Multistate Standard-Setting Technical Report

PRAXIS® MIDDLE SCHOOL MATHEMATICS (5164)

Licensure and Credentialing Research

ETS

Princeton, New Jersey

February 2021

EXECUTIVE SUMMARY

To support the decision-making process of education agencies establishing a passing score (cut score) for the *Praxis*® Middle School Mathematics (5164) test, research staff from Educational Testing Service (ETS) designed and conducted a distance-based multistate standard-setting study.

PARTICIPATING STATES

Panelists from 12 states and Washington, D.C., were recommended by their respective education agencies. The education agencies recommended panelists with (a) experience as either middle school mathematics teachers or college faculty who prepare middle school mathematics teachers and (b) familiarity with the knowledge and skills required of beginning middle school mathematics teachers.

RECOMMENDED PASSING SCORE

ETS provides a recommended passing score from the multistate standard-setting study to help education agencies determine an appropriate operational passing score. For the *Praxis* Middle School Mathematics test, the recommended passing score[1] is 38 out of a possible 60 raw-score points. The scale score associated with a raw score of 38 is 157 on a 100–200 scale.

[1] Results from the two panels participating in the study were averaged to produce the recommended passing score.

INTRODUCTION

To support the decision-making process for education agencies establishing a passing score (cut score) for the *Praxis*® Middle School Mathematics (5164) test, research staff from ETS designed and conducted a distance-based multistate standard-setting study in January 2021. Education agencies[2] recommended panelists with (a) experience as either middle school mathematics teachers or college faculty who prepare middle school mathematics teachers and (b) familiarity with the knowledge and skills required of beginning middle school mathematics teachers. Twelve states and Washington, D.C., were represented by 29 panelists, as listed in Table 1. (See Appendix A for the names and affiliations of the panelists.)

Table 1
Participating States, Washington, D.C., and Number of Panelists

Alabama (2 panelists)	Rhode Island (3 panelists)
Delaware (3 panelists)	South Carolina (2 panelists)
Idaho (3 panelists)	South Dakota (2 panelists)
Indiana (2 panelists)	Tennessee (2 panelists)
Kansas (2 panelists)	Washington, D.C. (2 panelists)
Kentucky (1 panelist)	West Virginia (2 panelists)
Mississippi (3 panelists)	

The following technical report contains three sections. The first section describes the content and format of the test. The second section describes the standard-setting processes and methods. The third section presents the results of the standard-setting study.

ETS provides a recommended passing score from the multistate standard-setting study to education agencies. In each state and D.C., the department of education, the board of education, or a designated educator licensure board is responsible for establishing the operational passing score in accordance with applicable regulations. This study provides a recommended passing score,[3] which represents the combined judgments of two panels of experienced educators. Each state and D.C., may want to consider the recommended passing score but also other sources of information when setting the final *Praxis* Middle School Mathematics passing score (see Geisinger & McCormick, 2010). A state and

[2] States and jurisdictions that currently use *Praxis* tests were invited to participate in the multistate standard-setting study.
[3] In addition to the recommended passing score averaged across the two panels, the passing scores for each panel are presented.

D.C., may accept the recommended passing score, adjust the score upward to reflect more stringent expectations, or adjust the score downward to reflect more lenient expectations. There is no *correct* decision; the appropriateness of any adjustment may only be evaluated in terms of its meeting the state and D.C.'s, needs.

Two sources of information to consider when setting the passing score are the standard error of measurement (SEM) and the standard error of judgment (SEJ). The former addresses the reliability of the *Praxis* Middle School Mathematics test score and the latter, the reliability of panelists' passing-score recommendation. The SEM allows a state and D.C., to recognize that any test score on any standardized test—including a *Praxis* Middle School Mathematics test score—is not perfectly reliable. A test score only *approximates* what a candidate truly knows or truly can do on the test. The SEM, therefore, addresses the question: How close of an approximation is the test score to the *true* score? The SEJ allows a state and D.C. to gauge the likelihood that the recommended passing score from a particular panel would be similar to the passing scores recommended by other panels of experts similar in composition and experience. The smaller the SEJ, the more likely that another panel would recommend a passing score consistent with the recommended passing score. The larger the SEJ, the less likely the recommended passing score would be reproduced by another panel.

In addition to measurement error metrics (e.g., SEM, SEJ), each state and D.C. should consider the likelihood of classification errors. That is, when adjusting a passing score, policymakers should consider whether it is more important to minimize a false-positive decision or to minimize a false-negative decision. A false-positive decision occurs when a candidate's test score suggests that he should receive a license/certificate, but his actual level of knowledge/skills indicates otherwise (i.e., the candidate does not possess the required knowledge/skills). A false-negative decision occurs when a candidate's test score suggests that she should not receive a license/certificate, but she actually does possess the required knowledge/skills. The state and D.C. need to consider which decision error is more important to minimize.

OVERVIEW OF THE *PRAXIS*® MIDDLE SCHOOL MATHEMATICS TEST

The Praxis® Middle School Mathematics *Study Companion* document (ETS, in press) describes the purpose and structure of the test. In brief, the test measures knowledge and competencies that are important for safe and effective beginning practice as a middle school mathematics teacher.

The three-hour assessment contains 66 selected-response items[4] covering five content areas: *Numbers and Operations* (approximately 16 items), *Algebra* (approximately 15 items), *Functions* (approximately 11 items), *Geometry and Measurement* (approximately 13 items), and *Statistics and Probability* (approximately 11 items).[5] The reporting scale for the *Praxis* Middle School Mathematics test ranges from 100 to 200 scale-score points.

PROCESSES AND METHODS

The design of the standard-setting study included two, independent expert panels of educators with experience with the test content and with new teachers or teacher candidates. Before the study, panelists received an email explaining the purpose of the standard-setting study and requesting that they review materials for the study, such as the test specifications and an overview presentation. This review helped familiarize the panelists with the general structure and content of the test. Additionally, panelists were asked to attend a brief, technology check meeting, to ensure that everyone could access the technology needed for the study.

For each panel, the first day of the standard-setting study began with a welcome by the meeting facilitator. After introductions of the panelists and ETS staff, the facilitator engaged the panel in a question and answer period about the overview presentation. Appendix B shows the agenda for the panel meeting.

[4] Six of the 66 selected-response items are pretest items and do not contribute to a candidate's score.
[5] The number of items for each content area may vary slightly from form to form of the test.

Reviewing the Test

Test familiarization was the first activity for the panel. The purpose of test familiarization is for the panelists to review the test and become familiar with the manner in which a candidate would take the test. After the facilitator described the purpose of the review and how to access the test[6], the standard-setting panelists took the test and had a discussion of the content measured. This discussion helped bring the panelists to a shared understanding of what the test measures.

The test discussion covered the major content areas being addressed by the test. Panelists were asked to remark on any content areas that would be particularly challenging for entry-level teachers or areas that address content particularly important for entry-level teachers. Overall, this discussion serves to reduce potential judgment errors later in the standard-setting process.

Defining the Just Qualified Candidate

Following the review of the test, panelists described the just qualified candidate. The *just qualified candidate description* plays a central role in standard setting (Perie, 2008); the goal of the standard-setting process is to identify the test score that aligns with this description.

Both panels worked together to create the final description of the just qualified candidate — the knowledge/skills that differentiate a *just* from a *not quite* qualified candidate. Each panel first worked separately by working in smaller and then a large group. Then both panels convened and, through whole-group discussion, combined the two descriptions in to the final version of the just qualified candidate to use for the remainder of the study.

The written description of the just qualified candidate summarized the panel discussion in a bulleted format. The description was not intended to describe all the knowledge and skills of the just qualified candidate but only highlight those that differentiate a *just* qualified candidate from a *not quite* qualified candidate. The written description was distributed to panelists to use during later phases of the study (see Appendix C for the just qualified candidate description).

[6] The computer-administered test items were available through the ETS IBIS Content Review Tool.

PANELISTS' JUDGMENTS

The standard-setting process for the *Praxis* Middle School Mathematics test was a probability-based Modified Angoff method (Brandon, 2004; Hambleton & Pitoniak, 2006). In this study, each panelist judged each item on the likelihood (probability or chance) that the just qualified candidate would answer the item correctly. Panelists made their judgments using the following rating scale: 0, .05, .10, .20, .30, .40, .50, .60, .70, .80, .90, .95, 1. The lower the value, the less likely it is that the just qualified candidate would answer the item correctly because the item is difficult for the just qualified candidate. The higher the value, the more likely it is that the just qualified candidate would answer the item correctly.

Panelists were asked to approach the judgment process in two stages. First, they reviewed both the description of the just qualified candidate and the item and determined what was the probability that the just qualified candidate would answer the question correctly. The facilitator encouraged the panelists to consider the following rules of thumb to guide their decision:

- Items in the 0 to .30 range were those the just qualified candidate would have a low chance of answering correctly.
- Items in the .40 to .60 range were those the just qualified candidate would have a moderate chance of answering correctly.
- Items in the .70 to 1 range were those that the just qualified candidate would have a high chance of answering correctly.

Next, panelists decided how to refine their judgment within the range. For example, if a panelist thought that there was a high chance that the just qualified candidate would answer the question correctly, the initial decision would be in the .70 to 1 range. The second decision for the panelist was to judge if the likelihood of answering it correctly is .70, .80, .90, .95 or 1.

After the training, panelists made practice judgments and discussed those judgments and their rationales. All panelists completed a post-training evaluation to confirm that they had received adequate training and felt prepared to continue; the standard-setting process continued only if all panelists confirmed their readiness.

Following this first round of judgments (*Round 1*), item-level feedback was provided to the panel. The panelists' judgments were displayed for each item and summarized across panelists. Items were highlighted to show when panelists converged in their judgments (at least two-thirds of the panelists located an item in the same difficulty range) or diverged in their judgments.

The panelists discussed their item-level judgments. These discussions helped panelists maintain a shared understanding of the knowledge/skills of the just qualified candidate and helped to clarify aspects of items that might not have been clear to all panelists during the Round 1 judgments. The purpose of the discussion was not to encourage panelists to conform to another's judgment, but to understand the different relevant perspectives among the panelists.

In Round 2, panelists discussed their Round 1 judgments and were encouraged by the facilitator (a) to share the rationales for their judgments and (b) to consider their judgments in light of the rationales provided by the other panelists. Panelists recorded their Round 2 judgments only for items when they wished to change a Round 1 judgment. Panelists' final judgments for the study, therefore, consist of their Round 1 judgments and any adjusted judgments made during Round 2.

Other than the description of the just qualified candidate, results from Panel 1 were not shared with Panel 2. The item-level judgments and resulting discussions for Panel 2 were independent of judgments and discussions that occurred with Panel 1.

RESULTS

EXPERT PANELS

Table 2 presents a summary of the panelists' demographic information. The panel included 29 educators representing 12 states and D.C. (See Appendix A for a listing of panelists.) Fourteen panelists were teachers, two were mathematics instructional coaches, ten were college faculty, two were administrators or department heads, and two held other positions. All of the faculty members' job responsibilities included the training, supervising, or mentoring of middle school mathematics teachers.

The number of experts by panel and their demographic information are presented in Appendix D (Table D1). One panelist was college faculty and a department head. Another panelist was college faculty and a coach (though not specifically described as a mathematics instructional coach). As such, those numbers will not sum to 29 and the percentages will exceed 100.

