Rich Mathematical Task – Algebra II – *Function of a Ride*

Task Overview/Description/Purpose:

- In this task students will explore function that is created from a section of a roller coaster. Their exploration will include describing key features of the function, describing the ride mathematically, and then creating an equation to match one section of the ride to make predictions.
- This task could be used as a summative assessment, but the task still allows for multiple strategies on some parts of the task, which could make this a formative task that would help the teacher inform further instruction.

Standards Alignment: Strand – Equations and Inequalities

Primary SOL: AII.6 For polynomial functions, the student will

a) recognize the general shape of function families; and
b) use knowledge of transformations to convert between equations and the corresponding graph of functions.

AII.7 The student will investigate and analyze quadratic and polynomial function families algebraically and graphically. Key concepts include

a) domain, range, and continuity;
b) intervals in which a function is increasing or decreasing;
c) extrema;
d) connections between and among multiple representations of functions using verbal descriptions, tables, equations, and graphs.

AII.8 The student will

a) collect and analyze data, determine the equation of the curve of best fit in order to make predictions, and solve practical problems, using mathematical models of quadratic functions.

Related SOL (within or across grade levels/courses): A.7, A.9

Learning Intentions:

- **Content (based on Essential Knowledge and Skills) –** I am learning to apply my understanding of function characteristics and transformations to analyze a real world problem.
- **Language –** I am learning to describe my analysis and predictions of real world problems given a model of functions.
- **Social –** I am learning how to communicate and justify my predictions when working collaboratively with others.

Success Criteria (Evidence of Student Learning):

- I can **identify** the domain and range of a function presented algebraically or graphically.
- I can **identify** intervals on which a function is increasing or decreasing.
- I can **identify** the location (x-value) of absolute maxima and absolute minima of a function over the domain of the function graphically.
- I can take one representation (verbal description, table, equation, or graphs) and **represent** the function in another form.
- I can **investigate** and **verify** transformations of quadratic functions using a graphing utility.
- I can **determine** an equation of the curve of best fit, using a graphing utility, given a set of no more than 20 data points in a table, graph, or practical situation.
- I can **make** predictions, using data, scatterplots, or the equation of the curve of best fit.
- I can **solve** practical problems involving an equation of the curve of best fit.
- I can **evaluate** the reasonableness of a mathematical model of a practical situation.
- I can describe my predictions and how I solve problems with functions.

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Mathematics Process Goals	
Problem Solving	• Students will choose an appropriate strategy to reach a solution to the problem.
Communication and Reasoning	• Students will provide work to show how they used their strategy to reach their solution. • Students will explain their reasoning using mathematical vocabulary about solving functions.
Connections and Representations	• Students will provide one or more representation of the situation: physical model, table, graph, equation.

Task Pre-Planning

Approximate Length/Time Frame*:* 55 minutes

Grouping of Students: Provide some individual think time for students to read the task and come up with a strategy that make sense to them. Then put students in small groups to discuss strategies, decide on a strategy, and solve the problem.

Materials and Technology:

- white boards
- markers
- graph paper
- graphing utility

Vocabulary:

- horizontal, vertical
- parabola
- function
- domain, range
- increasing, decreasing
- maxima, minima

Anticipate Responses: See Planning for Mathematical Discourse Chart (Columns 1-3)

Task Implementation (Before)

Task Launch:

- Work with your English colleagues to use reading strategies that will be familiar to your students.
- This task could be used as an introduction to systems of linear equations, so little vocabulary is needed prior to students beginning the task. The follow up discussion would be a great time to draw in the vocabulary for systems of linear equations.
- Present this task as a problem for students to solve in any manner that makes sense to them.
- Make sure students have access to a variety of materials.
- Allow students to pursue different strategies, and do not lead them to using a system of equations unless that is what they think of doing on their own.

Task Implementation (During)

Directions for Supporting Implementation of the Task

- Monitor – Teacher will listen and observe students as they work on task and ask assessing or advancing questions (see chart on next page)
- Select – Teacher will decide which strategies or thinking that will be highlighted (after student task implementation) that will advance mathematical ideas and support student learning
- Sequence – Teacher will decide the order in which student ideas will be highlighted (after student task implementation)
- Connect – Teacher will consider ways to facilitate connections between different student responses

Suggestions For Additional Student Support (possible supports or accommodations for individual student, as needed)
May include, among others:

- Possible use of sentences frames to support student thinking, predictions, and explanations:
  - I think ___ because ____.

- I found the interval by ….
- To find the maxima/minima/range/etc.), I …

- Provide highlighters to assist students in interacting with text
- Provide oral instructions
- Allow students to provide oral explanations
- Possible problem solving strategies questions for non-starters:
  - Can you just try a combination?
  - Could you draw a picture of the situation?
- To support mathematical skill development: Create or co-create an anchor chart (as building background) to describe how to determine values associated with function problems. The chart should model a couple of examples (words connected to the numerical representation of the equations and the graphic representations).

