Binomial Theorem through mathematical induction.

2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Set and Number Theory**	2023 *Standards of Learning* Knowledge and Skills (KS) **Set and Number Theory (SNT)**
[Moved from DM.9†; New Expectations]	**DM.SNT.1† The student will identify and use the properties of sets and set operations.** a) Compare and contrast sets, relations, and functions. b) Express relationships between sets using Venn diagrams. c) Describe a set using set-builder notation. d) Construct new sets using the set operations intersection, union, difference, and complement. e) Identify the laws of set theory (e.g., associative, commutative, distributive, De Morgan's Law). f) Use the principle of inclusion and exclusion to determine the size of a set. g) Use the properties of set operations to prove set equality.
[Moved from DM.13a-d]	**DM.SNT.2† The student will apply the formulas of combinatorics.** a) Create a tree diagram to represent relationships between independent events.

2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Set and Number Theory**	2023 *Standards of Learning* Knowledge and Skills (KS) **Set and Number Theory (SNT)**
	b) Use the Fundamental (Basic) Counting Principle to determine the number of possible outcomes of an event. c) Determine the number of combinations possible when subsets of r elements are selected from a set of n elements without regard to order. d) Determine the number of permutations possible when r objects selected from n objects are ordered. e) Use the pigeonhole principle to solve packing problems to facilitate proofs. f) Construct a proof by induction using principles of combinatorics.
[Moved from Mathematical Analysis]	**DM.SNT.3 The student will use Pascal's Triangle to analyze numerical patterns and relationships.** a) Construct Pascal's Triangle. b) Expand binomials having positive integral exponents, using the Binomial Theorem and Pascal's Triangle. c) Compare the binomial coefficient to the calculation of combinations. d) Identify the Fibonacci numbers within Pascal's Triangle.

2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Graph Theory (GT)**	2023 *Standards of Learning* Knowledge and Skills (KS) **Graph Theory (GT)**
DM.1† The student will model problems, using vertex-edge graphs. The concepts of valence, connectedness, paths, planarity, and directed graphs will be investigated. ● Determine the valence of each vertex in a graph. ● Use graphs to model situations in which the vertices represent objects, and edges (drawn between vertices) represent a particular relationship between objects. ● Represent the vertices and edges of a graph as an adjacency matrix, and use the matrix to solve problems. ● Investigate and describe valence and connectedness. ● Determine whether a graph is planar or nonplanar. ● Use directed graphs (digraphs) to represent situations with restrictions in traversal possibilities.	**DM.GT.1† The student will represent problems using vertex-edge graphs. The concepts of degree, connectedness, paths, planarity, and directed graphs will be analyzed.** a) Illustrate the basic terminology of graph theory (e.g., vertex, edge, graph, degree of a vertex). b) Use graphs to map situations in which the vertices represent objects, and edges represent a particular relationship between objects. c) Identify and describe degree and connectedness. d) Determine whether a graph is planar or nonplanar. e) Analyze the relationship between faces, edges, and vertices using Euler's formula $(F = E - V + 2)$. f) Use directed graphs (digraphs) to represent situations with restrictions in traversal possibilities. g) Determine when graphs are trees.
DM.2† The student will solve problems through investigation and application of circuits, cycles, Euler paths, Euler Circuits, Hamilton paths, and Hamilton circuits. Optimal solutions will be sought using existing algorithms and student-created algorithms. ● Determine whether a graph has an Euler circuit or path, and determine it, if it exists. ● Determine whether a graph has a Hamilton circuit or path, and determine it, if it exists. ● Count the number of Hamilton circuits for a complete graph with n vertices.	**DM.GT.2† The student will solve problems through analysis and application of circuits, cycles, Euler paths, Euler circuits, Hamilton paths, and Hamilton circuits. Optimal solutions will be determined using existing algorithms and student-created algorithms.** a) Determine whether a graph has an Euler circuit or path, and determine the circuit or path, if it exists. b) Determine whether a graph has a Hamilton circuit or path, and determine the circuit or path, if it exists. c) Count the number of Hamilton circuits for a complete graph with n vertices.