Table 2

Panel Member Demographics (Across Panels)

	N	%
Current position		
Teacher	14	48
Mathematics Instructional Coach	2	7
Administrator/Department head	2	7
College faculty	10	34
Other	2	7
Race		
White	24	83
Black or African American	3	10
American Indian or Alaskan Native	1	3
Middle Eastern	1	3
Gender		
Female	23	79
Male	6	21
Are you currently certified to teach middle school mathematics in your state?		
Yes	24	83
No	5	17
Are you currently teaching middle school mathematics in your state?		
Yes	14	48
No	15	52
Are you currently supervising or mentoring other teachers of middle school mathematics?		
Yes	15	52
No	14	48
At what K–12 grade level are you currently teaching middle school mathematics?		
Elementary and Middle school	1	3
Middle school (6–8 or 7–9)	11	38
Middle and High school	1	3
High school (9–12 or 10–12)	1	3
All Grades	1	3
Other	1	3
Not currently teaching at the K–12 level	13	45

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Table 2 (continued)

Panel Member Demographics (Across Panels)

	N	%
Including this year, how many years of experience do you have teaching middle school mathematics?		
3 years or less	4	14
4–7 years	4	14
8–11 years	10	34
12–15 years	4	14
16 years or more	7	24
Which best describes the location of your K–12 school?		
Urban	6	21
Suburban	5	17
Rural	5	17
Not currently working at the K–12 level	13	45
If you are college faculty, are you currently involved in the training/preparation of teacher candidates in this subject?		
Yes	8	28
No	2	7
Not college faculty	19	66

STANDARD-SETTING JUDGMENTS

Table 3 summarizes the standard-setting judgments (Round 2) of panelists. The table also includes estimates of the measurement error associated with the judgments: the standard deviation of the mean and the standard error of judgment (SEJ). The SEJ is one way of estimating the reliability or consistency of a panel's standard-setting judgments.[7] It indicates how likely it would be for several other panels of educators similar in makeup, experience, and standard-setting training to the current panel to recommend the same passing score on the same form of the test. The confidence intervals created by adding/subtracting two SEJs to each panel's recommended passing score overlap, indicating that they may be comparable.

Panelist-level results, for Rounds 1 and 2, are presented in Appendix D (Table D2).

[7] An SEJ assumes that panelists are randomly selected and that standard-setting judgments are independent. It is seldom the case that panelists are randomly sampled, and only the first round of judgments may be considered independent. The SEJ, therefore, likely underestimates the uncertainty of passing scores (Tannenbaum & Katz, 2013).

Table 3

Summary of Round 2 Standard-setting Judgments

	Panel 1	Panel 2
Average	37.96	36.28
Lowest	33.65	28.10
Highest	43.85	40.45
SD	3.57	3.06
SEJ	0.92	0.82

Round 1 judgments are made without discussion among the panelists. The most variability in judgments, therefore, is typically present in the first round. Round 2 judgments, however, are informed by panel discussion; thus, it is common to see a decrease both in the standard deviation and SEJ. The Round 2 average score is the panel's recommended passing score.

The panels' passing score recommendations for the *Praxis* Middle School Mathematics test are 37.96 for Panel 1 and 36.28 for Panel 2 (out of a possible 60 raw-score points). The values were rounded to the next highest whole number, to determine the functional recommended passing score — 38 for Panel 1 and 37 for Panel 2. The scale scores associated with 38 and 37 raw points are 157 and 155, respectively.

In addition to the recommended passing score for each panel, the average passing score across the two panels is provided to help education agencies determine an appropriate passing score. The panels' average passing score recommendation for the *Praxis* Middle School Mathematics test is 37.12 (out of a possible 60 raw-score points). The value was rounded to 38 (next highest raw score) to determine the functional recommended passing score. The scale score associated with 38 raw points is 157.

Table 4 presents the estimated conditional standard error of measurement (CSEM) around the recommended passing score. A standard error represents the uncertainty associated with a test score. The scale scores associated with one and two CSEM above and below the recommended passing score are provided. The conditional standard error of measurement provided is an estimate.

Table 4

Passing Scores Within 1 and 2 CSEM of the Recommended Passing Score[8]

Recommended passing score (CSEM)		Scale score equivalent
38 (3.76)		157
-2 CSEM	31	141
-1 CSEM	35	150
+ 1 CSEM	42	166
+ 2 CSEM	46	175

Note. **CSEM = conditional standard error(s) of measurement.**

FINAL EVALUATIONS

The panelists completed an evaluation at the conclusion of their standard-setting study. The evaluation asked the panelists to provide feedback about the quality of the standard-setting implementation and the factors that influenced their decisions. The responses to the evaluation provided evidence of the validity of the standard-setting process, and, as a result, evidence of the reasonableness of the recommended passing score.

Panelists were also shown the panel's recommended passing score and asked (a) how comfortable they are with the recommended passing score and (b) if they think the score was too high, too low, or about right. A summary of the final evaluation results is presented in Appendix D.

All panelists *strongly agreed* or *agreed* that they understood the purpose of the study and that the facilitator's instructions and explanations were clear. All panelists *strongly agreed* or *agreed* that they were prepared to make their standard-setting judgments. All panelists *strongly agreed* or *agreed* that the standard-setting process was easy to follow.

All panelists reported that the description of the just qualified candidate was at least *somewhat influential* in guiding their standard-setting judgments; 27 of the 29 panelists indicated the description was *very influential*. All of the panelists reported that between-round discussions were at least *somewhat influential* in guiding their judgments. More than half of the panelists (25 of the 29 panelists) indicated that their own professional experience was *very influential* in guiding their judgments.

All of the panelists indicated they were at least *somewhat comfortable* with the passing score they recommended; 24 of the 29 panelists were *very comfortable*. Twenty-seven of the 29 panelists indicated the recommended passing score was *about right* and two indicated that the passing score was *too low*.

[8] The unrounded CSEM value is added to or subtracted from the rounded passing-score recommendation. The resulting values are rounded up to the next-highest whole number and the rounded values are converted to scale scores.

SUMMARY

To support the decision-making process for education agencies establishing a passing score (cut score) for the *Praxis* Middle School Mathematics test, research staff from ETS designed and conducted a distance-based multistate standard-setting study.

ETS provides a recommended passing score from the multistate standard-setting study to help education agencies determine an appropriate operational passing score. For the *Praxis* Middle School Mathematics test, the recommended passing score[9] is 38 out of a possible 60 raw-score points. The scale score associated with a raw score of 38 is 157 on a 100–200 scale.

[9] Results from the two panels participating in the study were averaged to produce the recommended passing score.

REFERENCES

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APPENDIX A

PANELISTS' NAMES & AFFILIATIONS

Participating Panelists With Affiliation

Panelist	**Affiliation**
Ahmad Alhammouri	Jacksonville State University (AL)
Sandra Ammons	South Carolina Department of Education (SC)
Cathy Boutin	RI Math Teachers Association; Association of Teachers of Mathematics in New England (RI)
Nichole Bowman	Georgia Morse Middle School (SD)
Jeneva Clark	University of Tennessee (TN)
Beth Costner	Winthrop University (SC)
Darin DeNeal	Pendleton Heights Middle School (IN)
Ashley Digmann	Dakota Wesleyan University (SD)
Sherita Flake	Urban Teachers (DC)
Tekeeta Funchess	Hinds County Schools (MS)
Tracy Graham	North Providence School Department (RI)
Tiffany Hackendorn	Indian River School District (DE)
Shauna Hedgepeth	Purvis Middle School (MS)
Jennifer Heitman	Kamiah Middle School (ID)
Shannon Henderson	Putnam County Schools (WV)
Paul Johanson	Brigham young University - Idaho (ID)
Karen Lindsey	Germantown Middle School (MS)
Robin Magruder	Campbellsville University (KY)
Michelle Northshield	Red Clay Consolidated School District (DE)
William Reaves Jr.	Capital School District (DE)

(table continues)

Participating Panelists With Affiliation (continued)

Panelist	**Affiliation**
Terry Reed	West Virginia State University (WV)
Keri Richburg	Troy University (AL)
Diane Rodriguez	Bloomfield Jr./Sr. High School (IN)
Debra Scarpelli	Rhode Island Department of Education/Pawtucket School Department (RI)
Janet Stramel	Fort Hays State University (KS)
Jackie Vogel	Austin Peay State University (TN)
Kristopher Wallaert	Idaho State Department of Education (ID)
Katherine Wiechman	Maize Middle School (KS)

*One panelist did not wish to be listed in the final report.

APPENDIX B

STUDY AGENDA

AGENDA

Praxis® Middle School Mathematics (5164) Standard-Setting Study

Day 1

Welcome and Introduction

Overview of Standard Setting and the *Praxis* Middle School Mathematics Test

Review the *Praxis* Middle School Mathematics Test

Discuss the *Praxis* Middle School Mathematics Test

Lunch

Define the Knowledge/Skills of a Just Qualified Candidate

Break

Define the Just Qualified Candidate *(continued)*

End of Day 1

AGENDA

Praxis® Middle School Mathematics (5164) Standard-Setting Study

Day 2

Overview of Day 2

Define the Just Qualified Candidate *(continued)*

Standard-setting training presentation

Practice Round: Selected-response standard-setting judgments

Break

Practice Round: Data Discussion

Lunch Break

Round 1: Selected-response standard-setting judgments

Break

Round 1: Selected-response standard-setting judgments *(continued)*

End of Day 2

AGENDA

Praxis® Middle School Mathematics (5164) Standard-Setting Study

Day 3

Overview of Day 3

Round 1 Feedback and Round 2 Judgments

Break

Round 1 Feedback and Round 2 Judgments *(continued)*

Break

Feedback on Round 2 Recommended Cut Score

Complete Final Evaluation

End of Study

APPENDIX C

JUST QUALIFIED CANDIDATE DESCRIPTION

Description of the Just Qualified Candidate[10]

A just qualified candidate …

Numbers and Operations

1. Understands proportional reasoning and ratios relationships
2. Understands rational number operations and properties to solve problems (standard and real world)
3. Understand basic concepts of number theory (e.g., prime numbers, factors, exponential rules)
4. Knows how to recognizes the reasonableness of results within the context of a given problem
5. Is familiar with evaluating student work to identify misconceptions and valid explanations of mathematical concepts

Algebra

6. Understands linear equations and systems of two linear equations (solve; represent in multiple forms)
7. Understands representations of one-variable linear inequalities
8. Is familiar with systems of linear inequalities
9. Understands linear relationships in various forms (table, graph, description, equation, etc.)
10. Is familiar with representations of quadratic equations and expressions

Functions

11. Knows how to use and evaluate basic functions that model given information in a variety of contexts through multiple representations (sequence, function notation, tables, and graphs, etc.)
12. Is familiar with absolute value, quadratic, and exponential functions that model given information through multiple representations.
13. Understands the common characteristics and shape of the graph of basic functions, including domain, range, minimum/maximum, slope, and intercepts
14. Is familiar with common characteristics and shape of graph of absolute value, quadratic, and exponential functions

Geometry and Measurement

15. Understands multi-step applications of basic geometric concepts (including area, Pythagorean Theorem, surface area, perimeter, volume, angles/lines, characteristics of shapes, etc.)
16. Knows geometric relationships (e.g., basic transformations, distance, similarity, congruence, systems of measurement)
17. Knows the basic characteristics and properties of circles, as well as triangles, quadrilaterals, and other polygons

[10] Description of the just qualified candidate focuses on the knowledge/skills that differentiate a *just* from a *not quite* qualified candidate.

