

Rich Mathematical Task – Grade 4 – Exploring Quadrilaterals

Task Overview/Description/Purpose	
<ul style="list-style-type: none"> The purpose of this task is to deepen student understanding about the relationships between shape properties and quadrilaterals. Properties determine the way quadrilaterals look. Can you create quadrilaterals to fit different properties? In this task students will create quadrilaterals in order to develop their mathematical understanding of the properties of shapes. 	
Standards Alignment: Strand – Computation and Estimation	
<p>Primary SOL: 4.12 The student will classify quadrilaterals as parallelograms, rectangles, squares, rhombi, and/or trapezoids.</p> <ul style="list-style-type: none"> Develop definitions for parallelograms, rectangles, squares, rhombi, and trapezoids. Identify properties of quadrilaterals including parallel, perpendicular, and congruent sides. Identify parallel sides, congruent sides, and right angles using geometric markings to denote properties of quadrilaterals. <p>Related SOL: 3.12b, 3.13, 4.10b, 5.13a</p>	
Learning Intentions	
<ul style="list-style-type: none"> Content – I am learning about the relationship between the properties of shapes and quadrilaterals. Language – I am learning to describe and write about quadrilaterals and their properties. Social – I am learning to listen to my peers’ thinking and explain it in my own words. 	
Success Criteria (Evidence of Student Learning)	
<ul style="list-style-type: none"> I can develop definitions for parallelograms, rectangles, squares, rhombi, and trapezoids. I can identify properties of quadrilaterals. I can compare and contrast the properties of quadrilaterals. I can write about my mathematical thinking using vocabulary of the properties of quadrilaterals. I can listen to and explain my peer’s thinking. 	
Mathematics Process Goals	
Problem Solving	<ul style="list-style-type: none"> Students will choose an appropriate strategy or strategies to create quadrilaterals with specific properties. Students will accurately apply their strategy to obtain at least three valid solutions.
Communication and Reasoning	<ul style="list-style-type: none"> Students will communicate their thinking process of creating quadrilaterals to their peers and their learning community. Students will justify their solution steps in an organized and coherent matter. Students will use appropriate mathematical language, including shape and properties vocabulary, to express ideas with accuracy and precision verbally and in writing.
Connections and Representations	<ul style="list-style-type: none"> Students will show at least three representations to explore and model the problem and their solution steps. Students will describe connections between their quadrilaterals and strategies and those of their peers. Students will make connections to mathematical ideas, such as concept of shape properties, quadrilaterals, and symbolic notation.

Rich Mathematical Task – Grade 4 – Exploring Quadrilaterals

Task Pre-Planning	
Approximate Length/Time Frame: 60 minutes	
Grouping of Students: Students will begin working independently, then will be purposefully partnered based on teacher monitoring of strategies.	
Suggested Materials and Technology: <ul style="list-style-type: none"> Geoboards, Geostix, Exploragons Dot paper Online Geoboard Paper, pencil Copy of task for each student 	Vocabulary: <ul style="list-style-type: none"> parallel, congruent pair quadrilateral square, trapezoid, rhombus, rectangle, parallelogram opposite
Anticipate Responses: See the Planning for Mathematical Discourse Chart (columns 1-3).	

Task Implementation (Before) 10-15 minutes					
Task Launch: <ul style="list-style-type: none"> The teacher will display the Notice/Wonder blank grid for the whole group and then will create the following T-Chart to answer the questions, “What do you notice?” “What do you wonder?” 					
<ul style="list-style-type: none"> Next, students will engage in a notice/wonder group discussion while the teacher facilitates and writes notices and wonders on a T-Chart. Ask: What do you notice? What do you wonder? 					
<p>Example:</p> <table style="margin-left: auto; margin-right: auto; border-collapse: collapse;"> <tr> <td style="border-right: 1px solid black; border-bottom: 1px solid black; padding: 10px; text-align: center;">Notice</td> <td style="border-bottom: 1px solid black; padding: 10px; text-align: center;">Wonder</td> </tr> <tr> <td style="border-right: 1px solid black; height: 100px;"></td> <td style="height: 100px;"></td> </tr> </table>		Notice	Wonder		
Notice	Wonder				
<ul style="list-style-type: none"> Some important ideas to listen for and expand upon are: <ul style="list-style-type: none"> Definitions and graphic representations of pair, parallel, quadrilateral and congruent (it might be helpful to note these on an anchor chart that can be referred to by students throughout the task). The structure of the table and the intersections (i.e. what characteristics will the quadrilateral in this box need to have?) Geometric notation for indicating parallelism and congruency 					