2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Graph Theory (GT)**	2023 *Standards of Learning* Knowledge and Skills (KS) **Graph Theory (GT)**
● Use an Euler circuit algorithm to solve optimization problems.	d) Use an Euler circuit algorithm to solve optimization problems.
DM.3[†] The student will apply graphs to conflict-resolution problems, such as map coloring, scheduling, matching, and optimization. ● Model projects consisting of several subtasks, using a graph. ● Use graphs to resolve conflicts that arise in scheduling. ● Determine the chromatic number of a graph	**DM.GT.3[†] The student will apply graphs to conflict-resolution problems, such as graph coloring, scheduling, matching, and optimization.** a) Model projects consisting of several subtasks, using a graph. b) Use graphs to resolve conflicts that arise in scheduling. c) Use graph coloring to determine the chromatic number of a graph.
DM.4 The student will apply algorithms relating to trees, networks, and paths. Appropriate technology will be used to determine the number of possible solutions and generate solutions when a feasible number exists. ● Use Kruskal's algorithm to determine the shortest spanning tree of a connected graph. ● Use Prim's algorithm to determine the shortest spanning tree of a connected graph. ● Use Dijkstra's algorithm to determine the shortest spanning tree of a connected graph	**DM.GT.4 The student will recognize and apply algorithms to solve configuration, conflict-resolution, and sorting problems.** a) Recognize algorithms such as nearest neighbor, brute force, and cheapest link as they apply to graphs. b) Use Kruskal's algorithm to determine the shortest spanning tree of a connected graph. c) Use Prim's algorithm to determine the shortest spanning tree of a connected graph. d) Use Dijkstra's algorithm to determine the shortest spanning tree of a connected graph.
[Moved from DM.10]	**DM.GT.5 The student will use algorithms to schedule tasks to determine a minimum project time.** a) Specify in a digraph the order in which tests are to be performed. b) Identify the critical path to determine the earliest completion time (minimum project time). c) Use the list-processing algorithm to determine an optimal schedule.

2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Graph Theory (GT)**	2023 *Standards of Learning* Knowledge and Skills (KS) **Graph Theory (GT)**
	d) Create and test scheduling algorithms.

2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Election Theory and Fair Division**	2023 *Standards of Learning* Knowledge and Skills (KS)
DM.5[†] The student will analyze and describe the issue of fair division in discrete and continuous cases. ● Investigate and describe situations involving discrete division (e.g., estate division). ● Use an algorithm for fair division for a group of indivisible objects. ● Investigate and describe situations involving continuous division of an infinitely divisible set (e.g., cake cutting). ● Use an algorithm for fair division of an infinitely divisible set.	**[DELETED]**

DM.6[†] The student will investigate and describe weighted voting and the results of various election methods. These may include approval and preference voting as well as plurality, majority, runoff, sequential runoff, Borda count, and Condorcet winners.
 	
● Determine in how many different ways a voter can rank choices.
● Investigate and describe the following voting procedures:
  ○ weighted voting;
  ○ plurality;
  ○ majority;
  ○ sequential (winners runoff);
  o sequential (losers are eliminated);
	[DELETED]

2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Election Theory and Fair Division**	2023 *Standards of Learning* Knowledge and Skills (KS)
○ Borda count; and ○ Condorcet winner. ● Compare and contrast different voting procedures. ● Describe the possible effects of approval voting, insincere and sincere voting, a preference schedule, and strategic voting on the election outcome.	
DM.7 The student will identify apportionment inconsistencies that apply to issues such as salary caps in sports and allocation of representatives to Congress. Historical and current methods will be compared. ● Compare and contrast the Hamilton and Jefferson methods of political apportionment with the Hill-Huntington method (currently in use in the U.S. House of Representatives) and the Webster-Willcox method. ● Solve allocation problems, using apportionment methods. ● Investigate and describe how salary caps affect apportionment.	**[DELETED]**

2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Computer Mathematics**	2023 *Standards of Learning* Knowledge and Skills (KS) **Computational Methods (CM)**
DM.8 The student will describe and apply sorting algorithms and coding algorithms used in sorting, processing, and communicating information. ● Select and apply a sorting algorithm, such as a	**DM.CM.1† The student will describe and apply sorting and searching algorithms used in processing and communicating information.**

