Rich Mathematical Task – Grade *5– Trail Mix Time*

Task Overview/Description/Purpose:

- In this task, students will explore using a model to determine how to alter the amount of ingredients in a recipe based on serving size.
- The purpose of the task is for students to develop strategies for solving a practical problem in order to develop an understanding of multiplying whole numbers and fractions.

Standards Alignment: Strand – *Number and Number Sense*

Primary SOL: 5.6 The student will

a) solve single-step and multistep practical problems involving addition and subtraction with fractions and mixed numbers
b) solve single-step practical problems involving multiplication of a whole number, limited to 12 or less, and a proper fraction with models.*

Related SOL: 4.3d*, 4.5, 4.15, 5.2a*, 5.18, 6.5

*On the state assessment, items measuring this objective are assessed without the use of a calculator.

Learning Intention(s):

- **Content** - I am learning to choose from a range of strategies to solve practical problems involving multiplication of whole numbers and fractions.
- **Language** - I am learning to explain my model and my thinking clearly using the language of fractions.
- **Social** - I am learning to appreciate the contributions of each learner and the connections among others' reasoning and my own.

Success Criteria (Evidence of Student Learning):

- I can choose an efficient and effective strategy to solve a practical problem.
- I can use a model to help me multiply a whole number and a fraction.
- I can justify my thinking using fraction language, models, and representations.
- I can make connections between different problem-solving strategies and models for multiplication.

Mathematics Process Goals:

Problem Solving	• Students will apply their understanding of multiplying whole numbers and fractions to determine the amount of each ingredient needed for the recipe. • Students will choose an appropriate problem solving strategy and confirm the reasonableness of their solution.
Communication and Reasoning	• Students will communicate their thinking process of determining the amount of each ingredient needed and explain the connection to the model used. • Students will justify the strategy they used to solve the problem and explain solution steps. • Students will use appropriate mathematical language, including fraction vocabulary, to express ideas with accuracy and precision.
Connections and Representations	• Students will create and use a representation to explore and model the problem, as well as support the steps to a solution. • Students will describe mathematical connections between the reasoning behind determining the amount of ingredients and multiplying whole numbers and fractions.

Rich Mathematical Task – Grade *3* – *Trail Mix Time*

Task Pre-Planning

Approximate Length/Time Frame: 60 minutes

Grouping of Students: Students will begin working independently, then will be purposefully partnered based on teacher monitoring of strategies.

Materials and Technology:

- copy of task for each student
- pencil, grid paper
- dry erase boards/markers
- chart paper
- fraction circles or fraction strips
- measuring cups
- items to measure liquid volume (rice, beans, etc.)

Vocabulary:

- fraction
- mixed number/improper fraction
- whole number
- halves, fourths, eighths
- double/twice
- triple
- serving

Anticipate Responses: See the Planning for Mathematical Discourse Chart (columns 1-3).

Task Implementation (Before)

Task Launch:

- **Activate prior knowledge:** The teacher will bring students to a whole group setting for a number talk to access their prior knowledge of multiplying whole numbers and make connections to multiplying whole numbers and fractions. This thinking will support students in determining the correct amount of each ingredient needed to increase the recipe to provide enough servings for 8 people instead of 2.
    - Ask students to draw models to represent the following:
      - 3 groups of 3
      - 6 groups of 3
      - 9 groups of 3
    - Record on chart paper or dry erase board the models and equation for each, including repeated addition and multiplication, focusing on the words “groups of” in place of the multiplication symbol. Ask students to look for patterns and relationships between each problem (doubling/tripling of groups).
    - Ask students to draw models to represent the following:
      - 3 groups of 1/3
      - 3 groups of 2/3
    - Record on chart paper or dry erase board the models for each, focusing on understanding of 6/3 being equivalent to 3 and representations for both. Record equation for each problem, including repeated addition and multiplication.
- **Ensure understanding of task:** The teacher will share Learning Intentions and Success Criteria with the class. The teacher will give students the task and have a student read the task aloud. The teacher will ask questions to make sure task is understood: “What are we trying to figure out?” “What do you already know that can help you get started?” Give students the opportunity to “Turn and Talk” with a partner about the problem that needs to be solved.
- **Establish clear expectations:** Review rubric with students as a tool for monitoring their proficiency. Review classroom expectations for working independently and in groups. Support materials and manipulatives should be accessible for student use.

Task Implementation (During)

Directions for Supporting Implementation of the Task

- Monitor – Teacher will listen and observe students as they work on task and ask assessing or advancing questions (see the Planning for Mathematical Discourse chart on next page).
- Select – Teacher will decide which strategies will be highlighted (after student task implementation) that will advance mathematical ideas and support student learning.
- Sequence – Teacher will decide the order in which student ideas will be highlighted (after student task implementation).
- Connect – Teacher will consider ways to facilitate connections between different student responses.
    - Students work in purposefully planned groups for 20-25 minutes to explore strategies, share ideas and transfer their ideas to paper using pictures, words, and symbols.
        - As the teacher is monitoring, teacher will look for strategies used by students and record on the Mathematical Discourse Planning Chart.
        - The teacher should use questions to assess or advance student thinking.
- Students should be encouraged to explore different strategies for solving and evaluate effectiveness.

