Rich Mathematical Task – Grade 8 –*Longest Path*

Task Overview/Description/Purpose:

- In this task, students will determine who has the longest distance to travel in order to develop a mathematical understand of the Pythagorean Theorem in a real world context.
- This task is designed to deepen understanding of the application of the Pythagorean Theorem.

Standards Alignment: Strand – *Computation and Estimation*

Primary SOL: 8.9- The student will
a) verify the Pythagorean Theorem
b) apply the Pythagorean Theorem

Related SOL (within or across grade levels/courses): 8.3b, 7.6ab, 7.1d, 6.4

Learning Intention(s):

- **Content –** I am learning about the Pythagorean Theorem and how it can be used to solve practical problems.
- **Language-** I am learning to use mathematical vocabulary (hypotenuse, leg, square, distance) when solving practical problems related to the Pythagorean Theorem.
- **Social –** I am learning how to explain my strategy and work to others so I can refine my strategies for problem solving.

Success Criteria (Evidence of Student Learning);

- I can find the hypotenuse given the legs of a right triangle.
- I can find a leg given a leg and hypotenuse of a right triangle.
- I can justify my computational process and report my conclusions using appropriate mathematical vocabulary.
- I can make suggestions and utilize suggestions made by my peers to make revisions to my work and thinking.

Mathematics Process Goals

Problem Solving	• Students will use the Pythagorean Theorem to find a leg or hypotenuse of a right triangle in a practical situation.
Communication and Reasoning	• Students will justify verbally and with mathematical evidence how they know who has the longest route and if Austin is correct in his claim.
Connections and Representations	• Students will use graph paper, models of triangles, and the equation for the Pythagorean Theorem to represent the different paths the children take. • Students will make connections to prior geometric concepts, such as properties of rectangles, such as opposite sides of rectangles are parallel and congruent.

Task Pre-Planning

Approximate Length/Time Frame: 30-45 minutes

Grouping of Students: This task would be best completed with students working independently first to determine own paths, and then work collaboratively to discuss their ideas and come up with a solution.

Materials and Technology:	**Vocabulary:**
• Graph Paper	• Pythagorean Theorem

Task Pre-Planning	
	• Hypotenuse • Leg • Right Triangle • Diagonal • Rectangle • Square
Anticipate Responses: See Planning for Mathematical Discourse Chart (Columns 1-3)	

Task Implementation (Before)

Task Launch

- Ask questions about if students have had similar experiences to the one in the task. Did they ever have debates with their friends about who lived the closest or farthest distance from a location? How did they prove who lived the closest or farthest?
- To help students make sense of the task, do a "Three Reads" protocol of the task.
    - Show students a version of the task without the question.
    - Read the task to the students. Students should follow along and circle any words that are confusing to them. Have a class discussion about any words that students circled to ensure they all know what the words in the task mean.
    - Read the task a second time to students. Students should follow along and underline any important information.
    - Read the task a third time to students. Ask students what questions could be asked given the information provided in the task. After taking some suggestions, reveal the question.

Task Implementation (During)

Directions for Supporting Implementation of the Task

- Monitor – Teacher will listen and observe students as they work on task and ask assessing or advancing questions (see chart on next page)
- Select – Teacher will decide which strategies or thinking that will be highlighted (after student task implementation) that will advance mathematical ideas and support student learning
- Sequence – Teacher will decide the order in which student ideas will be highlighted (after student task implementation) during the closure discussion.
- Connect – Teacher will consider ways to facilitate connections between different student responses

Suggestions For Additional Student Support

- Question students, in both assessing and advancing formats, to help students refine their strategies.
- Have graph paper available to students who might benefit from drawing the map on a grid.
- Some students get stuck at one way of thinking and using one method. Asking questions like "How confident are you?" and "What would convince someone?" will help students get past this point.
- For students with motor processing difficulties, allow them to communicate the reasoning in other ways such as video recording or typing answers.
- For students with attention challenges ask student to restate the problem or important information.
- For students who need academic language support, consider the use of a visual word wall or reference sheet for students to use identifying hypotenuse, legs, right triangle, and Pythagorean Theorem Formula.
- For students who have weaknesses in executive function, provide them with this blank table to help them organize their work (see attached graphic organizers)
- For students who need more support in justifying their thinking, you may choose to provide them with the sentence frames below.