2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Computer Mathematics**	2023 *Standards of Learning* Knowledge and Skills (KS) **Computational Methods (CM)**
o bubble sort; o merge sort; and o network sort. ● Describe and apply a coding algorithm, such as o ISBN numbers; o UPC codes; o zip codes; and o banking codes.	a) Select and apply a sorting algorithm, such as a bubble sort, merge sort, or network sort. b) Describe the advantages and disadvantages of various sorting algorithms. c) Analyze the knapsack and bin-packing problems. d) Select and apply search algorithms to analyze problems. e) Determine the average, best-case, and worst-case reasoning for different searches.
[Moved from DM.12a,c,d,e]	**DM.CM.2† The student will use recursive processes.** a) Compare and contrast iterative and recursive processes. b) Use recursive processes to model growth and decay. c) Use recursive processes to create fractals. d) Use recursive processes to generate the Fibonacci sequence. e) Determine if a recursive solution is more efficient than an iterative solution.
[New Expectation]	**DM.CM.3 The student will identify and apply cryptographic methods.** a) Compare and contrast ciphers and codes. b) Describe the evolution of cipher systems. c) Identify the Fundamental Theorem of Arithmetic. d) Describe how the complexity of prime factorization is used in cryptography. e) Describe modular arithmetic in context (e.g., clocks, days of the week, measures of time). f) Analyze the relationship between divisibility and modulus.

2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Computer Mathematics**	2023 *Standards of Learning* Knowledge and Skills (KS) **Computational Methods (CM)**
	g) Determine congruence within modular arithmetic. h) Perform operations within modular arithmetic. i) Apply modular arithmetic to problems in context (e.g., cryptography, International Standard Book Number (ISBN), International Bank Account Number (IBAN)).
[New Expectation]	**DM.CM.4 The student will analyze the limitations of algorithms and their contextual relationships in computing.** a) Describe maximum complexity of an algorithm using Big O notation. b) Describe Turing machines and how they are used to test the limits of computation. c) Describe the halting problem and explain how it characterizes the fundamental limitations of computation and undecidability. d) Explain the P versus NP problem and defend a justification for equality, inequality, or undecidability. e) Analyze how the equivalence of P- and NP-class problems might impact society.
DM.9[†] The student will select, justify, and apply an appropriate technique to solve a logic problem. ● Generate truth tables that encode the truth and falsity of two or more statements. ● Use Venn diagrams to represent set relationships, such as intersection and union. ● Interpret Venn diagrams. ● Use Venn diagrams to codify and solve logic problems. ● Use matrices as arrays of data to solve logic problems.	**[Moved to DM.LR.1[†]; DM.LR.3[†]]**

2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Computer Mathematics**	2023 *Standards of Learning* Knowledge and Skills (KS) **Computational Methods (CM)**

2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Recursion and Optimization**	2023 *Standards of Learning* Knowledge and Skills (KS)
DM.10 The student will use algorithms to schedule tasks in order to determine a minimum project time. The algorithms will include critical path analysis, the list-processing algorithm, and student-created algorithms. ● Specify in a digraph the order in which tests are to be performed. ● Identify the critical path to determine the earliest completion time (minimum project time). ● Use the list-processing algorithm to determine an optimal schedule. ● Create and test scheduling algorithms.	**[Moved to DM.GT.5]**
DM.11 The student will solve linear programming problems. ● Model practical problems with systems of linear inequalities. ● Identify the feasibility region of a system of linear inequalities with no more than four constraints. ● Identify the coordinates of the corner points of a feasibility region. ● Determine the maximum or minimum value of the system. ● Describe the meaning of the maximum or minimum value in terms of the original problem.	**[Included in AFDA]**