Rich Mathematical Task – Grade 4 – Exploring Quadrilaterals

Task Implementation (Before) 10-15 minutes

- To help students understand the table, do an example with the students by talking through where to place a square. Consider using a Geoboard to model how to make the square and transfer it to the “2/2” location of the table. Ask, “What characteristics of a quadrilateral would go here (point to different location of the table)?” Have students share out the requirements of parallelism and congruency for different locations on the table.
- The teacher will then give the students the directions of the task alongside the “I Can” statements, with the expectation that after independent think time, students will be able to share their mathematical thinking with a partner. Partners will collaborate on their thinking, add to their charts, and explore the “Keep thinking!” part of the task.
- The teacher will ask the questions to make sure the task is understood: “What are we trying to figure out?” “What do you already know that can help you get started?”

Task Implementation (During) 25- 30 minutes

Directions for Supporting Implementation of the Task

- Monitor – Teacher will listen and observe students as they work on task and ask assessing or advancing questions (see chart on next page)
- Select – Teacher will decide which strategies (see examples in the “After” phase) or thinking that will be highlighted (after student task implementation) that will advance mathematical ideas and support student learning
- Sequence – Teacher will decide the order in which student ideas will be highlighted (after student task implementation)
- Connect – Teacher will consider ways to facilitate connections between different student responses
 - Students have independent think time for 10-15 minutes.
 - Students work in purposefully planned partnerships for 15-20 minutes to share strategies, add to their charts, and explore the extension questions.
 - As teacher is monitoring, teacher will look for partnerships that make sense depending on what students are doing independently.
 - Partnerships could be planned to counter misconceptions, move someone along in the sophistication of ideas, or to explore different ways to solve the same problem.

Suggestions For Additional Student Support

- Sentence frames for supporting student-to-student discourse:
 - I noticed _____, so I _____.
 - My partner noticed _____, so they _____.
 - First I _____, then I _____.
- Wide variety of manipulatives available for students to choose to use. Possible choices could be:
 - Geoboards
 - Geostix or Exploragons
 - Dot paper or [Online Geoboard](#)
- ELL students or students who have language disabilities may benefit from accessing prior vocabulary knowledge through pre-teaching and connecting visuals. Students may find a word bank or a display of the 4th grade VDOE Word Wall cards (pages 66-71) providing specific related vocabulary helpful.
- Students who have visual-processing disabilities may benefit from the use of highlighters or colored pencils to emphasize important parts of the task (i.e. the teacher might highlight the vocabulary words)
- Students who have memory disabilities may benefit from prompts to use anchor charts displaying vocabulary information.
- Students who have attention-deficit disabilities may benefit from suggested use of a concrete tool, online Geoboard, etc.

Rich Mathematical Task – Grade 4 – *Exploring Quadrilaterals*

Task Implementation (Before) 10-15 minutes

- Students who have cognitive and metacognitive disabilities may benefit from pre-teaching a modeled think-aloud for solving a different problem and/or generating a list of strategies the student uses to solve similar types of classifying shapes problems.

