Rich Mathematical Task – Geometry – *Sea Cities*

Task Overview/Description/Purpose:
• Sea level rise is affecting cities that lie on the coastline causing damages to infrastructures. Virginia is experiencing the fastest rate of sea level rise and residents will eventually need to be rehomed in sunken areas. The students will be involved with the planning of floating cities, an innovative solution to finding homes for those whose homes will be underwater. • The students will use properties of polygons and polygons, which tessellate, to justify their mathematical reasoning.

Standards Alignment: Strand – *Polygons and Circles*
Primary SOL: G.10 The student will solve problems, including practical problems, involving angles of convex polygons. This will include determining the a) sum of the interior and/or exterior angles; b) measure of an interior and/or exterior angle; and c) number of sides of a regular polygon. **Related SOL (within or across grade levels/courses):** 8.9a, 8.9b, 8.10, G.8a, G.8b, G.8c
Learning Intention(s): • **Content** - I am learning how to determine the measure of each interior and exterior angle of a regular polygon and determine angle measures of a regular polygon in a tessellation. • **Language** - I am learning to explain and justify my thinking and reasoning when applying properties of polygons and tessellating polygons in a plane. • **Social** - I am learning how to communicate my mathematical thinking to my peers and ask probing questions that help my peers and me advance our thinking about properties of polygons and tessellations.
Success Criteria (Evidence of Student Learning): • I can solve practical problems involving angles of convex polygons. • I can determine the measure of each interior and exterior angle of a regular polygon. • I can determine angle measures of regular polygons in a tessellation. • I can explain my thinking and process for solving practical problems involving angles of convex polygons. • I can logically communicate how my mathematical evidence supports my claims to my peers.

Mathematics Process Goals	
Problem Solving	• Students create and design a floating city through the application of tessellations as a solution to the floating city design. • Students apply the properties of polygons to determine interior and exterior angles measures of regular polygons.
Communication and Reasoning	• Students support arguments and claims with evidence when applying properties of polygons and tessellations. • Students use mathematical language to justify their reasoning.
Connections and Representations	• Students use multiple representations with accurate labels to explore and model the problem. • Students make mathematical connection that is relevant to the context of the problem using properties of polygons and tessellations.

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Task Pre-Planning

Approximate Length/Time Frame: 90 minutes

Grouping of Students:

Students will work on the task individually. When completed with the task, ask students to share their solutions with a shoulder partner. Finally, create small groups of students to share and discuss their sea city sketch as well as their solutions to prompts 1 and 4. Students will confer with a small group of students to compare and contrast possible solutions, with the teacher strategically grouping students who have used different methods or representations.

Materials and Technology:

- copy of task
- pencils
- calculators
- Desmos graphing calculator
- graph paper
- other materials necessary for sea city sketches (i.e. poster board, poster paper, graph paper)
- Geogebra

Vocabulary:

- Adjacent
- Congruent
- Exterior Angle
- Interior Angle
- Regular Polygon
- Tessellation
- Vertex

Anticipate Responses: See the Planning for Mathematical Discourse Chart (columns 1-3).

Task Implementation (Before)

Task Launch:

- Share this link with students on sea level rise in Virginia https://sealevelrise.org/states/virginia/. Have students read and answer the following questions.
  - How is sea level rise measured?
  - What are possible causes of sea level rise?
  - How many residential properties in Norfolk, Virginia Beach, Chesapeake, and Portsmouth are predicted to be affected by 2033?
  - What are possible solutions to address sea level rise?
- Students will use underlining, highlighting, cue words, or a visual vocabulary word wall to help make sense of the task.
- Students will access prior knowledge and vocabulary regarding the properties of polygons using the VDOE Word Wall Cards. Have students create a Frayer model for words they have not mastered.

Task Implementation (During)

Directions for Supporting Implementation of the Task

- Monitor – The teacher will observe students as they work independently on the task. The teacher will engage with students by asking assessing or advancing questions as necessary (see attached *Question Matrix).*
- Select – The teacher will select students to sketch and/or share their mathematical reasoning once everyone has had time to complete the task.
- Sequence – The teacher will select 2-3 student strategies to share with the whole group. One suggestion is to look for one common misconception and two correct responses to share.
- Connect – The teacher will consider ways to facilitate connections between different student representations.

Suggestions For Additional Student Support

- Possible use of sentences frames to support student thinking
  - SeaCitagon is using a hexagon to support the floating city because …

  - I chose ______________________________ as the polygon(s) to accommodate the three adjacent city buildings because …
  - I chose ______________________________ as the polygon to accommodate the four adjacent medical buildings because …
- Possible actions to support vocabulary development
  - Have students complete a Frayer model
  - Display VDOE Word Wall Cards.
  - Make word associations clear, e.g. focus on IN in Interior angles or EX in Exterior
  - Ensure when students are speaking/writing that they are utilizing the proper vocabulary terms
  - Pair vocabulary with visuals
  - Have students create a math vocabulary book
  - Ask students to speak with each other about what they know about polygons, especially regular polygons
- Possible problem solving strategies/graphic organizers
  - Students could use geogebra to easily create regular polygons and manipulate them to explore their mathematical reasoning
  - Draw sketches of regular polygons to visualize the tessellations
  - Consider providing regular polygons on paper for students to cut out and manipulate to explore tessellations

Task Implementation (After) ***20 minutes***

Connecting Student Responses (From Anticipating Student Response Chart) and Closure of the Task:

- Based on the actual student responses, sequence and select particular students to present their mathematical work during class discussion.
- Connect different students' responses and connect the responses to the key mathematical ideas to bring closure to the task.
- Consider ways to ensure that each student will have an equitable opportunity to share his/her thinking during task discussion.

Teacher Reflection About Student Learning:

- How will student understanding of the content through the use of the process goals be assessed (i.e., task rubric)?
- How will the evidence provided through student work inform further instruction?
- What was a common misconception and how can this be addressed in further instruction?
- Does vocabulary need further development?
- Are students able to explain their thinking in oral and written form?

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Planning for Mathematical Discourse

Mathematical Task: Sea Cities   Content Standard(s): G.10

Anticipated Student Response/Strategy *Provide examples of possible correct student responses along with examples of student errors/misconceptions*	**Assessing Questions** *Teacher questioning that allows student to explain and clarify thinking*	**Advancing Questions** *Teacher questioning that moves thinking forward*	**List of Students Providing Response** *Who? Which students used this strategy?*	**Discussion Order - sequencing student responses** • *Based on the actual student responses, sequence and select particular students to present their mathematical work during class discussion* • *Connect different students' responses and connect the responses to the key mathematical ideas* • *Consider ways to ensure that each student will have an equitable opportunity to share his/her thinking during task discussion*
Anticipated Student Response: Students may be unable to answer why a regular hexagon would be used for the base shape of the city.	• What is the angle measure of the interior angle of the regular hexagon? • How many hexagons could share a common vertex?	• What do you notice about tessellating an equilateral triangle about a point? • Why are certain structures in nature formed using regular hexagons?		
Anticipated Student Response: Students may only be able to divide the polygon into equilateral triangles.	• Could you include more than one side of the hexagon to form another polygon?	• Do the polygons you create have to be *regular* polygons?		
Anticipated Student Response: Students may not know how to measure side lengths or angles of their polygons.	• What tools are used in geometry to measure side lengths and angle measures?	• If you are unable to measure using a tool, what do you know about interior angles of polygons? • Are there any ways to find side lengths of certain polygons using right triangle properties?		
Anticipated Student Response:				

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Anticipated Student Response/Strategy *Provide examples of possible correct student responses along with examples of student errors/misconceptions*	**Assessing Questions** *Teacher questioning that allows student to explain and clarify thinking*	**Advancing Questions** *Teacher questioning that moves thinking forward*	**List of Students Providing Response** *Who? Which students used this strategy?*	**Discussion Order - sequencing student responses** • *Based on the actual student responses, sequence and select particular students to present their mathematical work during class discussion* • *Connect different students' responses and connect the responses to the key mathematical ideas* • *Consider ways to ensure that each student will have an equitable opportunity to share his/her thinking during task discussion*
Students may only be able to create four squares when designing the medical buildings.	• What are congruent polygons? • What are regular polygons?	• Do regular polygons with the same number of sides and angles have to be congruent? Do congruent polygons have to be regular?		

NAME ____ DATE ____________________

Sea Cities

Hampton Roads, Virginia is experiencing sea level rise at a rate of one inch per year and represents the highest sea level change along the coast. There are many causes of sea level rise to include sinking lands and high tides. There are also solutions such as filling underground voids by pumping ground water. One innovative solution to address the damage of property and the relocation of residents is to build on water. The company SeaCitagon has a goal of developing a floating city that can house over 1,500 residents. They will use a regular hexagon, the most efficient packing shape, as the base of their floating city.

1. What mathematical reason could SeaCitagon have to justify using a regular hexagon as its base shape for the city?

2. A regular hexagon can be divided into many other polygons. Sketch 2 regular hexagons that have been divided into other polygons. Measure each side length for the hexagons, and then label each angle measure and side length of the subdivided polygons.

3. Design a mini city for Seacitagon using at least 4 different *regular* polygons as building shapes and 16 polygons total. You may make your design on a poster, graph paper, geogebra, or any other form of display. For each polygon, label the building name, side lengths and each interior and exterior angle measure.

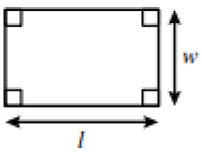
4. City officials want three buildings adjacent to each other in order to accommodate an office suite for the city. Which polygon(s) could be used to accommodate these three buildings so that each building is adjacent to the other two buildings? Why? Show and explain your mathematical reasoning.

5. The medical director wants the health center to be created with four congruent buildings such that they share a vertex to include a pediatrics center, a disease control center, an emergency center, and a general medical center. Which polygon could be used so that all four buildings are congruent and share a vertex? Show and explain your mathematical reasoning.

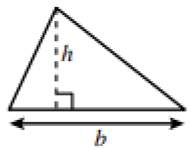
Extension --

I. If one inch of your drawing in prompt 3 equals 5 feet, what is the area of each of your buildings on your floating city?

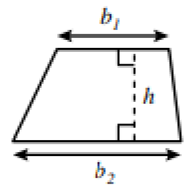
You may use these formulas to guide you:



$$p = 2l + 2w$$
$$A = lw$$



$$A = \frac{1}{2}bh$$



$$A = \frac{1}{2}h(b_1 + b_2)$$

II. Once you have calculated the areas of your buildings, calculate the cost of flooring for each building. You can find the cost of a particular price for flooring if one below is not the type of flooring you would like for your building.

- Laminate -- $3 per square foot
- Hardwood -- $9 per square foot
- Carpet -- $7 per square foot
- Linoleum -- $5 per square foot

III. The hospital needs to measure at least 10,000 square feet to accommodate enough rooms and the equipment necessary. With a scale of 1in: 5ft what area will the hospital have in the drawing?

IV. Using your city map, create the roads that connect the buildings. Identify angle measures, where transversals exist, and use angle relationships to identify 2-3 locations on your map.

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Rich Mathematical Task Rubric

	Advanced	Proficient	Developing	Emerging
Mathematical Understanding	Proficient Plus: • Uses relationships among mathematical concepts or makes mathematical generalizations	• Demonstrates an understanding of concepts and skills associated with task • Applies mathematical concepts and skills which lead to a valid and correct solution	• Demonstrates a partial understanding of concepts and skills associated with task • Applies mathematical concepts and skills which lead to an incomplete or incorrect solution	• Demonstrates no understanding of concepts and skills associated with task • Applies limited mathematical concepts and skills in an attempt to find a solution or provides no solution
Problem Solving	Proficient Plus: • Problem solving strategy is well developed or efficient	• Problem solving strategy displays an understanding of the underlying mathematical concept • Produces a solution relevant to the problem and confirms the reasonableness of the solution	• Problem solving strategy displays a limited understanding of the underlying mathematical concept • Produces a solution relevant to the problem but does not confirm the reasonableness of the solution	• A problem solving strategy is not evident • Does not produce a solution that is relevant to the problem
Communication and Reasoning	Proficient Plus: • Reasoning or justification is comprehensive • Consistently uses precise mathematical language to communicate thinking	• Demonstrates reasoning and/or justifies solution steps • Supports arguments and claims with evidence • Uses mathematical language to communicate thinking	• Reasoning or justification of solution steps is limited or contains misconceptions • Provides limited or inconsistent evidence to support arguments and claims • Uses limited mathematical language to partially communicate thinking	• Provides no correct reasoning or justification • Does not provide evidence to support arguments and claims • Uses no mathematical language to communicate thinking
Representations and Connections	Proficient Plus: • Uses representations to analyze relationships and extend thinking • Uses mathematical connections to extend the solution to other mathematics or to deepen understanding	• Uses a representation or multiple representations, with accurate labels, to explore and model the problem • Makes a mathematical connection that is relevant to the context of the problem	• Uses an incomplete or limited representation to model the problem • Makes a partial mathematical connection or the connection is not relevant to the context of the problem	• Uses no representation or uses a representation that does not model the problem • Makes no mathematical connections