2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Recursion and Optimization**	2023 *Standards of Learning* Knowledge and Skills (KS)
DM.12 The student will use the recursive process and difference equations with the aid of appropriate technology to generate a) **compound interest;** b) **sequences and series;** c) **fractals;** d) **population growth models; and** e) **the Fibonacci sequence.** ● Use finite differences and recursion to model compound interest and population growth situations. ● Model arithmetic and geometric sequences and series recursively. ● Compare and contrast the recursive process, and create fractals. ● Compare and contrast the recursive process and the Fibonacci sequence. ● Determine a recursive relationship that generates the Fibonacci sequence.	**[Moved to DM.CM.2[†]; Included in Mathematical Analysis]**
DM.13 The student will apply the formulas of combinatorics in the areas of a) **the Fundamental (Basic) Counting Principle;** b) **knapsack and bin-packing problems;** c) **permutations and combinations; and** d) **the pigeonhole principle** ● Determine the number of combinations possible when subsets of r elements are selected from a set of n elements without regard to order. ● Use the Fundamental (Basic) Counting Principle to determine the number of possible outcomes of an event.	**[Moved to DM.SNT.2[†]; DM.CM.1]**

2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Recursion and Optimization**	2023 *Standards of Learning* Knowledge and Skills (KS)
• Use the knapsack and bin-packing algorithms to solve practical problems. • Determine the number of permutations possible when *r* objects selected from *n* objects are ordered. • Use the pigeonhole principle to solve packing problems to facilitate proofs.	

2023 Discrete Mathematics SOL – Summary of Changes

Discrete Mathematics (2016 SOL to 2023 SOL Numbering)	Parameter Changes/Clarifications (2023 SOL)
• DM.1† → DM.GT.1† • DM.2† → DM.GT.2† • DM.3† → DM.GT.3† • DM.4 → DM.GT.4 • DM.5† → [Deleted] • DM.6† → [Deleted] • DM.7 → [Deleted] • DM.8 → DM.CM.1† • DM.9† → DM.LR.1; DM.LR.3†; DM.SNT.1† • [New Expectation] → DM.LR.2† • [Moved from Mathematical Analysis] → DM.LR.4 • DM.10 → DM.GT.5 • DM.11 → [Included in AFDA] • DM.12a,c,d,e → DM.CM.2† • DM.12b → [Included in Mathematical Analysis]	• DM.GT.1† - Analyze the relationship between faces, edges, and vertices using Euler's formula ($F = E - V + 2$) • DM.CM.2† - Compare and contrast both iterative and recursive processes

- DM.13a,c,d → DM.SNT.2† - DM.13b → DM.CM.1† [Moved from Mathematical Analysis] → DM.SNT.3 [New Expectation] → DM.CM.3 [New Expectation] → DM.CM.4	

Deletions from Discrete Mathematics (2016 SOL)	Additions to Discrete Mathematics (2023 SOL)
• DM.5† - Analyze and describe fair division in discrete and continuous cases [Deleted] • DM.6† - Investigate and describe weighted voting and the results of various election methods [Deleted] • DM.7 - Identify apportionment inconsistencies [Deleted] • DM.8 [KS] - Describe and apply a coding algorithm [Deleted] • DM.9† [KS] - Use matrices as arrays of data to solve logic problems [Included in Mathematical Analysis] • DM.11 – Solve linear programming problems [Included in AFDA] • DM.12b – Use recursive processes to generate sequences and series [Included in Mathematical Analysis]	• DM.LR.2† - Apply logic and proof techniques in the construction of a sound argument, including DeMorgan's Law • DM.LR.3† - Apply Boolean Algebra to represent and investigate the function of logical gates and circuits [New Expectation] • DM.LR.4 - Use Mathematical Induction to prove formulas and mathematical statements [Moved from Mathematical Analysis] • DM.SNT.1† - Investigate the properties of sets, their construction, and set operations [New Expectation] • DM.SNT.3 - Use Pascal's Triangle to explore numerical patterns and relationships [Moved from Mathematical Analysis] • DM.CM.3 - Investigate and apply cryptographic methods [New Expectation] • DM.CM.4 - Explore the limitations of algorithms and their contextual relationships in computing [New Expectation]

KEY: LG = Logical Reasoning; SNT = Set and Number Theory; GT= Graph Theory; CM = Computational Methods; EKS =Essential Knowledge and Skills (2016); KS = Knowledge and Skills (2023); US = Understanding the Standard; † This standard should be included in a semester course.

Mathematical Analysis *Standards of Learning* - 2023 Overview of Revisions

This overview includes a summary of the content embedded in three content strands.