Suggestions For Additional Student Support:

- Sentence frames to support student thinking and discourse:
    - I agree/disagree with _____'s strategy because _____.
    - The strategy I used to solve is _____.
    - I know that _____ cup(s) of __(ingredient)__ is needed because _____.
    - If this recipe made ____ serving(s), _____ cup(s) of __(ingredient)__ would be needed because _____.
- Input/Output table or chart to organize amounts of ingredients for various serving sizes (see attached)
- Variety of manipulatives available for students to choose to use:
    - Fraction strips, squares, or circles
    - Measuring cups
    - Items to measure liquid volume (i.e., rice, beans, etc.)

Task Implementation (After) *20 minutes*

Connecting Student Responses (From Anticipating Student Response Chart) and Closure of the Task:

- Based on the actual student responses, sequence and select particular students to present their mathematical work during a whole class discussion. Some possible big mathematical ideas to highlight could include:
    - A common misconception
    - Trajectory of sophistication in student ideas (i.e. concrete to abstract; learning trajectories for multiplication or division of decimals)
    - Connection between multiplication and division (could both operations provide the same outcome?)
- Connect different students' responses and connect the responses to the key mathematical ideas to bring closure to the task. Possible questions and sentence frames to connect student strategies:
    - How are these strategies alike? How are they different?
    - _____'s strategy is similar to _____'s strategy because _____
    - How do these connect to our Learning Intentions?
    - Why is this important?
- Highlight student strategies to show connections between different ideas for solutions, or to show the connection between levels of sophistication of student ideas (connect strategy of repeated addition to strategy of multiplication – similarities/differences). Allow students to ask clarifying questions.

Task Implementation (After) *20 minutes*

- Consider ways to ensure that each student will have an equitable opportunity to share his/her thinking during task discussion.
    - Students can participate in a Gallery Walk to view all strategies prior to coming together to discuss selected strategies.
    - Students can "Think, Pair, Share" strategies for solving.
- Close the lesson by revisiting the success criteria. Have students reflect on their progress toward the criteria.

Teacher Reflection About Student Learning:

- Student understanding of the content through the use of the process goals will be assessed using:
    - Task Rubric
    - Self-Assessment through I Can Statements
- Evidence provided through student work that will inform further instruction may include:
    - Teacher uses the chart with anticipated student solutions to monitor which students are using which strategies. Includes: possible misconceptions, learning trajectories and sophistication of student ideas, and multiple solution pathways. Next steps based on this information could include:
        - Informing sequence of tasks. What will come next in instruction to further student thinking in problem solving and fractional computation?
        - Informing small groups based on misconceptions not addressed in sharing.
        - Informing small groups based on movement along the learning trajectory/growing in sophistication of ideas (i.e. concrete to abstract)
- After task implementation, the teacher will use the Process Goals Rubric to assess students' strengths and areas for growth. This could be a focus on one category. Next steps based on this information could include:
    - Informing sequence of tasks. What will come next in instruction to further student engagement in the process goal(s)?
    - Informing small groups based on where students are in engagement in the process goal(s) (i.e. sentence frames for communication, graphic organizers, strategic grouping, manipulative support, etc.)

Planning for Mathematical Discourse

Mathematical Task: Trail Mix Time

Content Standard(s): SOL 5.6ab

Teacher Completes Prior to Task Implementation			**Teacher Completes During Task Implementation**	
Anticipated Student Response/Strategy *Provide examples of possible correct student responses along with examples of student errors/misconceptions*	**Assessing Questions** *Teacher questioning that allows student to explain and clarify thinking*	**Advancing Questions** *Teacher questioning that moves thinking forward*	**List of Students Providing Response** *Who? Which students used this strategy?*	**Discussion Order - sequencing student responses** • *Based on the actual student responses, sequence and select particular students to present their mathematical work during class discussion* • *Connect different students' responses and connect the responses to the key mathematical ideas* • *Consider ways to ensure that each student will have an equitable opportunity to share his/her thinking during task discussion*
Anticipated Student Response: *Common Misconception* Student multiplies all fractions by 8 instead of 4.	• Tell me about your thinking. • Why did you multiply by 8? • How many servings does this recipe make? • How many servings does he need? • Can you draw and label a model to represent your thinking?	• What if this recipe only made one serving? • How much of each ingredient would be needed for two servings? Four servings?		
Anticipated Student Response: Student uses repeated addition to solve problem correctly.	• Tell me about your thinking. • Do you see a pattern? • How did you know you could use addition to solve this problem? • Explain what is happening when you add these fractions together.	• Could you use another operation and get the same result? • Is there a faster or more efficient method you could use to solve? • Is there another method you could use that means the same as adding		

