Geometry
Vocabulary Word Wall Cards

Mathematics vocabulary word wall cards provide a display of mathematics content words and associated visual cues to assist in vocabulary development. The cards should be used as an instructional tool for teachers and then as a reference for all students.

Table of Contents

Reasoning, Lines, and Transformations

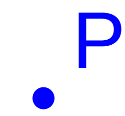
Triangles

Polygons and Circles

Three-Dimensional Figures

Basics of Geometry 1

Point – A point has no dimension. It is a location on a plane. It is represented by a dot.



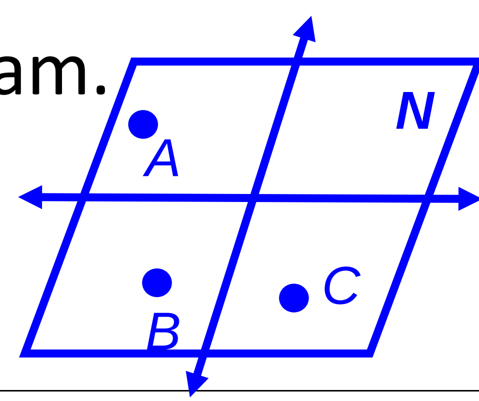
point P

Line – A line has one dimension. It is an infinite set of points represented by a line with two arrowheads that extend without end.

$\overleftrightarrow{AB}$ or $\overleftrightarrow{BA}$ or line *m*

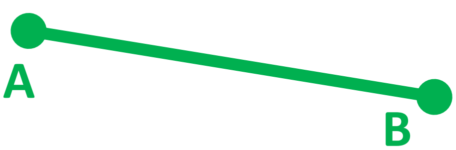
Plane – A plane has two dimensions extending without end. It is often represented by a parallelogram.

plane ABC or plane *N*



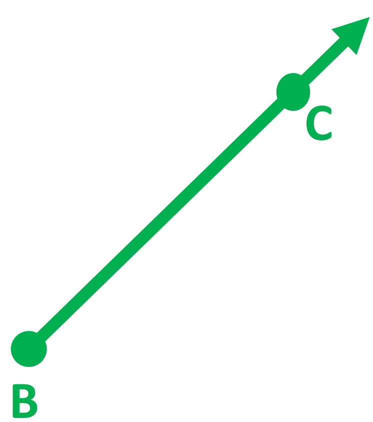
Basics of Geometry 2

Line segment – A line segment consists of two endpoints and all the points between them.



$\overline{AB}$ or $\overline{BA}$

Ray – A ray has one endpoint and extends without end in one direction.



$\overrightarrow{BC}$

Note: Name the endpoint first.
$\overrightarrow{BC}$ and $\overrightarrow{CB}$ are different rays.

Geometry Notation

Symbols used to represent statements or operations in geometry.

$\overline{BC}$	segment BC
$\overrightarrow{BC}$	ray BC
$\overleftrightarrow{BC}$	line BC
BC	length of BC
$\angle ABC$	angle ABC
$m\angle ABC$	measure of angle ABC
ΔABC	triangle ABC
$\parallel$	is parallel to
$\perp$	is perpendicular to
$\cong$	is congruent to
$\sim$	is similar to

Logic Notation

V	or
Λ	and
$\rightarrow$	read “implies”, if… then…
$\leftrightarrow$	read “if and only if”
iff	read “if and only if”
~	not
$\therefore$	therefore

Set Notation

{ }	empty set, null set
∅	empty set, null set
x **|**	read "*x* such that"
x**:**	read "*x* such that"
∪	union, disjunction, or
∩	intersection, conjunction, and

Conditional Statement

a logical argument consisting of
a set of premises,
hypothesis (p), and conclusion (q)

hypothesis

If an angle is a right angle,
then its measure is 90°.

conclusion

Symbolically:

if p, then q $\qquad$ $p \rightarrow q$

Converse

formed by interchanging the hypothesis and conclusion of a conditional statement

Conditional: If an angle is a right angle, then its measure is 90°.

Converse: If an angle measures 90°, then the angle is a right angle.

Symbolically:

if q, then p $\quad\quad$ $q \to p$

Inverse

formed by negating the hypothesis and conclusion of a conditional statement

Conditional: If an angle is a right angle, then its measure is 90°.

Inverse: If an angle is not a right angle, then its measure is not 90°.

Symbolically:

if ~p, then ~q   ~p→~q

Contrapositive

formed by interchanging and negating the hypothesis and conclusion of a conditional statement

Conditional: If an angle is a right angle, then its measure is 90°.

Contrapositive: If an angle does not measure 90°, then the angle is not a right angle.

Symbolically:

if ~q, then ~p $\sim q \rightarrow \sim p$

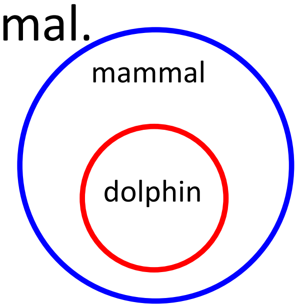
Symbolic Representations in Logical Arguments

Conditional	if p, then q	$p \rightarrow q$
Converse	if q, then p	$q \rightarrow p$
Inverse	if not p, then not q	$\sim p \rightarrow \sim q$
Contrapositive	if not q, then not p	$\sim q \rightarrow \sim p$

Conditional Statements and Venn Diagrams

Original Conditional Statement

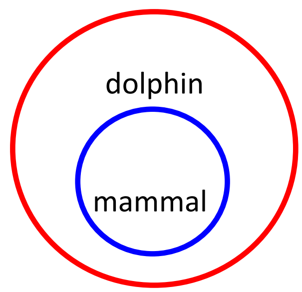
If an animal is a dolphin, then it is a mammal.