Characteristics of Functions
- Identify and analyze the properties of polynomial, rational, piecewise, absolute value, radical, and step functions and sketch the graphs of the functions
- Determine the limit of a function if it exists
- Analyze and describe the continuity of functions

Functional Relationships
- Analyze compositions of functions to determine and verify inverses of functions
- Analyze the characteristics of exponential and logarithmic functions, and sketch the graphs of the functions
- Analyze and represent sequences and finite series

Analytic Geometry
- Identify and analyze the properties of conic sections, and sketch a graph given an equation
- Use parametric equations to model and solve problems in context
- Perform operations with vectors in the coordinate plane
- Investigate and identify the characteristics of the graphs of polar equations
- Use matrices to organize data and add and subtract matrices, multiply matrices, multiply matrices by a scalar, and use matrices to solve systems of equations

Comparison of Mathematical Analysis *Standards of Learning* – 2016 to 2023

2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Functions**	2023 *Standards of Learning* Knowledge and Skills (KS) **Characteristics of Functions (CF)**

MA.1 The student will investigate and identify the properties of polynomial, rational, piecewise, and step functions and sketch the graphs of the functions.
 	
● Identify a polynomial, rational, piecewise, and step function, given an equation or graph.
● Given a graph or equation of a polynomial, rational, piecewise, or step function, identify:
  o domain and range;
  o zeros;
  o intercepts;
  o symmetry;
  o asymptotes (horizontal, vertical, and oblique/slant);
  o points of discontinuity;
  o intervals for which the function is increasing, decreasing or constant;
  o end behavior; and
  o relative and/or absolute maximum and minimum points.
● Sketch the graph of a polynomial, rational, piecewise, or step function.
● Investigate and verify characteristics of a polynomial, rational, piecewise, and step function, using a graphing utility.
● Rationalize the denominator of a rational function.
	MA.CF.1 The student will identify and analyze the properties of polynomial, rational, piecewise-defined, absolute value, radical, and step functions and sketch the graphs of the functions.
	
	a) Use mathematical reasoning to identify polynomial, rational, piecewise-defined, absolute value, radical, and step functions, given an equation or graph.
	b) Given multiple representations of a polynomial, rational, piecewise-defined, absolute value, radical, and step function, analyze:
	  i) domain and range;
	  ii) roots (including complex roots);
	  iii) intercepts;
	  iv) symmetry (including even and odd functions);
	  v) asymptotes (horizontal, vertical, and oblique/slant;
	  vi) points of discontinuity;
	  vii) intervals for which the function is increasing, decreasing or constant;
	  viii) end behavior; and
	  ix) relative and/or absolute maximum and minimum points.
	c) Sketch the graph of a polynomial, rational, piecewise-defined, absolute value, radical, and step function.

2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Functions**	2023 *Standards of Learning* Knowledge and Skills (KS) **Characteristics of Functions (CF)**
MA.2 The student will investigate and identify the characteristics of exponential and logarithmic functions to graph the function, solve equations, and solve practical problems. ● Identify exponential functions from an equation or a graph. ● Identify logarithmic functions from an equation or a graph. ● Define *e*, and know its approximate value. ● Convert between equations written in logarithmic and exponential form. ● Identify common and natural logarithms, given an equation or practical situation. ● Use laws of exponents and logarithms to solve equations and simplify expressions. ● Model practical problems, using exponential and logarithmic functions. ● Graph exponential and logarithmic functions and identify asymptotes, end behavior, intercepts, domain, and range.	**[Moved to MA.FR.2]**
MA.3 The student will apply compositions of functions and inverses of functions to practical situations and investigate and verify the domain and range of resulting functions. ● Determine the composition of functions algebraically and graphically. ● Determine the inverse of a function algebraically and graphically. ● Determine the domain and range of composite functions algebraically and graphically.	**[Moved to MA.FR.1]**

2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Functions**	2023 *Standards of Learning* Knowledge and Skills (KS) **Characteristics of Functions (CF)**
• Determine the domain and range of the inverse of a function algebraically and graphically.	

MA.4 The student will determine the limit of an algebraic function, if it exists, as the variable approaches either a finite number or infinity.
 	