Task Implementation (After)

Connecting Student Responses (From Anticipating Student Response Chart) and Closure of the Task:

- Based on the actual student responses, sequence and select particular students to present their mathematical work during class discussion.
- Connect different students' responses and connect the responses to the key mathematical ideas to bring closure to the task. Discuss similarities and differences between two strategies before adding additional strategies.
- Consider ways to ensure that each student will have an equitable opportunity to share his/her thinking during task discussion.
- Draw out any pertinent vocabulary, if possible, during the closure discussion and post word wall cards.

Teacher Reflection About Student Learning

- What strategies did students use and did they fit with what you expected them to do?
- What were the reoccurring student misconceptions?
- How will the evidence provided through student work inform further instruction?
- Does vocabulary need further development?
- Are students able to explain their work verbally (oral or written)?

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Planning for Mathematical Discourse

Teacher Completes Prior to Task Implementation			Teacher Completes During Task Implementation	
Anticipated Student Response/Strategy *Provide examples of possible correct student responses along with examples of student errors/misconceptions*	**Assessing Questions – Teacher Stays to Hear Response** *Teacher questioning that allows student to explain and clarify thinking*	**Advancing Questions – Teacher Poses Question and Walks Away** *Teacher questioning that moves thinking forward*	**List of Students Providing Response** *Who? Which students used this strategy?*	**Discussion Order - sequencing student responses** • *Based on the actual student responses, sequence and select particular students to present their mathematical work during class discussion* • *Connect different students' responses and connect the responses to the key mathematical ideas.* • *Consider ways to ensure that each student will have an equitable opportunity to share his/her thinking during task discussion*
Anticipated Student Response: Parts 1 - 4 Guess and check: Students can check by opening the graph in Desmos and trying different points. *This strategy would still require some estimation of values.	• What assumptions did you make about the function? • What's going on in this situation?	• How can you take your original trial and get closer without trying all options? • What are you noticing?	Student A Student D	
Anticipated Student Response: Parts 1 - 4 Analyzing the given graph and estimating. Students would need to explain their process.	• How did you decide what to draw? • What's going on in this situation?	• How could you simplify your process? • Can you explain what you did to a friend?	Student B Student C	
Anticipated Student Response: Part 5 The verbal description of the ride does not really have multiple strategies although students will use a variety of language.	• How did you decide where to begin your table? • How many parking spaces are you allowed?	• How can you take your original trial and get closer without trying all options? • Do you see any patterns in your table?	Student B	

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Possible Misconception: Students might write their description only using the vertical distances rather than the horizontal distances.				
Anticipated Student Response: Parts 6 & 7 Matching the graph on Desmos. Students might use the general form of a parabola with values or sliders and just make adjustments until their graph is a close match and then use that equation to make a prediction. Possible Misconception: Students might have difficulty with the transformations of a quadratic function.	• Explain how you came up with your equation. • How did you know what changes to make to the equation?	• Is this the only equation that would work? • Are you happy with how your equation's graph matches the original? • Can you find a closer match to the graph?		
Anticipated Student Response: Choosing points and performing a quadratic regression. Student could choose values for the	• How many points did you choose?	• Is this the only equation that would work?		

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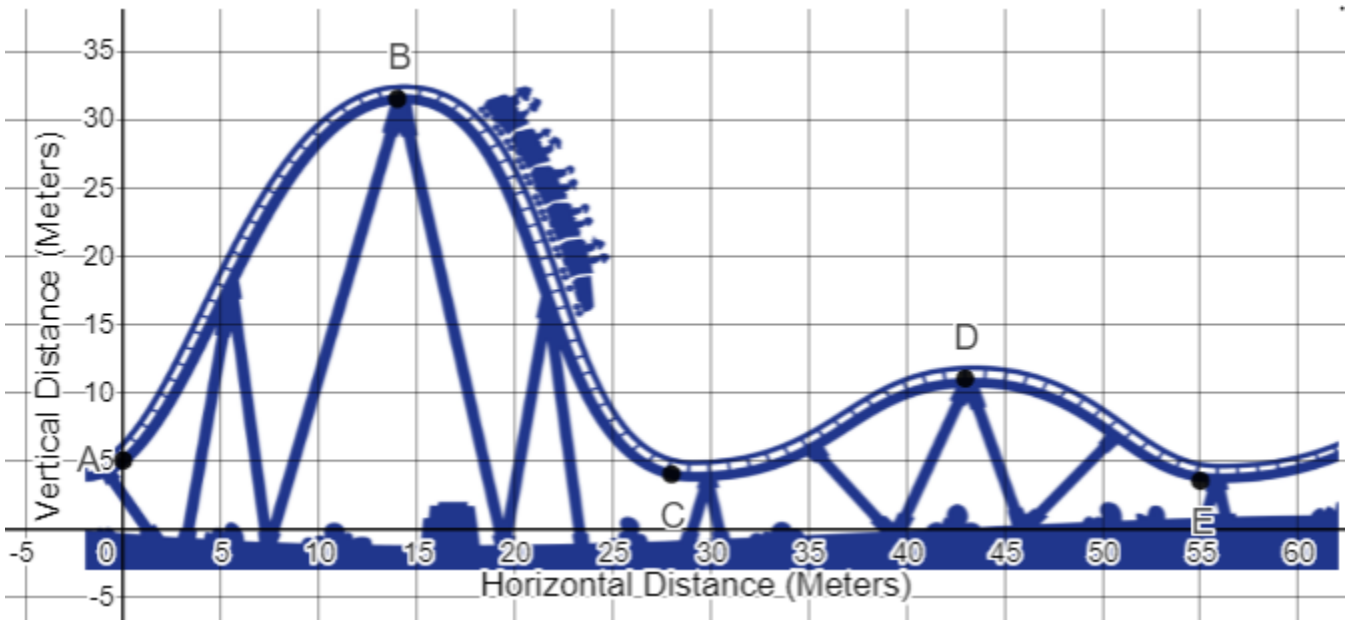
points A, B, and C to perform the regression and then use the equation to make a prediction. Possible Misconception: Students might struggle with the value to use for the points.	• How did you decide what values to use?	• Are you happy with how your equation's graph matches the original? • Can you find a closer match to the graph?		