Task Implementation (After) 15-20 minutes

Connecting Student Responses (From Anticipating Student Response Chart) and Closure of the Task:

- Based on student responses, sequence and select particular students to present their mathematical work during class discussion. Some possible big mathematical ideas to highlight could include:
 - Developing definitions of quadrilaterals
 - A common misconception
 - Vocabulary (applying names of trapezoid, square, rectangle, rhombus, parallelogram, and square to the shapes that they created)
 - Comparing/contrasting properties across quadrilaterals
 - The extension questions
 - Geometric notation
- Connect different students' responses and connect the responses to the key mathematical ideas to bring closure to the task. Possible questions and sentence frames to connect student strategies:
 - How is a trapezoid like a square? How is it different?
 - How could we name these shapes?
 - Why are parallel and congruent sides important to our definitions of quadrilaterals?
 - Can a square also be classified as a rhombus? Explain.
 - How do these connect to our mathematical, social, and language goals?
 - Why is this important?
- Point to/highlight/draw on student strategies to show the specific connections, either between different ideas for solutions or to show the connection between levels of sophistication of student ideas.
- Consider ways to ensure that each student will have an equitable opportunity to share his/her thinking during task discussion.
 - Students can engage in turn-and-talks and all students will be partnered or put in groups to share their thinking.
- Close the lesson by returning to the success criteria. Have students reflect on their progress toward the criteria.

Teacher Reflection About Student Learning:

- Teacher will use the chart with anticipated student solutions to monitor which students are using which strategies. This will include: possible misconceptions, quadrilaterals that students are creating, and multiple solution pathways. Next steps based on this information could include:
 - Informing sequence of tasks. What will come next in instruction to further student thinking in classifying quadrilaterals?
 - Informing small groups based on misconceptions that are not addressed in sharing.
 - Informing small groups based on movement along the learning trajectory/growing in sophistication of ideas (i.e. concrete to abstract, Van Hiele levels)
- After task implementation, the teacher will use the Task Rubric to assess where students are in demonstrating and understanding of the content and their use of the process goals. This could be a focus on one category. Next steps based on this information could include:
 - Informing sequence of tasks. What will come next in instruction to further student engagement in the content and/or process goal(s)?
 - Informing small groups based on where students are in engagement in the process goal(s) (i.e. think alouds, using specific sentence frames for communication, etc.).

Rich Mathematical Task – Grade 4 – Exploring Quadrilaterals

Planning for Mathematical Discourse

Mathematical Task: Exploring Quadrilaterals

Content Standard(s): SOL 4.12

Teacher Completes Prior to Task Implementation			Teacher Completes During Task Implementation	
<p>Anticipated Student Response/Strategy Provide examples of possible correct student responses along with examples of student errors/misconceptions</p>	<p>Assessing Questions – Teacher Stays to Hear Response Teacher questioning that allows student to explain and clarify thinking</p>	<p>Advancing Questions – Teacher Poses Question and Walks Away Teacher questioning that moves thinking forward</p>	<p>List of Students Providing Response Who? Which students used this strategy?</p>	<p>Discussion Order - sequencing student responses</p> <ul style="list-style-type: none"> Based on the actual student responses, sequence and select particular students to present their mathematical work during class discussion Consider ways to ensure that each student will have an equitable opportunity to share his/her thinking during task discussion
<p>Anticipated Student Response: <i>*Common misconception</i> Students include shapes that are not quadrilaterals (particularly in the 0/0 coordinate).</p>	<p>Tell me about your thinking. Can you explain how these shapes are quadrilaterals?</p>	<p>How will you change the shape so that it still fits the properties and is also a quadrilateral?</p>	<p>Student E put a triangle in the 0, 0 space.</p>	
<p>Anticipated Student Response: <i>*Common misconception</i> Students may only look at one characteristic and not the other. Example: For the coordinate that includes 0 congruent pairs and 2 parallel pairs, students might put a rectangle because it fulfills one of the criteria.</p>	<p>Tell me about your thinking. How do you know your shape fits in this box? How does it show _____ property?</p>	<p>How will you change the quadrilateral so that it fits <i>both</i> properties of the box?</p>	<p>Student F describes only one characteristic (parallelism) and uses the vocabulary of “doesn’t touch” to explain.</p>	
<p>Anticipated Student Response: <i>*Common misconception</i> Student lists something as “impossible” that is not impossible.</p>	<p>Tell me about your thinking. How does your representation/ thinking show that your solution is true? Why is this impossible?</p>	<p>How can you use a tool to explore and prove that this kind of quadrilateral is impossible?</p>		
<p>Anticipated Student Response: <i>*Misconception:</i> Student latches on to one shape and puts that shape in more than one place without understanding parallelism and/or congruency.</p>	<p>Tell me about your thinking. What do you know about a _____ and congruent sides? What do you know about a _____ and parallel sides?</p>	<p>What different quadrilateral can you create that will <i>not</i> have these properties? Where will it fit?</p>	<p>Student B uses disconnected language and puts a known shape of a rectangle in more than one place.</p>	