- What I know about the problem is…
- My method for solving the problem was…
- I know that ____ lives the shortest/farthest walking distance because…

- For ELs with first language literacy, try to provide prompt, or parts of prompt, in their home language.
- Provide just in time scaffolds such as the graphic organizer with the map labeled with names and the distances listed in the problem (see attached graphic organizers)
- For students who have visual impairment or visual motor weaknesses, provide a larger visual (see attached graphic organizers)

Task Implementation (After)

Connecting Student Responses (From Anticipating Student Response Chart) and Closure of the Task:

- Have students read the success criteria after task implementation to see how their knowledge of the Pythagorean Theorem and prior knowledge of geometry helped them meet the expectations for this task.
- Based on the actual student responses, sequence and select particular students to present their mathematical work during class discussion. Some possible big mathematical ideas to highlight could include:
  - Common misconceptions
  - Varying Strategies
- Connect different students' responses and connect the responses to the key mathematical ideas to bring closure to the task. Possible questions and sentence frames to connect student strategies:
  - How are these strategies alike? How are they different?
  - Where do you see _____'s strategy in _____'s strategy?
  - Why is this important?
- Consider ways to ensure that each student will have an equitable opportunity to share his/her thinking during task discussion. Some possible ways to do this are to-
  - Assign roles like time keeper, task master, material fetcher, and recorder of strategies to each member of the group.
  - Students will begin the task independently to give each student a chance to work through different solution pathways on their own. After students have had a chance to explore the task independently, they will join groups. Before students start collaborating, each student will take one minute to share their work to ensure that all students are actively engaged in the task and that they have had an opportunity to share their own unique pathway to solving the problem.

Teacher Reflection About Student Learning:

- Teacher should use the chart on the next page with the anticipated student solutions to monitor which students are using each strategy as well as record any additional strategies encountered. The sequence of tasks will inform what will come next in instruction to further student ideas and thinking. Form small groups to address misconceptions that are not addressed in the class debrief.
- Information gathered from the task rubric could identify small groups for later instruction, identifying specific students to partner with one another, and/or identifying students who need more teacher modeling and think alouds.

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Planning for Mathematical Discourse

Mathematical Task: Longest Path

Content Standard(s): SOL 8.9

Anticipated Student Response/Strategy *Provide examples of possible correct student responses along with examples of student errors/misconceptions*	**Assessing Questions:** *Teacher questioning that allows student to explain and clarify thinking*	**Advancing Questions:** *Teacher questioning that moves thinking forward*	**List of Students Providing Response** *Who? Which students used this strategy?*	**Discussion Order - sequencing student responses** • *Based on the actual student responses, sequence and select particular students to present their mathematical work during class discussion* • *Connect different students' responses and connect the responses to the key mathematical ideas.* • *Consider ways to ensure that each student will have an equitable opportunity to share his/her thinking during task discussion*
Anticipated Student Response: I don't know what to do?	• What is the question asking you? • What information do you have? • What do you notice? • What do you wonder? • What do you predict the solution might look like?	• What could you use to better visualize the problem? • What do you predict the solution might look like?		
Anticipated Student Response: Student finds Hypotenuse when they should be finding a leg (or a leg when they should find the Hypotenuse).	• Did you have a picture in mind when you read the problem? Can you share it with us so we can see what you saw? • How does the length of the hypotenuse compare to the other sides of the right triangle? Does it hold true	• Does your answer seem reasonable? • What do you know about right triangles? Does it hold true for what you have calculated?		

Anticipated Student Response/Strategy *Provide examples of possible correct student responses along with examples of student errors/misconceptions*	**Assessing Questions:** *Teacher questioning that allows student to explain and clarify thinking*	**Advancing Questions:** *Teacher questioning that moves thinking forward*	**List of Students Providing Response** *Who? Which students used this strategy?*	**Discussion Order - sequencing student responses** • *Based on the actual student responses, sequence and select particular students to present their mathematical work during class discussion* • *Connect different students' responses and connect the responses to the key mathematical ideas.* • *Consider ways to ensure that each student will have an equitable opportunity to share his/her thinking during task discussion*
	for what you have calculated?			
Anticipated Student Response: Student identifies longest straight-line distance from the park instead of longest walking distance.	• Can you read the problem aloud again? • Let's go back to the question. Is everything still making sense?	• Is there anything about this solution that doesn't make sense? • Would a person be able to walk this path without any obstacles?		
Anticipated Student Response: Student finds the missing side, but does not add it to other sides when finding the walking distance (i.e. Tyler and Austin)	• Describe _____'s path to the park. How would _____ walk there? • Explain how you came to your answer? • Let's go back to the question. Is everything still making sense?	• Would _____ be able to walk this path all the way from home to the park? • Is there anything about this solution that doesn't make sense?		
Anticipated Student Response: Student is stuck on Tyler's path because they believe the triangle is missing two leg lengths.	• What is some information given that could help you solve the problem? • What geometric figures do you see in this problem?	• Could you use a measurement from a different shape to help you determine unknown information?		