Teacher Completes Prior to Task Implementation			**Teacher Completes During Task Implementation**	
Anticipated Student Response/Strategy *Provide examples of possible correct student responses along with examples of student errors/misconceptions*	**Assessing Questions** *Teacher questioning that allows student to explain and clarify thinking*	**Advancing Questions** *Teacher questioning that moves thinking forward*	**List of Students Providing Response** *Who? Which students used this strategy?*	**Discussion Order - sequencing student responses** • *Based on the actual student responses, sequence and select particular students to present their mathematical work during class discussion* • *Connect different students' responses and connect the responses to the key mathematical ideas* • *Consider ways to ensure that each student will have an equitable opportunity to share his/her thinking during task discussion*
		multiple groups of the same amount?		
Anticipated Student Response: Student attempts to use repeated addition to solve the problem, but adds incorrectly.	• Explain your thinking. • Can you draw a model to show how you added the fractions together?	• Draw a picture to represent adding ½ + ½. Draw a picture to represent adding ¾ + ¾. What do you notice about adding both sets of these fractions? What do you wonder?		
Anticipated Student Response: Student doubles the recipe, doubles again, then doubles again-multiplying by 2 each time (2x2x2)	• Tell me about your thinking. • What does it mean to double? • Why did you double the amounts three times?	• Does that method always work? Why? • Is there a more efficient strategy you could use that would give you the same result?		
Anticipated Student Response: Student attempts to multiply each amount by 4 to solve the problem, but multiplies incorrectly.	• Tell me about your thinking. • How did you know you could use multiplication to solve this problem? • Draw a model to show how you multiplied 4 x ___	• What does it mean to have four groups of ½? What about four groups of ¾? • Look at this model of 4 x ___. What does it show?		

Teacher Completes Prior to Task Implementation			**Teacher Completes During Task Implementation**	
Anticipated Student Response/Strategy *Provide examples of possible correct student responses along with examples of student errors/misconceptions*	**Assessing Questions** *Teacher questioning that allows student to explain and clarify thinking*	**Advancing Questions** *Teacher questioning that moves thinking forward*	**List of Students Providing Response** *Who? Which students used this strategy?*	**Discussion Order - sequencing student responses** • *Based on the actual student responses, sequence and select particular students to present their mathematical work during class discussion* • *Connect different students' responses and connect the responses to the key mathematical ideas* • *Consider ways to ensure that each student will have an equitable opportunity to share his/her thinking during task discussion*
		Can you use the model to help you understand how much 4 x ___ = ?		

Name_____ Date_____

Trail Mix

The trail mix recipe below makes enough trail mix for two people.

Trail Mix Recipe

- 1/2 cup almonds
- 1/4 cup chocolate chips
- 4/8 cup raisins
- 3/4 cup granola
- 2/8 cup pumpkin seeds

*Makes 2 Servings

Josiah needs to make enough trail mix for eight people. How much of each ingredient will Josiah need? Explain and justify your thinking using pictures, numbers, and words.

Rich Mathematical Task Rubric

	Advanced	Proficient	Developing	Emerging
Mathematical Understanding	Proficient Plus: • Uses relationships among mathematical concepts or makes mathematical generalizations	• Demonstrates an understanding of concepts and skills associated with task • Applies mathematical concepts and skills which lead to a valid and correct solution	• Demonstrates a partial understanding of concepts and skills associated with task • Applies mathematical concepts and skills which lead to an incomplete or incorrect solution	• Demonstrates no understanding of concepts and skills associated with task • Applies limited mathematical concepts and skills in an attempt to find a solution or provides no solution
Problem Solving	Proficient Plus: • Problem solving strategy is well developed or efficient	• Problem solving strategy displays an understanding of the underlying mathematical concept • Produces a solution relevant to the problem and confirms the reasonableness of the solution	• Problem solving strategy displays a limited understanding of the underlying mathematical concept • Produces a solution relevant to the problem but does not confirm the reasonableness of the solution	• A problem solving strategy is not evident • Does not produce a solution that is relevant to the problem
Communication and Reasoning	Proficient Plus: • Reasoning or justification is comprehensive • Consistently uses precise mathematical language to communicate thinking	• Demonstrates reasoning and/or justifies solution steps • Supports arguments and claims with evidence • Uses mathematical language to communicate thinking	• Reasoning or justification of solution steps is limited or contains misconceptions • Provides limited or inconsistent evidence to support arguments and claims • Uses limited mathematical language to partially communicate thinking	• Provides no correct reasoning or justification • Does not provide evidence to support arguments and claims • Uses no mathematical language to communicate thinking
Representations and Connections	Proficient Plus: • Uses representations to analyze relationships and extend thinking • Uses mathematical connections to extend the	• Uses a representation or multiple representations, with accurate labels, to explore and model the problem	• Uses an incomplete or limited representation to model the problem • Makes a partial mathematical connection or the connection is not relevant to the context of the problem	• Uses no representation or uses a representation that does not model the problem • Makes no mathematical connections

	solution to other mathematics or to deepen understanding	• Makes a mathematical connection that is relevant to the context of the problem		