Rich Mathematical Task – Grade 4 – *Exploring Quadrilaterals*

Teacher Completes Prior to Task Implementation			Teacher Completes During Task Implementation	
Anticipated Student Response/Strategy <i>Provide examples of possible correct student responses along with examples of student errors/misconceptions</i>	Assessing Questions – Teacher Stays to Hear Response <i>Teacher questioning that allows student to explain and clarify thinking</i>	Advancing Questions – Teacher Poses Question and Walks Away <i>Teacher questioning that moves thinking forward</i>	List of Students Providing Response <i>Who? Which students used this strategy?</i>	Discussion Order - sequencing student responses <ul style="list-style-type: none"> ● <i>Based on the actual student responses, sequence and select particular students to present their mathematical work during class discussion</i> ● <i>Consider ways to ensure that each student will have an equitable opportunity to share his/her thinking during task discussion</i>
	Why does it make sense to put a _____ here?			
Anticipated Student Response: Student starts with square as a known shape but does not know where to go from there.	Tell me about your thinking. How does your representation/thinking show that your solution is true?	Can you think of a shape that still has two pairs of parallel sides but only one pair of congruent sides? Can you use the Geoboard (or other tool) to explore ideas for other parts of the grid?		
Anticipated Student Response: Students use Geoboards or dot paper to adjust the quadrilaterals and guess and check to fit the properties.	Tell me about your thinking. Can you explain where these quadrilateral properties are in your representation? How does your representation/thinking show that your solution is true?	How will you change the quadrilateral so that it ____? Explore the “keep thinking!” part of the task.	Student A explains “I had to play around with it and it worked” while thinking about other parts of the table.	
Anticipated Student Response: Student finds three correct solutions.	Tell me about your thinking. How does your representation/thinking show that your solutions are true?	Explore the “keep thinking!” part of the task. Tell me more about the patterns that you notice.	Student D finds 3 correct solutions and begins to consider more than one quadrilateral could fit in one part of the table.	

Rich Mathematical Task – Grade 4 – *Exploring Quadrilaterals*

Teacher Completes Prior to Task Implementation			Teacher Completes During Task Implementation	
Anticipated Student Response/Strategy <i>Provide examples of possible correct student responses along with examples of student errors/misconceptions</i>	Assessing Questions – Teacher Stays to Hear Response <i>Teacher questioning that allows student to explain and clarify thinking</i>	Advancing Questions – Teacher Poses Question and Walks Away <i>Teacher questioning that moves thinking forward</i>	List of Students Providing Response <i>Who? Which students used this strategy?</i>	Discussion Order - sequencing student responses <ul style="list-style-type: none"> ● <i>Based on the actual student responses, sequence and select particular students to present their mathematical work during class discussion</i> ● <i>Consider ways to ensure that each student will have an equitable opportunity to share his/her thinking during task discussion</i>
Anticipated Student Response: Student correctly places quadrilaterals on table but his or her explanations of <i>why</i> are not fully formed.	Tell me about your thinking. How is your writing connected to the quadrilateral you drew/made? Can you show me?	What can you add to your description?	Student G lists partial explanations by forgetting “pairs.”	