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	What do you know about these figures?			

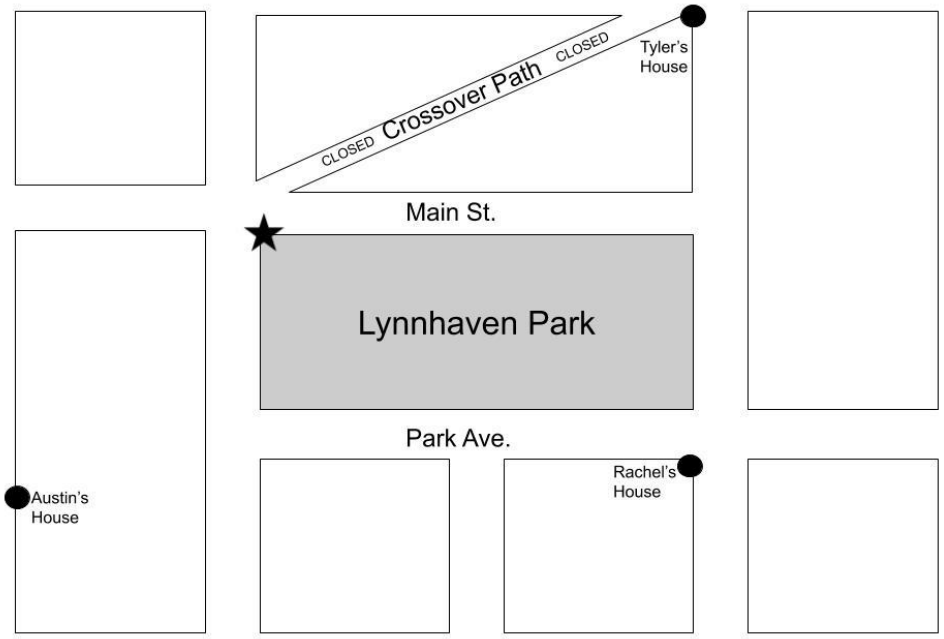
Name_____ Date_____

Longest Path

Tyler, Rachel, and Austin all live in the same neighborhood. On the weekends, they like to meet in the park to hang out. Austin is always the last to arrive, even though he leaves his house at the same time as the other two. He claims that his walk is a longer distance, therefore it takes him longer to arrive at the park. The friends decide to prove whether Austin's claim is true or not by determining the walking distance from each of their houses to the park.

Use the information and the map below to determine the distance from their houses to the meeting point at the park, which is marked with a star.

- Lynnhaven Park measures 100 meters long, and 35 meters wide.
- Rachel leaves her house and cuts diagonally through the park to walk to the meeting point. She knows that the street to cross from her house to the park is 4 meters wide.
- Tyler used to walk Crossover Path to get to the park and he knows that this path is 125 meters long. However, Crossover Path is closed for construction, so he must take another path to get to the park.
- Austin determines that his house is 70 meters away from Main Street. He also knows that the straight line distance from his house to the park entrance is 98 meters. However, he cannot walk that path because there are houses and fences in his way.
- Round your calculations to the tenths place as necessary.



Was Austin's claim correct? Does he have to walk the farthest to get to the meeting point at the park? If not, who walks the farthest? Provide evidence and explain your reasoning to either support or dispute Austin's claim.