Description of the Just Qualified Candidate (continued)

A just qualified candidate …

Probability and Statistics

18. Knows how to appropriately collect, interpret, analyze and represent data in various forms and identifies which form is most appropriate in a given situation (e.g., scatter plots, box and whisker plots, stem and leaf, etc.)
19. Understands measures of central tendency
20. Is familiar with variability and can compare two or more data sets
21. Understands how to use basic probability models including those in real world contexts

APPENDIX D

RESULTS

Table D1

Panel Member Demographics (by Panel)

	Panel 1		Panel 2	
	N	**%**	***N***	**%**
Current position				
Teacher	8	53	6	43
Mathematics Instructional Coach	2	13	0	0
Administrator/Department head	1	7	1	7
College faculty	4	27	5	36
Other	0	0	2	14
Race				
White	13	87	11	79
Black or African American	1	7	2	14
American Indian or Alaskan Native	0	0	1	7
Middle Eastern	1	7	0	0
Gender				
Female	13	87	10	71
Male	2	13	4	29
Are you currently certified to teach middle school mathematics in your state?				
Yes	11	73	13	93
No	4	27	1	7
Are you currently teaching middle school mathematics in your state?				
Yes	7	47	7	50
No	8	53	7	50
Are you currently supervising or mentoring other teachers of middle school mathematics?				
Yes	7	47	8	57
No	8	53	6	43
At what K–12 grade level are you currently teaching middle school mathematics?				
Elementary and Middle school	0	0	1	7
Middle school (6–8 or 7–9)	6	40	5	36
Middle and High school	1	7	0	0
High school (9–12 or 10–12)	1	7	0	0
All Grades	1	7	0	0
Other	1	7	0	0
Not currently teaching at the K–12 level	5	33	8	57

Table D1 (continued)

Panel Member Demographics (by Panel)

	Panel 1		Panel 2	
	N	**%**	***N***	**%**
Including this year, how many years of experience do you have teaching middle school mathematics?				
3 years or less	1	7	3	21
4–7 years	4	27	0	0
8–11 years	6	40	4	29
12–15 years	2	13	2	14
16 years or more	2	13	5	36
Which best describes the location of your K–12 school?				
Urban	4	27	2	14
Suburban	3	20	2	14
Rural	3	20	2	14
Not currently working at the K–12 level	5	33	8	57
If you are college faculty, are you currently involved in the training/preparation of teacher candidates in middle school mathematic?				
Yes	4	27	4	29
No	0	0	2	14
Not college faculty	11	73	8	57

Table D2

Passing Score Summary by Round of Judgments

	Panel 1		Panel 2	
Panelist	**Round 1**	**Round 2**	**Round 1**	**Round 2**
1	40.15	39.80	30.05	32.70
2	35.90	35.80	38.90	37.20
3	35.20	35.00	37.70	36.50
4	36.45	36.25	33.30	36.00
5	39.20	37.70	37.30	37.90
6	45.55	42.40	40.60	40.45
7	40.00	43.15	36.00	35.95
8	31.85	33.65	38.65	38.25
9	34.60	34.90	31.90	28.10
10	31.20	35.50	38.95	38.50
11	34.05	35.85	34.00	35.00
12	36.65	34.55	36.50	36.40
13	38.80	37.90	36.00	35.50
14	41.40	43.15	38.60	39.40
15	45.35	43.85		
Average	37.76	37.96	36.32	36.28
Lowest	31.20	33.65	30.05	28.10
Highest	45.55	43.85	40.60	40.45
SD	4.31	3.57	3.03	3.06
SEJ	1.11	0.92	0.81	0.82

Table D3

Final Evaluation: Panel 1

	Strongly agree		Agree		Disagree		Strongly disagree	
	N	**%**	***N***	**%**	***N***	**%**	***N***	**%**
• I understood the purpose of this study.	13	87	2	13	0	0	0	0
• The instructions and explanations provided by the facilitators were clear.	9	60	6	40	0	0	0	0
• The training in the standard-setting method was adequate to give me the information I needed to complete my assignment.	13	87	2	13	0	0	0	0
• The explanation of how the recommended passing score is computed was clear.	10	67	5	33	0	0	0	0
• The opportunity for feedback and discussion between rounds was helpful.	9	60	6	40	0	0	0	0
• The process of making the standard-setting judgments was easy to follow.	8	53	7	47	0	0	0	0

Table D3 (continued)

Final Evaluation: Panel 1

How influential was each of the following factors in guiding your standard-setting judgments?	Very influential		Somewhat influential		Not influential	
	N	%	*N*	%	*N*	%
• The description of the just qualified candidate	14	93	1	7	0	0
• The between-round discussions	8	53	7	47	0	0
• The knowledge/skills required to answer each test item	12	80	3	20	0	0
• The passing scores of other panel members	5	33	8	53	2	13
• My own professional experience	14	93	1	7	0	0

	Very comfortable		Somewhat comfortable		Somewhat uncomfortable		Very uncomfortable	
	N	%	*N*	%	*N*	%	*N*	%
• Overall, how comfortable are you with the panel's recommended passing score?	11	73	4	27	0	0	0	0

	Too low		About right		Too high	
	N	%	*N*	%	*N*	%
• Overall, the recommended passing score is:	2	13	13	87	0	0

Table D4

Final Evaluation: Panel 2

	Strongly agree		Agree		Disagree		Strongly disagree	
	N	**%**	***N***	**%**	***N***	**%**	***N***	**%**
• I understood the purpose of this study.	14	100	0	0	0	0	0	0
• The instructions and explanations provided by the facilitators were clear.	13	93	1	7	0	0	0	0
• The training in the standard-setting method was adequate to give me the information I needed to complete my assignment.	13	93	1	7	0	0	0	0
• The explanation of how the recommended passing score is computed was clear.	13	93	1	7	0	0	0	0
• The opportunity for feedback and discussion between rounds was helpful.	14	100	0	0	0	0	0	0
• The process of making the standard-setting judgments was easy to follow.	12	86	2	14	0	0	0	0

Table D4 (continued)

Final Evaluation: Panel 2

How influential was each of the following factors in guiding your standard-setting judgments?	Very influential		Somewhat influential		Not influential	
	N	%	*N*	%	*N*	%
• The description of the just qualified candidate	13	93	1	7	0	0
• The between-round discussions	9	64	5	36	0	0
• The knowledge/skills required to answer each test item	11	79	3	21	0	0
• The passing scores of other panel members	4	29	9	64	1	7
• My own professional experience	11	79	3	21	0	0

	Very comfortable		Somewhat comfortable		Somewhat uncomfortable		Very uncomfortable	
	N	%	*N*	%	*N*	%	*N*	%
• Overall, how comfortable are you with the panel's recommended passing score?	13	93	1	7	0	0	0	0

	Too low		About right		Too high	
	N	%	*N*	%	*N*	%
• Overall, the recommended passing score is:	0	0	14	100	0	0

TEST REVIEW WORKSHOP REPORT

Praxis Middle School Mathematics (5164) Test Review
Virginia
Thursday, April 29, 2021

OVERVIEW

This meeting was conducted using Zoom. The participants and agenda followed are provided in ATTENDEES and AGENDA below. RESULTS provides the panelists' feedback—captured through online polls—about the test and a recommended passing score.

The results include:

- Five of six panelists agreed that the test is appropriate for licensure as a middle school mathematics teacher, with the sixth indicating that the test assesses content beyond (i.e., above) what would be expected.
- The panel's recommended passing score—the median of their individual recommendations—is 157 on the Praxis scale (38 out of 60 points on the form reviewed). Five panelists were "very comfortable" with that recommendation and one panelist was "somewhat uncomfortable."

ATTENDEES

- Benjamin Baehr (ETS)
- Malik McKinley (ETS)
- Eric Steinhauer (ETS)
- Maggie Clemmons (VA DOE)

Panelists

1. Pamela Bailey
2. Dr. Angela Byrd-Wright
3. Shannon Fuhrman
4. Michele Giglio
5. Dena McElligott
6. Amy Siepka

AGENDA

Activity	*Approx. time*
Whole panel ▪ Welcome and Introductions ▪ Overview of the test and its development ▪ Overview of standard setting process **Agreement to proceed.**	30 mins
Breakout rooms ▪ Review of the test form (BREAK 5 mins) ▪ Discussion: What is measured?	1:50
Whole panel ▪ Discussion: What is measured? **Poll: Test review feedback.**	20 mins
Whole panel ▪ Review of standard setting results **Poll: Passing score recommendation.** ▪ Review of panel recommendation **Poll: Feedback on panel recommendation.**	20 mins

RESULTS

I. Following an overview of (i) the test's structure and content (ii) the test's development and (iii) the standard setting process, panelists responded to a poll indicating their agreement to proceed. All agreed to these two statements

(A) I understand that, by proceeding, I agree not to keep or disclose (1) secure test material provided and/or (2) any information provided specific to secure tests and/or (3) details of panel discussions of secure material, including panel recommendation.

(B) I understand that, by proceeding, I agree not to take this Praxis test any time within the next year.

II. Following a review of the test form used in standard setting, panelists discussed among themselves what they saw being measured including (1) what content they expect to be particularly challenging for candidates and (2) what content is especially important for beginning practice. Then panelists provided feedback about the test, starting with two questions to evaluate the test:

(C) How important are the knowledge and skills being assessed for effective beginning practice as a middle school mathematics teacher?

○ Very important	2
○ Important	3
○ Moderately important	1
○ Of some importance	–
○ Of little importance	–
Total	*6*

(D) Based on the knowledge and skills being assessed, is the test appropriate for licensure as a middle school mathematics teacher?

○ Yes	5
○ No	1
Total	*6*

Provided with an optional open-ended question

(E) About the knowledge and skills being assessed: Please provide any further feedback about alignment to the knowledge and skills a middle school mathematics teacher needs?

Two panelists provided further feedback:

- [from the panelist who answered "No" to (D)] *There were a number of questions that contained content well above what is expected of the middle school math teacher. A few of these type questions should be included to challenge the person being assessed and create a more rigorous test. It needs to be more balanced.*
- *There is a need for proportionality, exponents, and solving multi-step. All instructional questions are good*

III. The panel was provided an overview of the process and results of a multistate standard setting (MSSS) conducted for the test. *(Note: In advance of the meeting, panelists received a technical report describing the MSSS process and results in detail.)*

Results included:

- The recommended score value (RSV) from the study was 38 (out of 60 possible points) on the test form being reviewed
- The mean recommendation of 37.12 was rounded up to a whole number.
- The scale score associated with 38 raw points is 157.

Panelists were asked for their recommended passing score based on their review of the test, the panel's discussion of the test and the results of the MSSS. Choices presented were the MSSS RSV and raw scores ranging two conditional standard errors of measurement (CSEM) above and below the RSV:

(F) What passing score would you recommend [raw number correct

○ 46	Scale 175 (+2.0 CSEM)	–
○ 44	Scale 170	1
○ 42	Scale 166 (+1.0 CSEM)	1
○ 40	Scale 161	–
○ 38	Scale 157 (Recommended Value from MSSS)	3
○ 37	Scale 155	1
○ 35	Scale 150 (-1.0 CSEM)	–
○ 33	Scale 145	–
○ 31	Scale 141 (-2.0 CSEM)	–
Total		*6*

The panel was shown these results for a brief discussion and then asked for feedback on the panel's recommended passing score, identified as the median of the panelists' recommendations:

The panel's recommended passing score is 38, equivalent to a scaled score of 157.

Panelists provided feedback about this recommended passing score in answers to two questions:

(G) Overall, how comfortable are you with the panel's recommended cut score?

○ Very Comfortable	5
○ Somewhat Comfortable	–
○ Somewhat Uncomfortable	1
○ Very Uncomfortable	–
Total	*0*

(H) Overall, the panel's recommended cut score is:

○ Too high	–
○ About Right	5
○ Too Low	1
Total	*6*

Provided with an optional open-ended question

(I) Please provide any further comments about your recommendation

Two panelists provided further feedback:

- *Not too many higher level questions and would like to see more questions than 38 correct to actually have a passing score. I am glad I was able to participate - great meeting.*
- *I feel the cut score allows for a few careless mistakes as well as if potential teachers do not know above algebra I.*

ETS PRAXIS

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Middle School Mathematics (5164)

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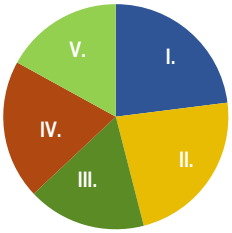
Table of Contents

Middle School Mathematics (5164)

Test at a Glance

The *Praxis*® Middle School Mathematics test is designed to measure knowledge and competencies that are important for safe and effective beginning practice as a middle school mathematics teacher. Test takers have typically completed a bachelor's degree program with appropriate coursework in mathematics and education.