• Verify estimates about the limit of a function using a graphing utility.
• Determine the limit of a function algebraically and verify with a graphing utility.
• Determine the limit of a function numerically and verify with a graphing utility.
• Use limit notation when describing end behavior of a function.
	MA.CF.2 The student will determine the limit of a function if it exists.
	
	a) Verify estimates about the limit of a function using graphing technology.
	b) Determine the limit of a function algebraically and verify with graphing technology.
	c) Determine the limit of a function numerically and verify with graphing technology.
	d) Use proper limit notation, including when describing the end behavior of a function.
	e) As the variable approaches a finite number,
	   i) determine the limit of a function numerically by direct substitution;
	   ii) determine the limit of a function using algebraic manipulation;
	   iii) estimate the limit of a function using a table; and
	   iv) determine the limit of a function from a given graph.
	f) As the variable approaches positive or negative infinity, analyze the limit of a function to describe the end behavior.

MA.5 The student will investigate and describe the continuity of functions. • Describe continuity of a function.	**MA.CF.3 The student will analyze and describe the continuity of functions.** a) Describe continuity of a function.

2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Functions**	2023 *Standards of Learning* Knowledge and Skills (KS) **Characteristics of Functions (CF)**
• Investigate the continuity of functions including absolute value, step, rational, and piecewise functions, using graphical and algebraic methods. • Classify types of discontinuity. • Prove continuity at a point, using the definition of limits.	b) Use mathematical notation to communicate and describe the continuity of functions including polynomial, rational, piecewise, absolute value, radical, and step function, using graphical and algebraic methods. c) Prove continuity at a point, using the definition. d) Classify types of discontinuity based on which condition of continuity is violated.

2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Equations**	2023 *Standards of Learning* Knowledge and Skills (KS) **Functional Relationships (FR)**
[Moved from MA.3]	**MA.FR.1 The student will analyze compositions of functions to determine and verify inverses of functions.** a) Construct the composition of functions algebraically and graphically. b) Determine the domain and range of composite functions algebraically and graphically. c) Develop the inverse of a function algebraically and graphically. d) Compare the domain and range of the inverse of a function with the original function, both algebraically and graphically. e) Use mathematical reasoning to generalize and communicate the criteria for an inverse function to exist.

2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Equations**	2023 *Standards of Learning* Knowledge and Skills (KS) **Functional Relationships (FR)**
[Moved from MA.2]	**MA.FR.2 The student will analyze the characteristics of exponential and logarithmic functions, and sketch the graphs of the functions.** a) Generalize characteristics of exponential and logarithmic functions from an equation or a graph. b) Define e and estimate its value. c) Convert between equations written in logarithmic and exponential form. d) Use laws of exponents and properties of logarithms to solve equations and simplify expressions. e) Represent contextual problems, using exponential and logarithmic functions, to include common and natural logarithms. f) Sketch the graph of exponential and logarithmic functions and identify asymptotes, end behavior, intercepts, domain, and range.
[Moved from MA.13 and AII.5]	**MA.FR.3 The student will analyze sequences and finite series, and model and solve problems in context using sequences and series.** a) Use and interpret the notation: $\sum$, n, n^{th}, and a_n. b) Derive the formulas associated with arithmetic and geometric sequences and series. c) Determine the nth term, a_n, for an arithmetic or geometric sequence. d) Determine the sum, S_n, if it exists, of an arithmetic or geometric series.

2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Equations**	2023 *Standards of Learning* Knowledge and Skills (KS) **Functional Relationships (FR)**
	e) Model and solve problems in context, using sequences and series. f) Distinguish between a convergent and divergent series. g) Describe convergent series in relation to the concept of a limit.

2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Analytic Geometry**	2023 *Standards of Learning* Knowledge and Skills (KS) **Analytic Geometry (AG)**
MA.6 The student will investigate, graph, and identify the properties of conic sections from equations in vertex and standard form. ● Given a translation or rotation matrix, determine an equation for the transformed function or conic section. ● Investigate and verify graphs of transformed conic sections, using a graphing utility. ● Graph conic sections from equations written in vertex or standard form using transformations. ● Identify properties of conic sections.	**MA.AG.1 The student will identify and analyze the properties of conic sections and sketch a graph given an equation.** a) Given a translation or rotation matrix, determine an equation for the transformed function or conic section. b) Convert between standard and general forms of conic equations by completing the square. c) Graph conic sections from equations written in general or standard form using transformations. d) Identify characteristics of conic sections including center, vertices, axes, symmetry, foci, directrix, eccentricity, and asymptotes. e) Represent applications of conic sections.
[Moved from MA.10]	**MA.AG.2 The student will use parametric equations to model and solve problems in context.** a) Graph and analyze parametric equations and use the graph to determine solutions.