Student Version of Task Description

Function of a Ride

Below you will find a graph comparing the horizontal and vertical distances of a portion of the roller coaster track, with key points labeled. Consider the point **A** to be the beginning of the roller coaster track. Also consider curves that look like parabolas, are parabolas (assume the curves are smooth).



stakeholder management | Productivity Steps
https://images.app.goo.gl/3sieQyMtH3sbWoRk6

View this graph in Desmos

1. What is the domain and range of the function?

   Domain Range

2. Find the intervals where the function in increasing and decreasing.

   Increasing

   Decreasing

   How did you find the intervals?

3. At what point on the coaster would you be going the fastest? The slowest? Explain why you chose these points.

   Fastest at Slowest at

4. What are the maxima and minima of the function?

   Maxima Minima

5. Where would you scream? Describe your ride as you travel the roller coaster. Include in your description your trip from point to point, whether you are moving up or down, and discuss what is happening to your speed.

6. Write the equation of the first "scream"! Find the equation of the first hill – the complete curve up and down again, from point A – C. Show all your work, with explanations when needed.

7. Looking at the first hill and its equation, how HIGH off the ground would you be after you have traveled 5 meters horizontally. Show your work and explain how you got the answer. How does the predicted height compare to the actual height of the roller coaster at that point?

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Rich Mathematical Task Rubric

	Advanced	Proficient	Developing	Emerging
Mathematical Understanding	Proficient Plus: • Uses relationships among mathematical concepts	• Demonstrates an understanding of concepts and skills associated with task • Applies mathematical concepts and skills which lead to a valid and correct solution	• Demonstrates a partial understanding of concepts and skills associated with task • Applies mathematical concepts and skills which lead to an incomplete or incorrect solution	• Demonstrates little or no understanding of concepts and skills associated with task • Applies limited mathematical concepts and skills in an attempt to find a solution or provides no solution
Problem Solving	Proficient Plus: • Problem solving strategy is efficient	• Problem solving strategy displays an understanding of the underlying mathematical concept • Produces a solution relevant to the problem and confirms the reasonableness of the solution	• Chooses a problem solving strategy that does not display an understanding of the underlying mathematical concept • Produces a solution relevant to the problem but does not confirm the reasonableness of the solution	• A problem solving strategy is not evident or is not complete • Does not produce a solution that is relevant to the problem
Communication and Reasoning	Proficient Plus: • Reasoning is organized and coherent • Consistent use of precise mathematical language and accurate use of symbolic notation	• Communicates thinking process • Demonstrates reasoning and/or justifies solution steps • Supports arguments and claims with evidence • Uses mathematical language to express ideas with precision	• Reasoning or justification of solution steps is limited or contains misconceptions • Provides limited or inconsistent evidence to support arguments and claims • Uses limited mathematical language to partially communicate thinking with some imprecision	• Provides little to no correct reasoning or justification • Does not provide evidence to support arguments and claims • Uses little or no mathematical language to communicate thinking
Representations and Connections	Proficient Plus: • Uses representations to analyze relationships and extend thinking • Uses mathematical connections to extend the solution to other mathematics or to deepen understanding	• Uses a representation or multiple representations, with accurate labels, to explore and model the problem • Makes a mathematical connection that is relevant to the context of the problem	• Uses an incomplete or limited representation to model the problem • Makes a partial mathematical connection or the connection is not relevant to the context of the problem	• Uses no representation or uses a representation that does not model the problem • Makes no mathematical connections