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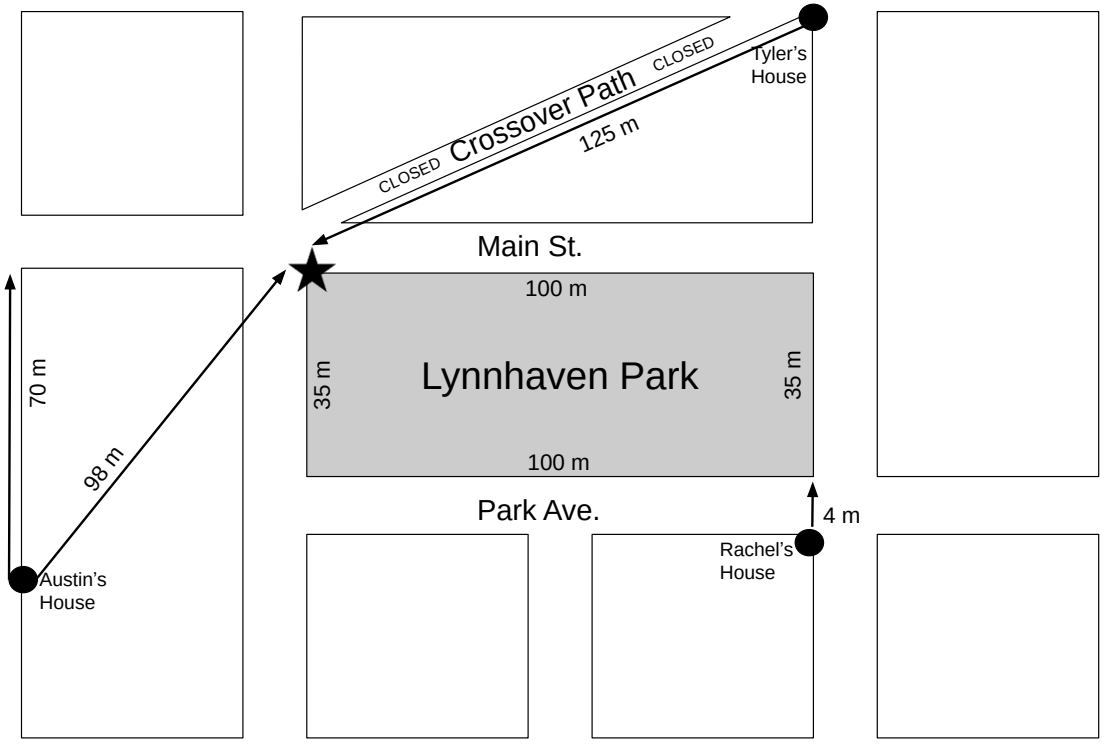
	Advanced	Proficient	Developing	Emerging
Mathematical Understanding	Proficient Plus: • Uses relationships among mathematical concepts	• Demonstrates an understanding of concepts and skills associated with task • Applies mathematical concepts and skills which lead to a valid and correct solution	• Demonstrates a partial understanding of concepts and skills associated with task • Applies mathematical concepts and skills which lead to an incomplete or incorrect solution	• Demonstrates little or no understanding of concepts and skills associated with task • Applies limited mathematical concepts and skills in an attempt to find a solution or provides no solution
Problem Solving	Proficient Plus: • Problem solving strategy is efficient	• Problem solving strategy displays an understanding of the underlying mathematical concept • Produces a solution relevant to the problem and confirms the reasonableness of the solution	• Chooses a problem solving strategy that does not display an understanding of the underlying mathematical concept • Produces a solution relevant to the problem but does not confirm the reasonableness of the solution	• A problem solving strategy is not evident or is not complete • Does not produce a solution that is relevant to the problem
Communication and Reasoning	Proficient Plus: • Reasoning is organized and coherent • Consistent use of precise mathematical language and accurate use of symbolic notation	• Communicates thinking process • Demonstrates reasoning and/or justifies solution steps • Supports arguments and claims with evidence • Uses mathematical language to express ideas with precision	• Reasoning or justification of solution steps is limited or contains misconceptions • Provides limited or inconsistent evidence to support arguments and claims • Uses limited mathematical language to partially communicate thinking with some imprecision	• Provides little to no correct reasoning or justification • Does not provide evidence to support arguments and claims • Uses little or no mathematical language to communicate thinking
Representations and Connections	Proficient Plus: • Uses representations to analyze relationships and extend thinking Uses mathematical connections to extend the solution to other mathematics or to deepen understanding	• Uses a representation or multiple representations, with accurate labels, to explore and model the problem • Makes a mathematical connection that is relevant to the context of the problem	• Uses an incomplete or limited representation to model the problem • Makes a partial mathematical connection or the connection is not relevant to the context of the problem	• Uses no representation or uses a representation that does not model the problem • Makes no mathematical connections

Possible Graphic Organizers

For students who are having a hard time organizing the information in the problem, provide them with this blank table to help them:

	Length of a Leg	Length of a Leg	Hypotenuse
Rachel			
Tyler			
Austin			

For students who are struggling to label the map with the distances provided in the problem, provide them with this pre-labeled map:



For students who have visual impairment or might benefit from a larger visual: