Rich Mathematical Task – Algebra I – *Full Parking Lot*

Task Overview/Description/Purpose:

- In this task students will explore a situation that includes cars and motorcycles parked in a lot in order to develop the mathematical concept of systems of linear equations.
- This task could be used to introduce systems of linear equations because it will show students that there are multiple ways of solving the problem, but a system of linear equations could be more efficient. It could also be used at the end of a unit in order to see if students tend to use a graphing or one of the algebraic approaches to solving systems of linear equations.

Standards Alignment: Strand – Equations and Inequalities

Primary SOL: A.4 The student will
  d) solve systems of two linear equations in two variables algebraically and graphically.
  e) solve practical problems involving equations and systems of equations.

Related SOL (within or across grade levels/courses): 8.17, 8.4, A.4a, AII.4

Learning Intentions:

- **Content (based on Essential Knowledge and Skills) –** I am learning to apply my understanding of systems of linear equations to make informed decisions about a real world problem.
- **Language –** I am learning to explain my reasoning with mathematical language about solving systems of equations.
- **Social –** I am working toward mathematical and logical consensus with my collaborative team.

Success Criteria (Evidence of Student Learning):

- I can mathematically model a situation with a system of linear equations.
- I can solve a system of two linear equations using my preferred method (algebraically or graphically).
- I can use my math as evidence to collaboratively construct a claim about a real-world situation.
- I can logically communicate how my mathematical evidence supports my claim.
- I can describe how to solve systems of linear equations.

Mathematics Process Goals

Problem Solving	• Students will choose an appropriate strategy to reach a solution to the problem.
Communication and Reasoning	• Students will provide work to show how they used their strategy to reach their solution. • Students will explain their reasoning using mathematical vocabulary.
Connections and Representations	• Students will provide one or more representation of the situation: physical model, table, graph, equation.

Task Pre-Planning

**Approximate Length/Time Frame*:* 55 minutes

Grouping of Students: Provide some individual think time for students to read the task and come up with a strategy that make sense to them. Then put students in small groups to discuss strategies, decide on a strategy, and solve the problem.

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Materials and Technology:	Vocabulary:
- two color counters - white boards - markers - graph paper - graphing utility	- system of linear equations - point of intersection - solution

Anticipate Responses: See Planning for Mathematical Discourse Chart (Columns 1-3)

Task Implementation (Before)

Task Launch:
- Work with your English colleagues to use reading strategies that will be familiar to your students.
- This task could be used as an introduction to systems of linear equations, so little vocabulary is needed prior to students beginning the task. The follow up discussion would be a great time to draw in the vocabulary for systems of linear equations.
- Present this task as a problem for students to solve in any manner that makes sense to them.
- Make sure students have access to a variety of materials.
- Allow students to pursue different strategies, and do not lead them to using a system of equations unless that is what they think of doing on their own.

Task Implementation (During)

Directions for Supporting Implementation of the Task
- Monitor – Teacher will listen and observe students as they work on task and ask assessing or advancing questions (see chart on next page)
- Select – Teacher will decide which strategies or thinking that will be highlighted (after student task implementation) that will advance mathematical ideas and support student learning
- Sequence – Teacher will decide the order in which student ideas will be highlighted (after student task implementation)
- Connect – Teacher will consider ways to facilitate connections between different student responses

Suggestions For Additional Student Support (possible supports or accommodations for individual student, as needed)
May include, among others:
- Possible use of sentences frames to support student thinking and justification
    - I used ___ and ___ as the variables.
    - What I already know about the equations is ….
    - Before I can solve, I need to decide which variable I will solve for. I chose ___ because___.
- Provide highlighters to assist students in interacting with text
- Provide oral instructions
- Allow students to provide oral explanations
- Possible problem solving strategies questions for non-starters:
    - Can you just try a combination?
    - Could you draw a picture of the situation?
- Allow students to make connections or share cognates to key terms in another language (e.g., intersection and system).
- To support mathematical skill development: Create or co-create an anchor chart to describe how to solve systems of equations. Include sequence words. The chart should model a couple of examples (words connected to the numerical representation of the equations).

Task Implementation (After)

Connecting Student Responses (From Anticipating Student Response Chart) and Closure of the Task:
- Based on the actual student responses, sequence and select particular students to present their mathematical work during class discussion.
- Connect different students' responses and connect the responses to the key mathematical ideas to bring closure to the task. Discuss similarities and differences between two strategies before adding additional strategies.
- Consider ways to ensure that each student will have an equitable opportunity to share his/her thinking during task discussion.
- Draw out any pertinent vocabulary, if possible, during the closure discussion and post word wall cards.

Teacher Reflection About Student Learning:

- What strategies did students use and did they fit with what you expected them to do? **Note: Guess and check is not a course appropriate strategy for mathematical understanding even though it can lead to a correct solution.**
- What were the reoccurring student misconceptions?
- How will the evidence provided through student work inform further instruction?
- Does vocabulary need further development?
- Are students able to explain their work verbally (oral or written)?

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Planning for Mathematical Discourse

Mathematical Task: _____ Full Parking Lot _____   Content Standard(s): _____A.4(d), A.4(e)_____

Anticipated Student Response/Strategy *Provide examples of possible correct student responses along with examples of student errors/misconceptions*	Assessing Questions – Teacher Stays to Hear Response *Teacher questioning that allows student to explain and clarify thinking*	Advancing Questions – Teacher Poses Question and Walks Away *Teacher questioning that moves thinking forward*	List of Students Providing Response *Who? Which students used this strategy?*	Discussion Order - sequencing student responses
				• *Based on the actual student responses, sequence and select particular students to present their mathematical work during class discussion* • *Connect different students' responses and connect the responses to the key mathematical ideas.* • *Consider ways to ensure that each student will have an equitable opportunity to share his/her thinking during task discussion*
Anticipated Student Response: Guess and check. *This strategy will obtain a correct solution, but would not be course appropriate for mathematical understanding.	• What assumptions did you make about the number of wheels? • What's going on in this situation?	• How can you take your original trial and get closer without trying all options? • What are you noticing?		
Anticipated Student Response: Drawing a picture – 20 spaces w/ number of wheels per space. *This strategy will obtain a correct solution, but would not be course appropriate for mathematical understanding unless there is further support.	• How did you decide what to draw? • What's going on in this situation?	• How could you simplify your picture? • How many wheels have to be in every space?	**Student E** **Student F**	

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Anticipated Student Response: <table><tr><td>Cars (wheels)</td><td>Motorcycles (wheels)</td><td>Total Wheels</td></tr><tr><td>10 (40)</td><td>10 (20)</td><td>60</td></tr><tr><td>11 (44)</td><td>9 (18)</td><td>62</td></tr><tr><td>12 (48)</td><td>8 (16)</td><td>64</td></tr><tr><td>13 (52)</td><td>7 (14)</td><td>66</td></tr></table> Possible Misconception: students might start with 10 and 10, but go up in both columns, forgetting there are only 20 parking spaces.	• How did you decide where to begin your table? • How many parking spaces are you allowed?	• How can you take your original trial and get closer without trying all options? • Do you see any patterns in your table?	**Student E** *This student does not actually make a table, but explains how they did begin their process with 10 of each type of vehicle.*	
Anticipated Student Response: Solve an equation/or a system of equations using substitution: $4x + 2(20 - x) = 66$ Possible Misconception: students may struggle to define their second vehicle type in terms of the first.	• Explain how you came up with the parts of your equation. • Why did you choose to define motorcycles (or cars) in terms of the other vehicle?	• Is this the only equation that would work? • How can you use the number of parking spaces within the equation?	**Student A** **Student C**	

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Anticipated Student Response: Write and solve a system of equations by graphing: $\begin{cases} x + y = 20 \\ 4x + 2y = 66 \end{cases}$ Possible Misconception: Students often see how to relate the number of wheels, but struggle with the first equation.	• What do your variables represent? • How many parking spaces do you have to work with?	• Can you solve this system in more than one way? • How can you incorporate the number of parking spaces in an equation?	**Student D**	
Anticipated Student Response: Write and solve a system of equations by elimination: $\begin{cases} x + y = 20 \\ 4x + 2y = 66 \end{cases}$ Possible Misconception: Students often see how to relate the number of wheels, but struggle with the first equation.	• What do your variables represent? • How many parking spaces do you have to work with?	• Can you solve this system in more than one way? • How can you incorporate the number of parking spaces in an equation?	**Student B**	

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All 20 spaces in my favorite parking lot are filled by vehicles. Some are occupied by two-wheeled motorcycles and others by cars. Each space has only one vehicle occupying it. To calm myself, I counted the wheels in the parking lot and there were 66. How many cars and how many motorcycles have invaded my lot?

Show all work and explain how you arrived at your final solution.

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Rich Mathematical Task Rubric

	Advanced	Proficient	Developing	Emerging
Mathematical Understanding	Proficient Plus: • Uses relationships among mathematical concepts	• Demonstrates an understanding of concepts and skills associated with task • Applies mathematical concepts and skills which lead to a valid and correct solution	• Demonstrates a partial understanding of concepts and skills associated with task • Applies mathematical concepts and skills which lead to an incomplete or incorrect solution	• Demonstrates little or no understanding of concepts and skills associated with task • Applies limited mathematical concepts and skills in an attempt to find a solution or provides no solution
Problem Solving	Proficient Plus: • Problem solving strategy is efficient	• Problem solving strategy displays an understanding of the underlying mathematical concept • Produces a solution relevant to the problem and confirms the reasonableness of the solution	• Chooses a problem solving strategy that does not display an understanding of the underlying mathematical concept • Produces a solution relevant to the problem but does not confirm the reasonableness of the solution	• A problem solving strategy is not evident or is not complete • Does not produce a solution that is relevant to the problem
Communication and Reasoning	Proficient Plus: • Reasoning is organized and coherent • Consistent use of precise mathematical language and accurate use of symbolic notation	• Communicates thinking process • Demonstrates reasoning and/or justifies solution steps • Supports arguments and claims with evidence • Uses mathematical language to express ideas with precision	• Reasoning or justification of solution steps is limited or contains misconceptions • Provides limited or inconsistent evidence to support arguments and claims • Uses limited mathematical language to partially communicate thinking with some imprecision	• Provides little to no correct reasoning or justification • Does not provide evidence to support arguments and claims • Uses little or no mathematical language to communicate thinking
Representations and Connections	Proficient Plus: • Uses representations to analyze relationships and extend thinking • Uses mathematical connections to extend the solution to other mathematics or to deepen understanding	• Uses a representation or multiple representations, with accurate labels, to explore and model the problem • Makes a mathematical connection that is relevant to the context of the problem	• Uses an incomplete or limited representation to model the problem • Makes a partial mathematical connection or the connection is not relevant to the context of the problem	• Uses no representation or uses a representation that does not model the problem • Makes no mathematical connections