Test Name	Middle School Mathematics
Test Code	5164
Time	180 minutes
Number of Questions	66 selected-response and numeric-entry questions
Format	The test consists of a variety of selected-response questions, where you select one or more answer choices; questions where you enter a numeric answer in a box; and other types of questions. You can review the possible question types in Understanding Question Types.
Calculator	An on-screen graphing calculator is provided.
Test Delivery	Computer Delivered



Content Categories	Approximate Number of Questions	Approximate Percentage of Examination
I. Numbers and Operations	16	23%
II. Algebra	15	23%
III. Functions	11	17%
IV. Geometry and Measurement	13	20%
V. Statistics and Probability	11	17%

All questions assess content from the above Middle School Mathematics domains.

Approximately 30% of questions assess content applied to a Task of Teaching Mathematics.

About This Test

The Middle School Mathematics content topics span the middle school mathematics curriculum, including content related to (I) Numbers and Operations, (II) Algebra, (III) Functions, (IV) Geometry and Measurement, and (V) Statistics and Probability. A full list of the mathematics topics covered is provided in Content Topics.

Test takers will find that approximately 30% of the questions call for application of mathematics within a teaching scenario or an instructional task. Such questions—designed to measure applications of mathematics knowledge and skills to the kinds of decisions and evaluations a teacher must make during work with students, curriculum, and instruction—situate mathematics content questions in tasks that are critical for teaching. A full list of the teaching tasks covered, which have been identified based on research on mathematics instruction and are a routine part of mathematics instruction, is provided in Tasks of Teaching Mathematics.

Test takers have access to an on-screen graphing calculator and a list of selected unit conversions and formulas. This list is also provided in the Middle School Mathematics (5164) Sample Test Questions section.

The assessment is designed and developed through work with practicing teachers and teacher educators to reflect the mathematics curriculum as well as state and national standards for mathematics, including the Standards for the Preparation of Middle Level Mathematics Teachers (2020), by the National Council of Teachers of Mathematics (NCTM) and the Council for the Accreditation of Educator Preparation (CAEP).

This test may contain some questions that will not count toward your score.

On-Screen Graphing Calculator

An on-screen graphing calculator is provided for the computer-delivered test. Please consult the *Praxis*® Calculator Use web page (http://www.ets.org/praxis/test_day/policies/calculators/) for further information and for a link to download the calculator and view tutorials on using the calculator.

You are expected to know how and when to use the calculator since it will be helpful for some questions. The calculator is available as a free download for a 30-day trial period. You are expected to become familiar with its functionality before taking the test. The calculator may be used to perform calculations (e.g., division, exponents, roots, finding the mean of a data set), to graph and analyze functions, to find numerical solutions to equations, and to generate a table of values for a function.

Using Your Calculator

Take time to download the trial version of the calculator. View the tutorials on the website. Practice with the calculator so that you are comfortable using it on the test.

There are only some questions on the test for which a calculator is helpful or necessary. First, decide how you will solve a problem, then determine if you need a calculator. For many questions, there is more than one way to solve the problem. Don't use the calculator if you don't need to; you may waste time.

Sometimes answer choices are rounded, so the answer that you get might not match the answer choices in the question. Since the answer choices are rounded, substituting the choices into the question might not produce an exact answer.

Don't round any intermediate calculations. For example, if the calculator produces a result for the first step of a solution, keep the result in the calculator and use it for the second step. If you round the result from the first step and the answer choices are close to each other, you might choose the incorrect answer.

Read the question carefully so that you know what you are being asked to do. Sometimes a result from the calculator is NOT the final answer. If an answer you get is not one of the choices in the question, it may be that you didn't answer the question being asked. Read the question again. It might also be that you rounded at an intermediate step in solving the problem.

Think about how you are going to solve the question before using the calculator. You may only need the calculator in the final step or two. Don't use it more than necessary.

Check the calculator modes (degree versus radian, floating decimal versus scientific notation) to see that these are correct for the question being asked.

Make sure that you know how to perform the basic arithmetic operations and calculations (e.g., division, exponents, roots). Your test may involve questions that require you to do some of the following: graph functions and analyze the graphs, find zeros of functions, find points of intersection of graphs of functions, find minima/maxima of functions, find numerical solutions to equations, and generate a table of values for a function.

Content Topics

This list details the topics that may be included on the test. All test questions will cover one or more of these topics.

Discussion Questions

In this section, discussion questions are open-ended questions or statements intended to help test your knowledge of fundamental concepts and your ability to apply those concepts to classroom or real-world situations. **We do not provide answers for the discussion questions** but thinking about the answers will help improve your understanding of fundamental concepts and may help you answer a broad range of questions on the test. Most of the questions require you to combine several pieces of knowledge to formulate an integrated understanding and response. They are written to help you gain increased understanding and facility with the test's subject matter. You may want to discuss these questions and possible areas with a teacher or mentor.

I. Numbers and Operations

A. Understands operations and properties of the real number system

1. Represents and solves word problems involving addition, subtraction, multiplication, and division of real numbers
2. Represents and identifies the effect that an operation has on a given number (e.g., adding a negative, adding the inverse, dividing by a nonzero fraction)
3. Uses the order of operations to simplify computations and solve problems
4. Identifies and applies properties of operations on a number system (e.g., commutative, associative, distributive, identity)
5. Compares and orders real numbers, including absolute values of real numbers
6. Classifies real numbers (e.g., natural, whole, integer, rational, irrational)
7. Identifies whether the sum or product of rational and/or irrational numbers must be rational, must be irrational, or can be rational or irrational (e.g., the sum of two rational numbers must be rational, the product of two irrational numbers can be rational or irrational)
8. Performs operations involving integer exponents
9. Approximates the value of a radical expression
10. Uses scientific notation to represent and compare numbers and to perform calculations

B. Understands the relationships among fractions, decimals, and percents

1. Converts among fractions, decimals, and percents
2. Represents repeating decimals as fractions
3. Represents fractions, decimals, and percents with models (e.g., area models, base-10 blocks, set models, colored rods)

C. Understands how to use ratios and proportional relationships to solve problems

1. Uses the language of ratio and rate to describe relationships between two quantities
2. Identifies and represents proportional relationships and uses them to solve problems (e.g., unit rates, scale factors, constant of proportionality)
3. Solves percent problems (e.g., expressing a percent as a ratio per 100, discounts, markups, taxes, tips, simple interest, percent error)

D. Understands how to reason quantitatively and use units to solve problems

1. Chooses and interprets units consistently in formulas
2. Chooses and interprets the scale in graphs and data displays
3. Solves problems involving dimensional analysis (e.g., feet per second to miles per hour, feet per second to kilometers per hour)

E. Understands how to use basic concepts of number theory (e.g., divisibility, prime factorization, multiples) to solve problems

1. Uses the definitions of prime and composite numbers to solve problems
2. Solves problems involving factors, multiples, and divisibility

Discussion Questions: Numbers and Operations

Note that the use of "e.g." to start a list of examples implies that only a few examples are offered and not an exhaustive list.

- Be able to convert repeating decimals into fractions (e.g., $0.58\overline{3} = \frac{7}{12}$).
- Be able to distinguish between a ratio and a rate.
- Be able to calculate percent change, percent (relative) error, and percents of percents.
- Be able to determine the correct units in an answer based on the units of the initial measurements given in a problem.
- Be able to identify a scale for a graph that allows an entire set of data to be represented on the graph.
- Be familiar with what unit conversions are given on the math reference sheet. Note that some other common unit conversions (e.g., 1 yard = 3 feet, 1 minute = 60 seconds) are expected to be known, and other unit conversions (e.g., 1 mile = 1,760 yards, 1 gallon = 128 fluid ounces, 1 hour = 3,600 seconds) are expected to be determined based on what is known or what is given on the math reference sheet.

II. Algebra

A. Understands how to create, evaluate, and manipulate algebraic expressions, equations, and formulas

1. Adds, subtracts, and multiplies linear and quadratic polynomials, including polynomials with rational coefficients

2. Evaluates, manipulates, and compares algebraic expressions involving rational exponents (e.g., radicals, negative exponents)
3. Uses variables to construct and solve equations and inequalities in real-world contexts
4. Translates verbal relationships into algebraic equations or expressions
5. Interprets parts of expressions and equations in terms of a real-world setting
6. Rewrites linear, quadratic, and exponential expressions in equivalent forms to reveal properties of the quantity represented by the expression
7. Determines the nature of the solutions of a quadratic equation (e.g., interprets the graph, finds the discriminant, writes the equation in factored form)
8. Rearranges formulas to solve for a specified variable (e.g., solve $d = rt$ for t)

B. Understands how to recognize and represent linear relationships algebraically

1. Determines the equation of a line from information presented in various forms (e.g., table, graph, description)
2. Recognizes and is able to extract information about a linear equation when it is presented in various forms (e.g., slope-intercept, point-slope, standard)
3. Converts among various forms of linear equations (e.g., slope-intercept, point-slope, standard)

C. Understands how to solve equations and inequalities

1. Solves one-variable linear equations and inequalities
2. Solves one-variable nonlinear equations and inequalities (e.g., absolute value, quadratic)
3. Represents solutions to equations and inequalities (e.g., on a number line, in the xy-plane)
4. Justifies each step in solving equations and inequalities

D. Understands how to solve systems of equations and inequalities

1. Solves a system of two linear equations or inequalities in two variables algebraically and graphically
2. Solves a system consisting of a linear equation and a quadratic equation in two variables graphically
3. Finds the solutions of $f(x) = g(x)$ approximately (e.g., uses technology to graph the functions); includes cases where $f(x)$ and /or $g(x)$ are linear, quadratic, or exponential functions
4. Graphs the solution set to a system of linear inequalities in two variables in the xy-plane
5. In a modeling context, represents constraints by systems of equations and/or inequalities and interprets solutions as viable or nonviable options

Discussion Questions: Algebra

Note that the use of "e.g." to start a list of examples implies that only a few examples are offered and not an exhaustive list.

- Be able to identify expressions that are equivalent to expressions such $x^{\frac{2}{3}}$, $x^{\frac{5}{2}}$, x^{-4}, $\left(x^3\right)^{-1}$, and $\sqrt[5]{x^2}$.
- Be able to write and solve equations, inequalities, and systems of equations or inequalities that represent real-world problems.
- Be able to identify what parts of expressions and equations (e.g., coefficients, terms, factors) represent in the context of a real-world situation.
- Be able to use the quadratic formula, which is given on the math reference sheet.
- Be able to determine the equation of a line given two points on the line or one point on the line and the slope of the line.
- Be able to determine the slope of a line or points on a line when an equation of the line is given in standard form, slope-intercept form, or point-slope form.
- Be able to solve one-variable linear equations and inequalities that have variables on both sides, involve combining like terms, and involve using the distributive property.
- Be able to graph the solutions to linear equations, linear inequalities, systems of linear equations, and systems of linear inequalities in two variables in the xy-plane.
- Be able to graph the solutions to one-variable inequalities on the number line.
- Be able to identify the properties (e.g., commutative property, distributive property) that justify each step in a given method for solving an equation or inequality.
- Be able to solve systems of linear equations graphically, by substitution, or by elimination.
- Remember that the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$.