2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Analytic Geometry**	2023 *Standards of Learning* Knowledge and Skills (KS) **Analytic Geometry (AG)**
	b) Use parametric equations to model contextual problems, including motion over time.
MA.7 The student will perform operations with vectors in the coordinate plane and solve practical problems using vectors. ● Use vector notation. ● Perform the operations of addition, subtraction, scalar multiplication, and inner (dot) product on vectors. ● Graph vectors and resultant vectors. ● Express complex numbers in vector notation. ● Identify properties of vector addition, scalar multiplication, and dot product. ● Determine the components of a vector. ● Determine the norm (magnitude) of a vector. ● Use vectors in simple geometric proofs. ● Solve problems, including practical problems, using vectors.	**MA.AG.3 The student will perform operations with vectors in the coordinate plane.** a) Use vector notation. b) Perform the operations of addition, subtraction, and scalar multiplication, graphically and algebraically on vectors. c) Find the dot (inner) product of two vectors and use it to determine the angle between two vectors. d) Determine if two vectors are orthogonal. e) Express complex numbers in vector notation. f) Verify properties of the dot product. g) Determine the components of a vector. h) Determine the norm (magnitude) of a vector. i) Find a unit vector in the same direction of a given vector. j) Apply vectors to problems in context.
MA.8 The student will identify, create, and solve practical problems involving triangles. ● Solve and create problems, including practical problems, using trigonometric functions. ● Solve and create problems, including practical problems, using the Pythagorean Theorem. ● Solve and create problems, including practical problems, using the Law of Sines and the Law of Cosines.	**[Included in Trigonometry]**

2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Analytic Geometry**	2023 *Standards of Learning* Knowledge and Skills (KS) **Analytic Geometry (AG)**
● Solve problems, including practical problems, where triangles are formed from vectors.	
MA.9 The student will investigate and identify the characteristics of the graphs of polar equations. ● Classify polar equations (rose, cardioid, limaçon, lemniscate, spiral, and circle), given the graph or the equation. ● Determine the effects of changes in the parameters of polar equations on the graph, using a graphing utility. ● Convert between complex numbers written in rectangular form and polar form. ● Determine and verify the intersection of the graphs of two polar equations, using a graphing utility.	**MA.AG.4 The student will investigate and identify the characteristics of the graphs of polar equations.** a) Classify polar equations (rose, cardioid, limaçon, lemniscate, spiral, and circle), given the graph or the equation. b) Determine the effects of changes in the parameters of polar equations on the graph, using graphing technology. c) Convert between polar and rectangular forms of coordinates. d) Convert between complex numbers written in rectangular form and polar form. e) Convert equations between polar and rectangular forms. f) Determine and verify the intersection of the graphs of two polar equations, using graphing technology.
MA.10 The student will use parametric equations to model and solve practical problems. ● Graph parametric equations. ● Use parametric equations to model practical problems, including motion over time. ● Determine solutions to parametric equations graphically. ● Use a graphing utility to graph and analyze parametric equations.	**[Moved to MA.AG.2]**
MA.11 The student will use matrices to organize data and will add and subtract matrices, multiply matrices, multiply matrices by a scalar, and use matrices to solve systems of equations. ● Multiply matrices by a scalar.	**MA.AG.5 The student will use matrices to organize data and will add and subtract matrices, multiply matrices, multiply matrices by a scalar, and use matrices to solve systems of equations.** a) Multiply matrices by a scalar.