The following worksheets and graphic organizers are intended for student use in the classroom.

Name _____

Date _____

Exploring Quadrilaterals

Create a quadrilateral to place in at least 3 different boxes on the attached grid based on the number of pairs of parallel sides and congruent sides. Create drawings that represent your shapes and explain why they fit using mathematical vocabulary. Use any available tools to assist with your drawings.

Shape ____ makes sense because _____

Shape ____ makes sense because _____

Shape ____ makes sense because _____

Keep thinking!

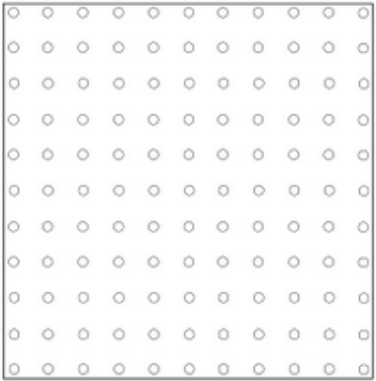
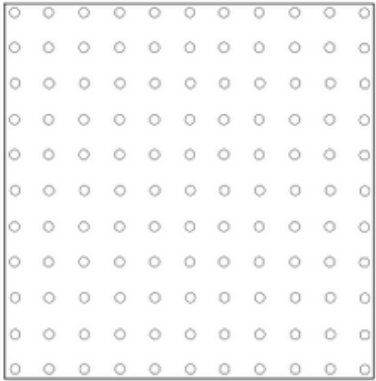
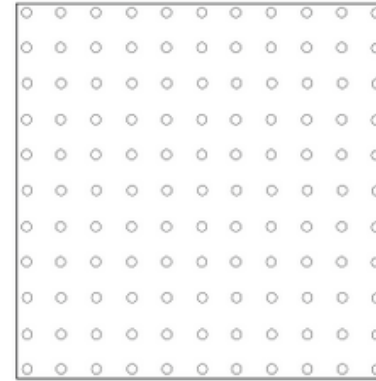
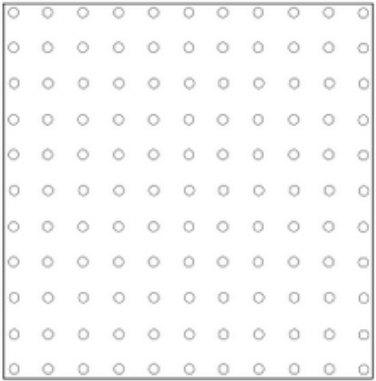
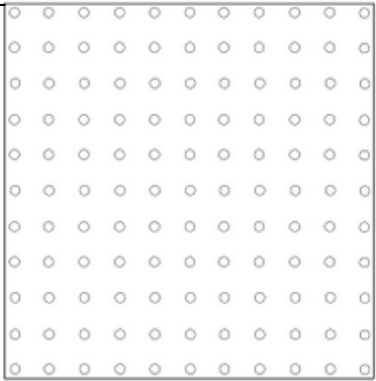

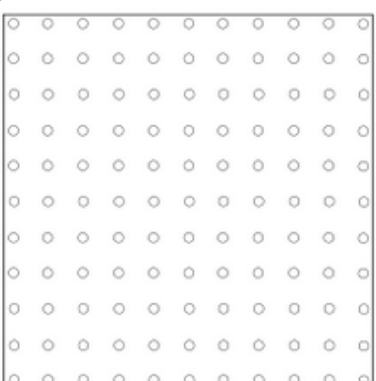
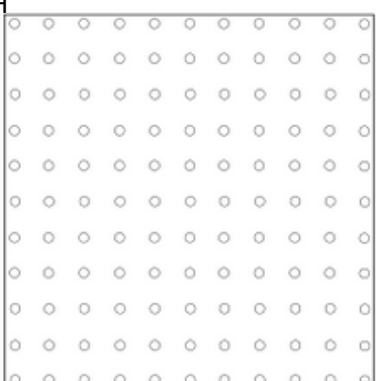
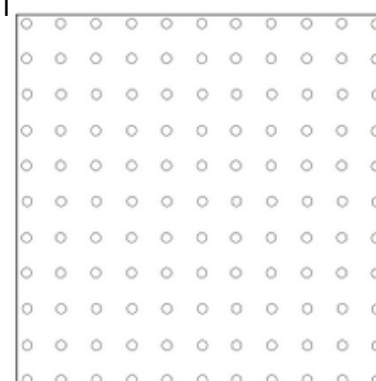
Are there any boxes in the grid that could have more than one quadrilateral?