III. Functions

A. Understands how to identify, define, and evaluate functions

1. Determines whether a relation is a function
2. Given a function (presented as a table of values, algebraically, or graphically), determines if the function is linear, quadratic, or exponential
3. Determines the value of a function for a specified value in its domain

B. Knows how to determine and interpret the domain and the range of a function presented as a table of values, algebraically, or graphically

1. Determines the domain and range of a function
2. Interprets domain and range in real-world settings

C. Understands basic characteristics of linear functions (e.g., intercepts, slope)

1. Calculates the intercepts of a line and interprets them in a modeling context
2. Calculates the slope of a line presented as a table of values, algebraically, or graphically and interprets it in a modeling context
3. Interprets what a point (x, y) on the graph of a proportional relationship means in terms of the situation, with special attention to the points $(0,0)$ and $(1,r)$, where r is the unit rate

D. Understands the relationships among functions, tables, and graphs

1. Determines an equation to represent a linear or quadratic function presented graphically
2. Determines the type of equation that best represents a given graph
3. Sketches a graph, given an equation of a function (e.g., square root, absolute value)
4. Compares properties (e.g., intercepts, slope, maximum) of two functions presented as tables of values, algebraically, graphically, or by verbal descriptions
5. Identifies the symbolic representation of a linear function that is created when a graph is translated horizontally or vertically or reflected across the *x*-axis

E. Knows how to analyze and represent functions (i.e., linear, quadratic, exponential) that model given information

1. Interprets statements that use function notation in terms of a context
2. Interprets the parameters in a linear or exponential function in terms of a context
3. Calculates the rate of change of a function over a given interval and interprets it in a context
4. Determines and interprets the *x*- and *y*-intercepts of quadratic functions
5. Develops a function—represented by a graph, equation, or table—to model a given set of conditions
6. Evaluates whether a particular mathematical model (e.g., graph, equation, table) can be used to describe a given set of conditions
7. Interprets a particular mathematical model (e.g., graph, equation, table) in a given context

F. Understands differences between linear, quadratic, and exponential models, including how their equations are created and used to solve problems

1. Identifies situations in which one quantity changes at a constant rate per unit interval relative to another
2. Identifies situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another

3. Observes that a quantity increasing exponentially eventually exceeds a quantity increasing linearly or quadratically

G. Is familiar with how to represent arithmetic sequences as functions

1. Writes arithmetic sequences both recursively and with an explicit formula and uses them to model situations

Discussion Questions: Functions

Note that the use of "e.g." to start a list of examples implies that only a few examples are offered and not an exhaustive list, whereas the use of "i.e." to start a list of examples implies that the given list of examples is complete.

- Remember that a function assigns exactly one element of its range to each element of its domain.
- Be able to identify features of a function represented as a table, equation, or graph that indicate whether the function is linear, quadratic, or exponential (e.g., the second differences of a quadratic function are constant, a quadratic function has degree 2, a quadratic function has either a maximum or a minimum).
- Be able to determine the domain (*x*-values) and range (*y*-values) of a function.
- Be able to determine the domain and range of a function that is reasonable in the context of a given real-world situation (e.g., the domain in a certain situation consists of positive integers, the domain in a certain situation cannot include values that would result in a negative value for the height of an object).
- Be able to determine the intercepts and slope of a line represented as a table, equation, or graph.
- Be able to interpret the meaning of the intercepts and slope of a line in the context of a real-world situation.
- Be able to interpret the meaning of a point on the graph of a proportional relationship in the context of a real-world situation.
- Be able to determine whether a given graph is best represented using a linear equation, quadratic equation, exponential equation, absolute value equation, etc.
- Be able to write an equation of the function that results after an existing function is translated horizontally, translated vertically, or reflected across the *x*-axis.
- Be able to interpret the meaning of *m* and *b* in a function of the form $f(x) = mx + b$ in the context of a real-world situation.
- Be able to interpret the meaning of *a* and *b* in a function of the form $f(x) = a \cdot b^x$ in the context of a real-world situation.
- Be able to calculate the rate of change of a function *f* on the interval $[a, b]$ by calculating $\frac{f(b) - f(a)}{b - a}$, and be able to interpret the meaning of the rate of change in the context of the problem.
- Be able to interpret the meaning of the intercepts of a quadratic function in the context of a real-world situation.
- Given a linear, quadratic, or exponential function represented as a table, equation, graph, or description, be able to determine a different representation of the function (e.g., determine the function that best matches a description of a real-world situation).

- Be able to interpret the meaning of a feature of a function (e.g., the maximum on a graph, an ordered pair in a table of values) in the context of a real-world situation.
- Be able to identify real-world situations that are best modeled by linear functions or that are best modeled by exponential functions.
- Be able to find the value of a term in an arithmetic sequence.
- Be able to write an expression, equation, or function that represents an arithmetic sequence.
- Be familiar with the differences between recursive and explicit rules for arithmetic sequences.
- Consider becoming familiar with the arithmetic sequence formula on the math reference sheet.

IV. Geometry and Measurement

A. Knows the properties of types of lines (e.g., parallel, perpendicular, intersecting) and angles

1. Solves problems involving parallel, perpendicular, and intersecting lines
2. Applies angle relationships (e.g., supplementary, vertical, alternate interior) to solve problems

B. Understands the properties of triangles

1. Solves problems involving the Pythagorean theorem in two dimensions
2. Identifies characteristics of special triangles (e.g., equilateral, isosceles, right) and uses them to solve problems
3. Determines whether given side lengths or angle measures would produce a triangle (e.g., triangle inequality theorem) and classifies triangles by their sides or angles
4. Determines whether given conditions would produce a unique triangle, no triangle, or more than one triangle

C. Knows the properties of quadrilaterals and other polygons

1. Identifies the relationships among various quadrilaterals (e.g., parallelogram, rectangle, rhombus)
2. Solves problems involving sides and angles of polygons

D. Knows the concepts of transformations (i.e., translations, reflections, rotations, dilations)

1. Applies properties of translations, reflections, and rotations (e.g., line segments are taken to congruent line segments, angles are taken to congruent angles, parallel lines are taken to parallel lines)
2. Applies properties of dilations (e.g., angles are taken to congruent angles, parallel lines are taken to parallel lines)
3. Identifies a sequence of transformations that maps a preimage onto an image
4. Given a figure, describes the transformations that map the figure onto itself, including reflection over a line of symmetry
5. For a given transformation, determines the coordinates of a point on an image

E. Understands the concepts of congruence and similarity

1. Determines whether two figures are congruent or similar
2. Uses congruence and similarity to solve problems with two-dimensional and three-dimensional figures

F. Understands the properties of circles

1. Solves problems involving circles (e.g., circumference, area)

G. Knows how to interpret relationships between geometric objects in the *xy*-plane (e.g., distance, midpoint)

1. Uses coordinate geometry to represent and identify the properties of geometric shapes and to solve problems (e.g., Pythagorean theorem, perimeter, area)
2. Determines the distance between two points
3. Determines the midpoint of a segment

H. Understands how to solve problems involving perimeter and area of polygons

1. Calculates and interprets perimeter and area of polygons that can be composed of triangles and quadrilaterals, including in real-world situations
2. Calculates changes in perimeter and area as the dimensions of a polygon change

I. Knows how to solve problems involving solids

1. Calculates and interprets surface area and volume of solids (e.g., prisms, pyramids, cylinders, spheres) and composite solids, including in real-world situations
2. Calculates changes in surface area and volume as the dimensions of a solid change
3. Uses two-dimensional representations (e.g., nets) of three-dimensional objects to visualize and solve problems

J. Understands systems of measurement (i.e., metric, United States customary)

1. Solves measurement, estimation, and conversion problems involving time, length, temperature, volume, and mass in standard measurement systems
2. Uses appropriate units of measurement in a given context

Discussion Questions: Geometry and Measurement

Note that the use of "e.g." to start a list of examples implies that only a few examples are offered and not an exhaustive list, whereas the use of "i.e." to start a list of examples implies that the given list of examples is complete.

- Be familiar with the relationship between the slopes of parallel lines and the relationship between the slopes of perpendicular lines.
- Be able to identify congruent and supplementary angles given two parallel lines and a transversal.

- Be able to distinguish among acute, right, and obtuse triangles.
- Be able to identify and use special characteristics of triangles (e.g., equilateral, isosceles, right) to solve problems involving lengths of sides and measures of angles.
- Be able to distinguish among different types of quadrilaterals.
- Be able to identify and use special characteristics of squares, rectangles, parallelograms, rhombuses, and trapezoids to solve problems involving lengths of sides and measures of angles.
- Be able to find missing side lengths or angle measures in polygons with more than four sides.
- Be able to find the measures of interior and exterior angles of regular polygons.
- Be familiar with the effects of translations, reflections, rotations, and dilations on figures.
- Be able to translate, reflect, rotate, and dilate figures.
- Be able to distinguish between congruent and similar figures and use corresponding parts of congruent or similar figures to solve problems.
- Be familiar with what geometric formulas are given on the math reference sheet, and be able to apply these formulas to solve problems.
- Be able to solve problems that involve the circumference or area of a circle and the perimeter or area of a polygon (e.g., finding the difference between the area of a square and the area of a circle inscribed in the square).
- Be able to find the distance between any two points in the *xy*-plane by using a formula or the Pythagorean theorem.
- Be able to find the midpoint of a line segment in the *xy*-plane by using a formula or another approach.
- Be familiar with the effect on the perimeter, area, or volume of a figure as the dimensions of the figure change by different factors.
- Be able to use a net to find the surface area and volume of a solid.
- Be familiar with what unit conversions are given on the math reference sheet. Note that some other common unit conversions (e.g., 1 yard = 3 feet, 1 minute = 60 seconds) are expected to be known, and other unit conversions (e.g., 1 mile = 1,760 yards, 1 gallon = 128 fluid ounces, 1 hour = 3,600 seconds) are expected to be determined based on what is known or what is given on the math reference sheet.
- Be able to identify units that measure length, area, volume, weight, etc.

V. Statistics and Probability

A. Understands statistical processes and how to evaluate them

1. Recognizes a statistical question as one that anticipates variability in the data related to the question and accounts for it in the answers
2. Uses statistics to make inferences about population parameters based on a sample from that population
3. Distinguishes between random and biased sampling

B. Understands how to interpret, analyze, and represent data presented in a variety of displays

1. Represents and analyzes data in various displays (e.g., bar graphs, line graphs, circle graphs, boxplots, histograms, scatterplots, stem-and-leaf plots, two-way tables)
2. Calculates relative frequencies for rows or columns in two-way tables and uses the calculations to describe possible associations between the two variables
3. Uses the equation of a linear model to solve problems in the context of bivariate measurement data (e.g., interpreting the slope and intercept, interpolation)
4. Describes how two quantitative variables are related (e.g., fit a function to data, association, correlation)
5. Chooses appropriate graphs based on data type (e.g., numerical, categorical)

C. Understands concepts associated with measures of central tendency and dispersion

1. Solves for the mean and weighted average of given sets of data
2. Determines and interprets measures of center (e.g., mean, median, mode) and spread (e.g., range, interquartile range) in a variety of problems
3. Summarizes a given numerical data set in relation to its context
4. Describes the distribution of a set of data by its center and spread
5. Uses statistics appropriate to the shape of the data distribution to compare center and spread of two or more different data sets
6. Interprets differences in center and spread in the context of the data sets, accounting for possible effects of outliers

D. Knows how to use and evaluate probability models

1. Uses counting techniques (e.g., the counting principle, tree diagrams) to answer questions involving a finite sample space
2. Solves probability problems involving simple events
3. Solves probability problems involving compound events
4. Interprets a probability model and uses it to find probabilities of events
5. Compares probabilities from a model to observed frequencies and identifies possible sources of the discrepancy if the agreement is not good
6. Interprets a uniform probability model and uses it to determine probabilities of events

Discussion areas: Statistics and Probability

Note that the use of "e.g." to start a list of examples implies that only a few examples are offered and not an exhaustive list.

- Be able to make inferences about a population based on a random sample from the population (e.g., estimate the number of people in a population for which a certain characteristic is true).
- Be able to interpret a line of best fit (trend line) and use it to solve problems.

- Be familiar with how to summarize numerical data sets in relation to their context (e.g., describe any overall pattern and any notable differences from the overall pattern, relate the chosen measures of center and variability to the shape of the data).
- Be able to express the difference between the centers of two data sets as a multiple of a measure of variability.
- Be able to solve counting problems by using counting techniques or by counting individual outcomes (e.g., construct or interpret a tree diagram that models a sample space).
- Be able to solve probability problems involving independent events or dependent events.
- Be able to solve problems involving a probability model (which may not be uniform) by observing frequencies in data generated from a chance process.
- Be able to solve problems involving a uniform probability model by assigning equal probability to all outcomes.