2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Analytic Geometry**	2023 *Standards of Learning* Knowledge and Skills (KS) **Analytic Geometry (AG)**
● Add, subtract, and multiply matrices. ● Model problems with a system of no more than three linear equations. ● Express a system of linear equations as a matrix equation. ● Solve a system of equations using matrices. ● Determine the inverse of a two-by-two or three-by-three matrix using paper and pencil. ● Verify two matrices are inverses using matrix multiplication. ● Verify the commutative and associative properties for matrix addition and multiplication.	b) Add, subtract, and multiply matrices. c) Represent problems with a system of no more than three linear equations. d) Express a system of linear equations as a matrix equation. e) Solve a system of equations using matrices. f) Determine the inverse of a two-by-two or three-by-three matrix using paper and pencil. g) Verify two matrices are inverses using matrix multiplication. h) Verify the commutative and associative properties for matrix addition and multiplication.

2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Discrete Mathematics**	2023 *Standards of Learning* Knowledge and Skills (KS)
MA.12 The student will expand binomials having positive integral exponents. ● Expand binomials having positive integral exponents. ● Use the Binomial Theorem, the formula for combinations, and Pascal's Triangle to expand binomials.	**[Moved to Discrete Mathematics]**

2016 *Standards of Learning* Essential Knowledge and Skills (EKS) **Discrete Mathematics**	2023 *Standards of Learning* Knowledge and Skills (KS)
MA.13 The student will determine the sum of finite and infinite convergent series. ● Use and interpret the notation: $\sum$, n, n^{th}, and a_n. ● Derive the formulas associated with arithmetic and geometric sequences and series. ● Given the formula, determine the nth term, a_n, for an arithmetic or geometric sequence. ● Given the formula, determine the sum, S_n, if it exists, of an arithmetic or geometric series. ● Model and solve problems, using sequence and series information. ● Distinguish between a convergent and divergent series. ● Discuss convergent series in relation to the concept of a limit.	**[Moved to MA.FR.3]**
MA.14 The student will use mathematical induction to prove formulas and mathematical statements. ● Compare inductive and deductive reasoning. ● Prove formulas and mathematical statements, using mathematical induction.	**[Moved to Discrete Mathematics]**

2023 Mathematical Analysis SOL – Summary of Changes

Mathematical Analysis (2016 SOL to 2023 SOL Numbering)	Parameter Changes/Clarifications (2023 SOL)
- MA.1 → MA.CF.1 - MA.2 → MA.FR.2 - MA.3 → MA.FR.1 - MA.4 → MA.CF.2 - MA.5 → MA.CF.3 - MA.6 → MA.AG.1 - MA.7 → MA.AG.3 - MA.8 → [Included in Trigonometry] - MA.9 → MA.AG.4 - MA.10 → MA.AG.2 - MA.11 → MA.AG.5 - MA.12 → [Moved to Discrete Mathematics] - MA.13 → MA.FR.3 - MA.14 → [Moved to Discrete Mathematics]	- MA.CF.2 [KS] - More specific parameters defined for determining and estimating the limit of a function and using limits to describe end behavior - MA.CF.3 [KS] - Continuity of functions addresses polynomial and radical functions; classify types of discontinuity based on which condition of continuity is violated - MA.FR.1 [KS] - Generalize and communicate the criteria for an inverse function to exist - MA.AG.1 [KS] - Convert between standard and general forms of conic equations by completing the square; characteristics of conics to be identified are specified; model applications of conics

Deletions from Mathematical Analysis (2016 SOL)	Additions to Mathematical Analysis (2023 SOL)
- MA.1 [EKS] Rationalize the denominator of a rational function [Deleted] - MA.6 [EKS] Given a translation or rotation matrix, determine an equation for a transformed function or conic section [Deleted] - MA.8 - Identify, create, and solve practical problems involving triangles [included in Trigonometry] - MA.12 - Expand binomials having positive integral exponents [included in Discrete Mathematics] - MA.13 [EKS] - Determine the sum of infinite convergent series [Deleted] - MA.14 - Use mathematical induction to prove formulas and mathematical statements [Included in Discrete Mathematics]	- MA.CF.1 [KS] - Investigate and identify the properties of absolute value and radical functions given an equation or graph and sketch the graphs of the functions - MA.AG.3 [KS] - Determine if two vectors are orthogonal

KEY: CF = Characteristics of Functions; FR = Functional Relationships; AG= Analytic Geometry; EKS =Essential Knowledge and Skills (2016); KS = Knowledge and Skills (2023); US = Understanding the Standard