Explain.

Were there any boxes for which no quadrilateral would fit? Explain.

Rich Mathematical Task – Grade 4 – *Exploring Quadrilaterals*

Pairs of Congruent Sides

		0	1	2
Pairs of Parallel Sides	0	A 	B 	C 
	1	D 	E 	F 
	2	G 	H 	I 

Rich Mathematical Task – Grade 4 – Exploring Quadrilaterals

Rich Mathematical Task Rubric

	Advanced	Proficient	Developing	Emerging
Mathematical Understanding	Proficient Plus: <ul style="list-style-type: none"> • Uses relationships among mathematical concepts 	<ul style="list-style-type: none"> • Demonstrates an understanding of concepts and skills associated with task • Applies mathematical concepts and skills which lead to a valid and correct solution 	<ul style="list-style-type: none"> • Demonstrates a partial understanding of concepts and skills associated with task • Applies mathematical concepts and skills which lead to an incomplete or incorrect solution 	<ul style="list-style-type: none"> • Demonstrates little or no understanding of concepts and skills associated with task • Applies limited mathematical concepts and skills in an attempt to find a solution or provides no solution
Problem Solving	Proficient Plus: <ul style="list-style-type: none"> • Problem solving strategy is efficient 	<ul style="list-style-type: none"> • Problem solving strategy displays an understanding of the underlying mathematical concept • Produces a solution relevant to the problem and confirms the reasonableness of the solution 	<ul style="list-style-type: none"> • Chooses a problem solving strategy that does not display an understanding of the underlying mathematical concept • Produces a solution relevant to the problem but does not confirm the reasonableness of the solution 	<ul style="list-style-type: none"> • A problem solving strategy is not evident or is not complete • Does not produce a solution that is relevant to the problem
Communication and Reasoning	Proficient Plus: <ul style="list-style-type: none"> • Reasoning is organized and coherent • Consistent use of precise mathematical language and accurate use of symbolic notation 	<ul style="list-style-type: none"> • Communicates thinking process • Demonstrates reasoning and/or justifies solution steps • Supports arguments and claims with evidence • Uses mathematical language to express ideas with precision 	<ul style="list-style-type: none"> • Reasoning or justification of solution steps is limited or contains misconceptions • Provides limited or inconsistent evidence to support arguments and claims • Uses limited mathematical language to partially communicate thinking with some imprecision 	<ul style="list-style-type: none"> • Provides little to no correct reasoning or justification • Does not provide evidence to support arguments and claims • Uses little or no mathematical language to communicate thinking
Representations and Connections	Proficient Plus: <ul style="list-style-type: none"> • Uses representations to analyze relationships and extend thinking • Uses mathematical connections to extend the solution to other mathematics or to deepen understanding 	<ul style="list-style-type: none"> • Uses a representation or multiple representations, with accurate labels, to explore and model the problem • Makes a mathematical connection that is relevant to the context of the problem 	<ul style="list-style-type: none"> • Uses an incomplete or limited representation to model the problem • Makes a partial mathematical connection or the connection is not relevant to the context of the problem 	<ul style="list-style-type: none"> • Uses no representation or uses a representation that does not model the problem • Makes no mathematical connections