Tasks of Teaching Mathematics

This list includes instructional tasks that teachers engage in that are essential for effective teaching of middle school mathematics. Approximately 30% of test questions will measure content knowledge by assessing how that content knowledge is applied in the context of one or more of these tasks.

Mathematical explanations, justifications, and definitions

1. Identifies valid explanations of mathematical concepts (e.g., explaining why a mathematical idea is considered to be true), procedures, representations, or models
2. Evaluates or compares explanations and justifications for their validity, generalizability, coherence, or precision, including identifying flaws in explanations and justifications
3. Determines the changes that would improve the validity, generalizability, coherence, and/or precision of an explanation or justification
4. Evaluates whether counterarguments address a critique of a given justification
5. Evaluates definitions or other mathematical language for validity, generalizability, precision, usefulness in a particular context, or support of key ideas

Mathematical problems, tasks, examples, and procedures

6. Identifies problems or tasks that fit a particular structure, address the same concept, demonstrate desired characteristics, or elicit particular student thinking
7. Identifies two or more problems that systematically vary in difficulty or complexity
8. Evaluates the usefulness of examples for introducing a concept, illustrating an idea, or demonstrating a strategy, procedure, or practice
9. Identifies examples that support particular strategies or address particular student questions, misconceptions, or challenges with content
10. Identifies nonexamples or counterexamples that highlight a mathematical distinction or demonstrate why a student conjecture is incorrect or partially incorrect

11. Evaluates procedures for working with mathematics content to identify special cases in which the procedure might be problematic or for validity, appropriateness, or robustness

Mathematical representations, models, manipulatives, and technology

12. Evaluates representations and models (e.g., concrete, pictorial) in terms of validity, generalizability, usefulness for supporting students' understanding, or fit to the concept, calculation, etc. to be represented
13. Evaluates how representations and models (e.g., concrete, pictorial) have been used to show particular ideas, relationships between ideas, processes, or strategies
14. Evaluates the use of technology (e.g., graphing tools, software) for its appropriateness or its support of key ideas

Students' mathematical reasoning

15. Identifies likely misconceptions about or partial understanding of particular mathematics content and practices
16. Identifies how new mathematics content and practices can build on or connect to students' prior knowledge, including misconceptions and errors
17. Evaluates or compares student work (e.g., solutions, explanations, justifications, representations) in terms of validity, generalizability, coherence, and/or precision
18. Evaluates student work to identify the use of a particular concept, idea, or strategy
19. Identifies how a student's reasoning would replicate across similar problems
20. Identifies different pieces of student work that demonstrate the same reasoning
21. Identifies situations in which student work that seems valid might mask incorrect thinking

Middle School Mathematics (5164) Sample Test Questions

Information about Questions That Is Specific to the Middle School Mathematics Test

General

- All numbers used are real numbers.
- Unless otherwise stated, the domain of a given function *f* is the set of all real numbers *x* for which $f(x)$ is a real number.
- Rectangular coordinate systems are used unless otherwise stated.
- Figures that accompany questions are intended to provide information that is useful in answering questions.
  - Figures are drawn to scale unless otherwise stated.
  - Lines shown as straight are straight, and angle measures are positive. Positions of points, angles, regions, etc., exist in the order shown.

Types of questions that may be on the test

- Selected-response questions—select one answer choice
  - These are questions that ask you to select only one answer choice from a list of four choices.
  - Note that in most selected-response questions that ask for numerical values, the exact answer should be found. If a selected-response question includes a word or phrase like "approximately," "best approximates," or "is closest to," it generally indicates that the correct option will **not** be an exact value.
- Selected-response questions—select one or more answer choices
  - These are questions that ask you to select one or more answer choices from a list of choices. A question may or may not specify the number of choices to select. These questions are marked with square boxes beside the answer choices, not circles or ovals. See question 16 in the Sample Test Questions.
  - If a question of this type has exactly three answer choices, one, two, or three of the choices may be correct.
  - If a question of this type has more than three answer choices, the number of correct choices will be at least 2 but fewer than the number of choices. For example, if a question of this type has six answer choices, there will be two, three, four, or five correct choices.
- Selected-response questions—select an area
  - These are questions that ask you to select one or more locations on a picture or a figure (e.g., the *xy*-plane).

- Numeric-entry questions
  - Some of these questions ask you to enter your answer as an integer or a decimal in a single answer box. Equivalent forms of the correct answer, such as 2.5 and 2.50, are all correct. See questions 10 and 14 in the Sample Test Questions. Note that in these questions, the exact answer should be entered unless the question asks you to round your answer. Therefore, if one of these questions does **not** ask you to round your answer, you should be able to enter the exact answer in the numeric-entry box. If you are unable to do so, this may indicate that your answer is incorrect.
  - Some of these questions ask you to enter your answer as a fraction in two separate boxes—one for the numerator and one for the denominator. A negative sign can be entered in either box. Equivalent forms of the correct answer, such as $\frac{1}{2}$ and $\frac{6}{12}$, are all correct, though there may be cases where you need to simplify your fraction so it fits in the boxes. See question 9 in the Sample Test Questions.
- Drag-and-drop questions
  - These questions ask you to pair up given phrases or expressions by dragging (with your computer mouse) phrases from one location and matching them with given phrases or expressions in another location. See question 8 in the Sample Test Questions.
- Table grid questions
  - These questions refer to a table in which statements appear in the first column. For each statement, select the correct properties by selecting the appropriate cell(s) in the table. See question 2 in the Sample Test Questions.
- Text completion questions
  - These questions ask you to select one or more answer choices to complete one or more sentences. The choices may be located in drop-down menus in the sentences or in columns at the end of the question. You will select one answer choice from each drop-down menu or column.

Unit Conversions

1 mile = 5,280 feet	1 mile ≈ 1.61 kilometers	1 inch = 2.54 centimeters
1 pound = 16 ounces	1 ton = 2,000 pounds	1 kilogram ≈ 2.2 pounds
1 cup = 8 fluid ounces	1 quart = 2 pints	1 gallon ≈ 3.785 liters
1 pint = 2 cups	1 gallon = 4 quarts	1 liter = 1,000 cubic centimeters

Formulas

Area

Rectangle with length ℓ and width w: ..... $A = \ell w$

Parallelogram with height h and base of length b: ..... $A = bh$

Triangle with height h and base of length b: ..... $A = \frac{1}{2}bh$

Trapezoid with height h and bases of length b_1 and b_2: ..... $A = \frac{1}{2}(b_1 + b_2)h$

Circle with radius r: ..... $A = \pi r^2$

Perimeter

Rectangle with length ℓ and width w: ..... $P = 2\ell + 2w$

Circumference

Circle with radius r: ..... $C = 2\pi r$

Volume

Right rectangular prism with length ℓ, width w, and height h: ..... $V = \ell wh$

Right prism with height h and base of area B: ..... $V = Bh$

Pyramid with height h and base of area B: ..... $V = \frac{1}{3}Bh$

Right circular cylinder with height h and base of radius r: ..... $V = \pi r^2 h$

Right circular cone with height h and base of radius r: ..... $V = \frac{1}{3}\pi r^2 h$

Sphere with radius r: ..... $V = \frac{4}{3}\pi r^3$

Surface Area

Cube with side of length s: ..... $A = 6s^2$

Right rectangular prism with length ℓ, width w, and height h: ..... $A = 2\ell w + 2\ell h + 2wh$

Right circular cylinder with height h and base of radius r: ..... $A = 2\pi rh + 2\pi r^2$

Sphere with radius r: ..... $A = 4\pi r^2$

Other Formulas

Quadratic formula: ..... $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Arithmetic sequence: ..... $a_n = a_1 + (n-1)d$

Pythagorean theorem: ..... $a^2 + b^2 = c^2$

Sum of the measures of the interior angles of a polygon with n sides: ..... $S = 180°(n-2)$

Mathematics (5164) Sample Test Questions

1. A student used the same reasoning to evaluate four expressions. The four expressions and the student's answers are given as follows. The student incorrectly evaluated the first two expressions but correctly evaluated the next two expressions.

    1. $7 \times 2 - 6 + 3 = 5$
    2. $9 - 5 + 16 \div 8 = 2$
    3. $9 + 24 \div 3 - 1 = 16$
    4. $7 \times 2 - 18 \div 6 = 11$

    If the student continues to use the same reasoning, which of the following expressions is the student most likely to evaluate **INCORRECTLY**?

    A. $8 + 7 - 12 \div 3$
    B. $13 - 3 \times 2 + 5$
    C. $10 \times 6 \div 15 - 3$
    D. $4 \times 5 + 10 - 12$

2. Ms. Kress asked her students to compare $\frac{1}{3}$ and $\frac{7}{8}$. Four of her students correctly answered that $\frac{7}{8}$ is greater than $\frac{1}{3}$, but they gave different explanations when asked to describe their strategies to the class.

   Indicate whether each of the following student explanations provides evidence or does not provide evidence of a mathematically valid strategy for comparing $\frac{1}{3}$ and $\frac{7}{8}$.

Student Explanation	Provides Evidence	Does Not Provide Evidence
When you look at the numbers, you see that 7 is bigger than 1, so $\frac{7}{8}$ is the bigger fraction.		
In the first fraction, 1 is less than half of 3, but in the second, 7 is more than half of 8, so $\frac{7}{8}$ is larger than $\frac{1}{3}$.		
I multiplied 1 times 7 and 3 times 7, so $\frac{1}{3}$ is the same as $\frac{7}{21}$. This means that $\frac{7}{8}$ is bigger than $\frac{1}{3}$ because $\frac{1}{8}$ is bigger than $\frac{1}{21}$.		
I wanted to make a fraction equal to $\frac{1}{3}$ with the same bottom number as $\frac{7}{8}$, so I added 5 to 3 and got 8. Then I added 5 to 1 and got 6, but 7 is greater than 6, so $\frac{7}{8}$ is greater.		

3. Ernesto bought 2 sport coats for $88.95 each. One of the coats needed alterations that cost $15.50, and a 6% sales tax is applied to the cost of the coats but not to the alterations.

   Which of the following values is closest to the total cost for the sport coats and the alterations?

   A. $190
   B. $200
   C. $205
   D. $215

4. The maximum speed at which a horse can run is 36 miles per hour.

   What is the maximum speed of the horse in feet per second?

   A. 2.4
   B. 24.5
   C. 37.5
   D. 52.8

5. A teacher wants to give an example in which the distributive property must be used to solve a literal equation for a given variable.

   Which of the following examples best serves the teacher's purpose?

   A. Solve $A = \frac{(a+b)h}{2}$ for b.
   B. Solve $A = P(1+rt)$ for r.
   C. Solve $P = 2\ell + 2w$ for w.
   D. Solve $S = 2\ell w + 2\ell h + 2wh$ for h.

6. A line in the xy-plane passes through the point $(4,5)$ and is parallel to the graph of $3x + y = 4$.

   What is an equation of the line?

   A. $y = -3x + 17$
   B. $y = -3x + 7$
   C. $y = 3x - 7$
   D. $y = 3x - 17$

7. Mr. Keller's class is learning about algebraic equations. In his teacher's edition of the textbook, Mr. Keller finds a page that suggests he ask students to critique the following two solutions to determine whether they are valid.

$4x+2 = 66$	$5 = 2x+3$
$\frac{6x}{6} = \frac{66}{6}$	$\frac{5}{5} = \frac{5x}{5}$
$x = 11$	$x = 1$

Which of the following is best addressed by the preceding task?

A. Misunderstanding of the properties of operations
B. Misunderstanding of the meaning of the equal sign
C. Misunderstanding of how to identify and combine like terms
D. Misunderstanding of how to use inverse operations to solve equations

8. The steps in a solution method for the equation $\frac{1}{3}(11x+20)=2x$ follow.

Provide the justification for the result shown in each step in the solution method.

Addition/Subtraction Property of Equality

Multiplication/Division Property of Equality

Distributive Property

Step	Justification
$\frac{1}{3}(11x+20)=2x$	Given
$11x+20=6x$	
$20=-5x$	
$-4=x$	

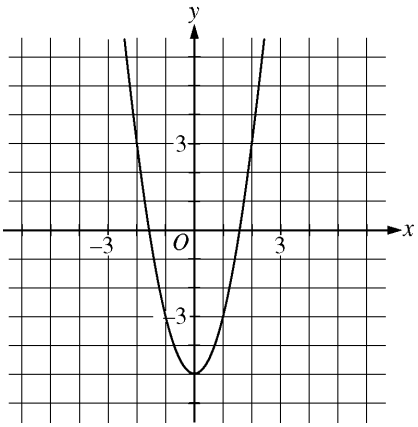
9. The graph of linear function f passes through the points $(-3,11)$ and $(7,-4)$.

   What is the slope of the graph of f?

   Give your answer as a fraction.

   $\frac{\square}{\square}$

10. The graph of the quadratic equation $y = ax^2 + c$ is shown in the following xy-plane.



   If a and c are integers, what are the values of a and c?

   $a =$ ☐

   $c =$ ☐

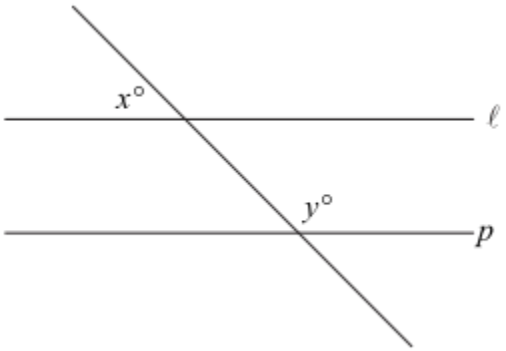
11. A teacher wants to show the students in an Algebra I class two examples of functions that are both linear and continuous. The teacher thinks of the following example.

    The temperature in degrees Fahrenheit is a function of the temperature in degrees Celsius.

    Which of the following is also an example of a function that is both linear and continuous?

    A. The height in inches of a person is a function of the person's age in years throughout the person's life.
    B. The number of calories consumed is a function of the volume, in ounces, of an energy drink that is consumed.
    C. The total cost in dollars for purchasing hot dogs is a function of the number of hot dogs purchased for $2.00 each.
    D. The total number of games in a tournament is a function of the number of teams in the tournament when each team plays every other team once.

12. In the following figure, line ℓ and line p are parallel, and $y = 3x$.



    What is the value of x?

    A. 75
    B. 60
    C. 45
    D. 30

13. When Ms. Slater gave her students a problem to assess their prior knowledge at the start of a lesson on finding the side length of a square given its area, several students incorrectly answered that the side length of a square with area 36 square units is 9 units. At the end of the lesson, Ms. Slater wanted to give a similar problem to assess whether her students were still making the same error.

    Which of the following areas would be **LEAST** useful for assessing student learning in this situation?

    A. 4 square units
    B. 16 square units
    C. 64 square units
    D. 100 square units

14. Reggie hiked 3,500 meters along a trail at a nearby park each day for the last 14 days.

    How many **kilometers** did Reggie hike in the last 14 days?

    ☐ kilometers

15. An automobile company sold 6 different models in the United States in a certain year. The following table shows the percent of total sales in the United States for each of the 6 models that year.

Model	Percent of Sales
B	49.5%
C	24.7%
D	16.1%
E	5.0%
F	3.5%
G	1.2%

    If a circle graph is constructed using the data in the table, which of the following values best approximates the measure, in degrees, of the central angle for the sector representing the sales of Model D ?

    A. 4°
    B. 16°
    C. 29°
    D. 58°

16. Each of the integers in list *K* (not shown) is greater than 75, and integers may appear more than once in the list. List *M* consists of the integers in list *K* and 4 additional integers that are each less than 75.

    Which of the following statements about the centers or spreads of lists *K* and *M* must be true?

    Select **ALL** that apply.

    A. The mean of the integers in list *M* is less than the mean of the integers in list *K*.
    B. The median of the integers in list *M* is less than the median of the integers in list *K*.
    C. The mode of the integers in list *M* is less than the mode of the integers in list *K*.
    D. The range of the integers in list *M* is greater than the range of the integers in list *K*.
    E. The interquartile range of the integers in list *M* is greater than the interquartile range of the integers in list *K*.

17. In a survey, 50 people were asked how many hours per day, *h*, they watched television. The survey results are shown in the following table.

Hours of Television Watched per Day

Number of Hours Watched per Day	Number of People
$h < 1$	5
$1 \le h < 2$	12
$2 \le h < 3$	16
$3 \le h < 4$	14
$h \ge 4$	3

If a person is selected at random from those surveyed, what is the probability that the person selected will have watched at least 2 hours but less than 4 hours of television per day?

A. $\frac{3}{10}$
B. $\frac{8}{25}$
C. $\frac{1}{2}$
D. $\frac{3}{5}$

Middle School Mathematics (5164) Answers

1. The correct answer is (B). A common misconception about the order of operations is that multiplication always comes before division and that addition always comes before subtraction. In the first expression, the student evaluated $7 \times 2 - (6 + 3)$ instead of $7 \times 2 - 6 + 3$, and in the second expression, the student evaluated $9 - (5 + 16 \div 8)$ instead of $9 - 5 + 16 \div 8$. In the next two expressions, the misconception described does not interfere with a student's ability to correctly evaluate the expressions, and the student obtained the correct answers, so it is likely that this misconception is the basis for the student's incorrect answers. In the expressions in (A), (C), and (D), this misconception does not interfere with a student's ability to correctly evaluate the expressions either, but in the expression in (B), it is incorrect to add before subtracting, so the expression in (B) is the one that the student is most likely to evaluate incorrectly.

Task of Teaching Topic	Students' mathematical reasoning
Task of Teaching Subtopic	19. Identifies how a student's reasoning would replicate across similar problems
Category	I. Numbers and Operations
Topic	A. Understands operations and properties of the real number system
Subtopic	3. Uses the order of operations to simplify computations and solve problems

2. The first and fourth explanations do not provide evidence of a mathematically valid strategy for comparing $\frac{1}{3}$ and $\frac{7}{8}$, but the second and third explanations do. In the first explanation, the student compares only the numerators of the fractions, which is not a valid strategy because it does not take into account the effect of the denominator on the size of the pieces. In the second explanation, the student compares both fractions to the benchmark fraction $\frac{1}{2}$, which is a valid strategy since $\frac{1}{3}$ is less than $\frac{1}{2}$ and $\frac{7}{8}$ is greater than $\frac{1}{2}$. In the third explanation, the student uses multiplicative reasoning to find a common numerator, and then the student compares the fractions by reasoning about the sizes of the unit

fractions $\frac{1}{8}$ and $\frac{1}{21}$. This is a valid strategy. In the fourth explanation, the student uses additive reasoning to try to find a fraction equivalent to $\frac{1}{3}$ that has a denominator of 8, but $\frac{6}{8}$ is not equivalent to $\frac{1}{3}$, so this strategy is not valid.

Task of Teaching Topic	Students' mathematical reasoning
Task of Teaching Subtopic	17. Evaluates or compares student work (e.g., solutions, explanations, justifications, representations) in terms of validity, generalizability, coherence, and/or precision
Category	I. Numbers and Operations
Topic	A. Understands operations and properties of the real number system
Subtopic	5. Compares and orders real numbers, including absolute values of real numbers

3. The correct answer is (C). Based on the information in the question, the total cost can be calculated as $(\$88.95)(2)(1.06)+\$15.50=\$204.07$. The choice that is closest to the total cost is \$205.

Category	I. Numbers and Operations
Topic	C. Understands how to use ratios and proportional relationships to solve problems
Subtopic	3. Solves percent problems (e.g., expressing a percent as a ratio per 100, discounts, markups, taxes, tips, simple interest, percent error)

4. The correct answer is (D). To find the maximum speed of the horse in feet per second, multiply the maximum speed of the horse in miles per hour by the appropriate conversion factors. Note that certain conversion factors are provided on the test, such as the conversion from miles to feet, but other conversion factors are not provided, such as the conversions from hours to minutes and from minutes to seconds.

Since 1 mile = 5,280 feet,
1 hour = 60 minutes, and
1 minute = 60 seconds, the maximum speed of the horse in feet per second is equal to

$$\frac{36 \text{ miles}}{1 \text{ hour}} \cdot \frac{5{,}280 \text{ feet}}{1 \text{ mile}} \cdot \frac{1 \text{ hour}}{60 \text{ minutes}} \cdot \frac{1 \text{ minute}}{60 \text{ seconds}},$$

which is equal to 52.8 feet per second. Remember that when calculations like this are performed, the units in the numerators and denominators of the fractions need to be divided out so that only the required units remain.

Category	I. Numbers and Operations
Topic	D. Understands how to reason quantitatively and use units to solve problems
Subtopic	3. Solves problems involving dimensional analysis (e.g., feet per second to miles per hour, feet per second to kilometers per hour)

5. The correct answer is (D). Remember that both $a(b+c) = ab + ac$ and $ab + ac = a(b+c)$ demonstrate the distributive property. To solve $S = 2\ell w + 2\ell h + 2wh$ for *h*, first subtract $2\ell w$ from both sides of the equation to obtain $S - 2\ell w = 2\ell h + 2wh$. Then use the distributive property to factor the right-hand side of the equation to obtain $S - 2\ell w = h(2\ell + 2w)$. Finally, divide both sides of the equation by $2\ell + 2w$ to obtain $\frac{S - 2\ell w}{2\ell + 2w} = h$. In this example, since *h* appears in two of the terms on the right-hand side of the equation but does not appear in the third term, the distributive property must be used to isolate *h*, which means (D) is the example that best serves the teacher's purpose. Each of the other examples can be solved for the given variable without using the distributive property.

Task of Teaching Topic	Mathematical problems, tasks, examples, and procedures
Task of Teaching Subtopic	9. Identifies examples that support particular strategies or address particular student questions, misconceptions, or challenges with content
Category	II. Algebra
Topic	A. Understands how to create, evaluate, and manipulate algebraic expressions, equations, and formulas
Subtopic	8. Rearranges formulas to solve for a specified variable (e.g., solve $d = rt$ for *t*)

6. The correct answer is (A). Since $3x + y = 4$ is equivalent to $y = -3x + 4$, the slope of the graph of $3x + y = 4$ is -3. This means that the slope of the line that passes through the point $(4,5)$ is also -3 since the line is parallel to the graph of $3x + y = 4$. Substituting the slope of -3 and the point $(4,5)$ into the point-slope form of the equation of a line yields $y - 5 = -3(x - 4)$. Applying the distributive property yields

$y-5=-3x+12$, and then adding 5 to both sides of the equation yields $y=-3x+17$, which is the equation in (A).

Category	II. Algebra
Topic	B. Understands how to recognize and represent linear relationships algebraically
Subtopic	1. Determines the equation of a line from information presented in various forms (e.g., table, graph, description)

7. The correct answer is (C). In the first solution, $4x$ and 2 are added to get $6x$, but the $4x$ term contains a variable, whereas the 2 is a constant term; it is incorrect to add $4x$ and 2 because they are not like terms. Similarly, in the second solution, $2x$ and 3 are added to get $5x$, but $2x$ and 3 are not like terms, so this strategy is not valid. Therefore, a misunderstanding of how to identify and combine like terms is the option that is best addressed by asking students to critique the two invalid strategies.

Task of Teaching Topic	Mathematical problems, tasks, examples, and procedures
Task of Teaching Subtopic	8. Evaluates the usefulness of examples for introducing a concept, illustrating an idea, or demonstrating a strategy, procedure, or practice
Category	II. Algebra
Topic	C. Understands how to solve equations and inequalities
Subtopic	1. Solves one-variable linear equations and inequalities

8. The correct answer, from top to bottom, is the Multiplication/Division Property of Equality, the Addition/Subtraction Property of Equality, and the Multiplication/Division Property of Equality. The equation $\frac{1}{3}(11x+20)=2x$ is multiplied by 3 on both sides to obtain $11x+20=6x$, so this step is justified by the Multiplication/Division Property of Equality. Then, $11x$ is subtracted from both sides of the equation $11x+20=6x$ to obtain $20=-5x$, so this step is justified by the Addition/Subtraction Property of Equality. Finally, both sides of the equation $20=-5x$ are divided by -5 to obtain $-4=x$, so this step is justified by the Multiplication/Division Property of Equality.

Category	II. Algebra
Topic	C. Understands how to solve equations and inequalities
Subtopic	4. Justifies each step in solving equations and inequalities

9. The correct answer is $-\frac{3}{2}$. The slope of a linear function can be found by substituting into the formula $m=\frac{y_2-y_1}{x_2-x_1}$, where m is the slope and (x_1,y_1) and (x_2,y_2) are two points on the linear function. Substituting the given points into the formula gives $m=\frac{-4-11}{7-(-3)}=\frac{-15}{10}=-\frac{3}{2}$.

Category	III. Functions
Topic	C. Understands basic characteristics of linear functions (e.g., intercepts, slope)
Subtopic	2. Calculates the slope of a line presented as a table of values, algebraically, or graphically and interprets it in a modeling context

10. The correct answer is $a=2$ and $c=-5$. Since the graph of the equation intersects the *y*-axis at the point $(0,-5)$, the value of c must be -5. Then, one method to find the value of a is to substitute the coordinates from another point on the graph into the equation and solve for a. Using the point $(2,3)$ and the fact that $c=-5$, it can be determined that $3=a(2^2)-5$, so $4a-5=3$. To solve this equation for a, add 5 to both sides of the equation, and then divide both sides of the equation by 4, which leads to the answer $a=2$.

Category	III. Functions
Topic	D. Understands the relationships among functions, tables, and graphs
Subtopic	1. Determines an equation to represent a linear or quadratic function presented graphically

11. The correct answer is (B). The function is linear because the number of calories consumed is proportional to the number of ounces of the energy drink that are consumed, and the function is continuous because the volume of the energy drink that is consumed can be measured with any level of accuracy (e.g., to the nearest hundredth of an ounce, not only the nearest ounce). The function in (A) is continuous but it is not linear because a person does not grow linearly over time. The function in (C) is linear but it is not continuous because it can be assumed that one can only buy a whole number of hot dogs. The function

in (D) is neither linear nor continuous because the function is quadratic and there can only be a whole number of teams in the tournament.

Task of Teaching Topic	Mathematical problems, tasks, examples, and procedures
Task of Teaching Subtopic	8. Evaluates the usefulness of examples for introducing a concept, illustrating an idea, or demonstrating a strategy, procedure, or practice
Category	III. Functions
Topic	F. Understands differences between linear, quadratic, and exponential models, including how their equations are created and used to solve problems
Subtopic	1. Identifies situations in which one quantity changes at a constant rate per unit interval relative to another

12. The correct answer is (C). The properties of angles associated with parallel and transversal lines can be used to show that the angle with measure x degrees and the angle with measure y degrees are supplementary angles. The sum of the measures of supplementary angles is $180°$, so $x + y = 180$. It is given that $y = 3x$. Substituting $3x$ for y in the equation $x + y = 180$ yields $4x = 180$. Hence, $x = 45$.

Category	IV. Geometry and Measurement
Topic	A. Knows the properties of types of lines (e.g., parallel, perpendicular, intersecting) and angles
Subtopic	2. Applies angle relationships (e.g., supplementary, vertical, alternate interior) to solve problems

13. The correct answer is (B). At the start of the lesson, several students answered that the side length of a square with area 36 square units is 9 units instead of giving the correct side length of 6 units. Since the side length of a square with **perimeter** 36 units is 9 units, the students are probably confusing area and perimeter. Therefore, the area that would be the least useful for assessing student learning in this situation is the one that would allow a student to find the correct answer by dividing the number of square units by 4, since one would divide the perimeter of a square by 4 to find the side length of the square. In (B), $\sqrt{16} = 4$ and $16 \div 4 = 4$. Since the answers are the same, Ms. Slater would have no way of knowing whether students were thinking about area or thinking about perimeter when finding the answer, so this is not a useful problem for assessing student learning in this situation. In each of (A), (C), and (D), the square root of the number of square units is not equal to the result when the number of square units is divided by 4, so these areas would be useful for assessing student learning in this situation.

Task of Teaching Topic	Mathematical problems, tasks, examples, and procedures
Task of Teaching Subtopic	6. Identifies problems or tasks that fit a particular structure, address the same concept, demonstrate desired characteristics, or elicit particular student thinking
Category	IV. Geometry and Measurement
Topic	H. Understands how to solve problems involving perimeter and area of polygons
Subtopic	1. Calculates and interprets perimeter and area of polygons that can be composed of triangles and quadrilaterals, including in real-world situations

14. The correct answer is 49 kilometers. Reggie hiked 3,500 meters along a trail each day for 14 days, so Reggie hiked $3{,}500 \times 14 = 49{,}000$ meters during that time. Since there are 1,000 meters in 1 kilometer, dividing 49,000 by 1,000 gives the final answer: that Reggie hiked 49 kilometers in the last 14 days.

Category	IV. Geometry and Measurement
Topic	J. Understands systems of measurement (i.e., metric, United States customary)
Subtopic	1. Solves measurement, estimation, and conversion problems involving time, length, temperature, volume, and mass in standard measurement systems

15. The correct answer is (D). There are $360°$ in a circle, and 16.1% of 360 is equal to $0.161 \times 360 = 57.96$. So of the values given, $58°$ is the value that best approximates the measure of the central angle of the sector representing the sales of Model D.

Category	V. Statistics and Probability
Topic	B. Understands how to interpret, analyze, and represent data presented in a variety of displays
Subtopic	1. Represents and analyzes data in various displays (e.g., bar graphs, line graphs, circle graphs, boxplots, histograms, scatterplots, stem-and-leaf plots, two-way tables)

16. The correct answers are (A) and (D). Since the 4 additional integers that list *M* contains are each less than each of the integers in list *K*, then each of the 4 additional integers that list *M* contains must be less than the mean of the integers in list *K*, so the statement in (A) must be true. Also, since the greatest integer in list *M* is equal to the greatest integer in list *K*, but the least integer in list *M* is less than the least integer in list *K*, the statement in (D) must be true. To demonstrate that the statements in (B), (C), and (E) may not be true, suppose that list *K* consists of the integer 76 listed 15 times and that list *M* consists of the integers 71, 72, 73, 74, and the integer 76 listed 15 times. In this case, the median of each list is 76 and the mode of each list is 76, so the statements in (B) and (C) are not true for this example. In addition, the first quartile of each list is 76 and the third quartile of each list is 76, which means the interquartile range of each list is 0, so the statement in (E) is not true for this example. Therefore, the statements in (B), (C), and (E) may not be true.

Category	V. Statistics and Probability
Topic	C. Understands concepts associated with measures of central tendency and dispersion
Subtopic	2. Determines and interprets measures of center (e.g., mean, median, mode) and spread (e.g., range, interquartile range) in a variety of problems

17. The correct answer is (D). Based on the data in the table, a total of $16+14=30$ people surveyed watched at least 2 hours of television but less than 4 hours of television per day. If a person is selected at random from those surveyed, the probability that the person selected will have watched at least 2 hours but less than 4 hours per day is $\frac{30}{50}=\frac{3}{5}$.

Category	V. Statistics and Probability
Topic	D. Knows how to use and evaluate probability models
Subtopic	2. Solves probability problems involving simple events

Understanding Question Types

The *Praxis*® assessments include a variety of question types: constructed response (for which you write a response of your own); selected response, for which you select one or more answers from a list of choices or make another kind of selection (e.g., by selecting a sentence in a text or by selecting part of a graphic); and numeric entry, for which you enter a numeric value in an answer field. You may be familiar with these question formats from taking other standardized tests. If not, familiarize yourself with them so you don't spend time during the test figuring out how to answer them.

Understanding Selected-Response and Numeric-Entry Questions

For most questions, you respond by selecting an oval to select a single answer from a list of answer choices.

However, interactive question types may also ask you to respond by:

- Selecting more than one choice from a list of choices.
- Typing in a numeric-entry box. When the answer is a number, you may be asked to enter a numerical answer. Some questions may have more than one entry box to enter a response.
- Selecting parts of a graphic. In some questions, you will select your answers by selecting a location (or locations) on a graphic such as a map or chart, as opposed to choosing your answer from a list.
- Selecting sentences. In questions with reading passages, you may be asked to choose your answers by selecting a sentence (or sentences) within the reading passage.
- Dragging and dropping answer choices into targets on the screen. You may be asked to select answers from a list of choices and to drag your answers to the appropriate location in a table, paragraph of text or graphic.
- Selecting answer choices from a drop-down menu. You may be asked to choose answers by selecting choices from a drop-down menu (e.g., to complete a sentence).

Remember that with every question you will get clear instructions.

Understanding Constructed-Response Questions

Constructed-response questions require you to demonstrate your knowledge in a subject area by writing your own response to topics. Essays and short-answer questions are types of constructed-response questions.

For example, an essay question might present you with a topic and ask you to discuss the extent to which you agree or disagree with the opinion stated. You must support your position with specific reasons and examples from your own experience, observations, or reading.

Review a few sample essay topics:

- *Brown v. Board of Education of Topeka*

  "We come then to the question presented: Does segregation of children in public schools solely on the basis of race, even though the physical facilities and other 'tangible' factors may be equal, deprive the children of the minority group of equal educational opportunities? We believe that it does."

A. What legal doctrine or principle, established in Plessy v. Ferguson (1896), did the Supreme Court reverse when it issued the 1954 ruling quoted above?
B. What was the rationale given by the justices for their 1954 ruling?

- *In his self-analysis, Mr. Payton says that the better-performing students say small-group work is boring and that they learn more working alone or only with students like themselves. Assume that Mr. Payton wants to continue using cooperative learning groups because he believes they have value for all students.*
  - Describe TWO strategies he could use to address the concerns of the students who have complained.
  - Explain how each strategy suggested could provide an opportunity to improve the functioning of cooperative learning groups. Base your response on principles of effective instructional strategies.
- *"Minimum-wage jobs are a ticket to nowhere. They are boring and repetitive and teach employees little or nothing of value. Minimum-wage employers take advantage of people because they need a job."*
  - Discuss the extent to which you agree or disagree with this opinion. Support your views with specific reasons and examples from your own experience, observations, or reading.

Keep these things in mind when you respond to a constructed-response question:

1. **Answer the question accurately**. Analyze what each part of the question is asking you to do. If the question asks you to describe or discuss, you should provide more than just a list.
2. **Answer the question completely**. If a question asks you to do three distinct things in your response, you should cover all three things for the best score. Otherwise, no matter how well you write, you will not be awarded full credit.
3. **Answer the question that is asked**. Do not change the question or challenge the basis of the question. You will receive no credit or a low score if you answer another question or if you state, for example, that there is no possible answer.
4. **Give a thorough and detailed response**. You must demonstrate that you have a thorough understanding of the subject matter. However, your response should be straightforward and not filled with unnecessary information.
5. **Take notes on scratch paper** so that you don't miss any details. Then you'll be sure to have all the information you need to answer the question.
6. **Reread your response**. Check that you have written what you thought you wrote. Be sure not to leave sentences unfinished or omit clarifying information.

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