True!

Converse - Reversing the Clauses

If an animal is a mammal, then it is a dolphin.

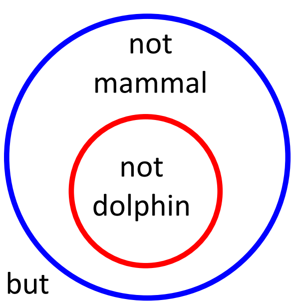


False!

(Counterexample: An elephant is a mammal but is not a dolphin)

Inverse - Negating the Clauses

If an animal is not a dolphin, then it is not a mammal.

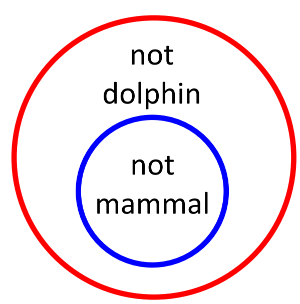


False!

(Counterexample: A whale is not a dolphin but is still a mammal)

Contrapositive - Reversing and Negating the Clauses

If an animal is not a mammal, then it is not a dolphin.



True!

Deductive Reasoning

method using logic to draw conclusions based upon definitions, postulates, and theorems

Example of Deductive Reasoning:

Statement A: If a quadrilateral contains only right angles, then it is a rectangle.

Statement B: Quadrilateral *P* contains only right angles.

Conclusion: Quadrilateral *P* is a rectangle.

Inductive Reasoning

method of drawing conclusions from a limited set of observations

Example:

Given a pattern, determine the next figure (set of dots) using inductive reasoning.

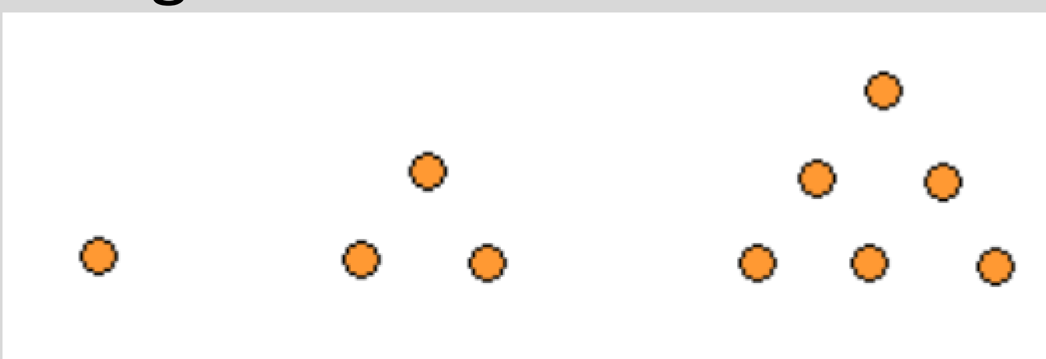


Figure 1 Figure 2 Figure 3

The next figure should look like this:

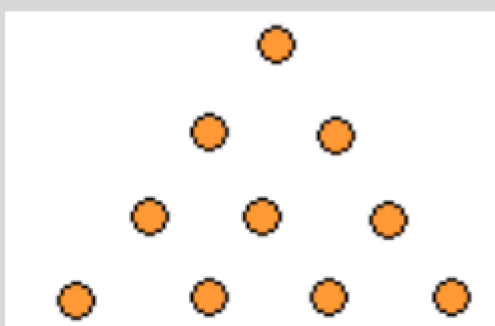


Figure 4

Direct Proofs

a justification logically valid and based on initial assumptions, definitions, postulates, and theorems

Example: (two-column proof)

Given: $\angle 1 \cong \angle 2$

Prove: $\angle 2 \cong \angle 1$

Statements	Reasons
$\angle 1 \cong \angle 2$	Given
$m\angle 1 = m\angle 2$	Definition of congruent angles
$m\angle 2 = m\angle 1$	Symmetric Property of Equality
$\angle 2 \cong \angle 1$	Definition of congruent angles

Example: (paragraph proof)

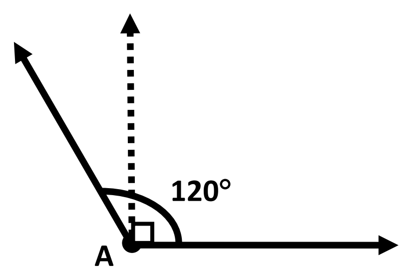
It is given that $\angle 1 \cong \angle 2$. By the Definition of congruent angles, $m\angle 1 = m\angle 2$. By the Symmetric Property of Equality, $m\angle 2 = m\angle 1$. By the Definition of congruent angles, $\angle 2 \cong \angle 1$.

Properties of Congruence

Reflexive Property	$\overline{AB} \cong \overline{AB}$
	$\angle A \cong \angle A$
Symmetric Property	If $\overline{AB} \cong \overline{CD}$, then $\overline{CD} \cong \overline{AB}$.
	If $\angle A \cong \angle B$, then $\angle B \cong \angle A$
Transitive Property	If $\overline{AB} \cong \overline{CD}$ and $\overline{CD} \cong \overline{EF}$, then $\overline{AB} \cong \overline{EF}$.
	If $\angle A \cong \angle B$ *and* $\angle B \cong \angle C$, then $\angle A \cong \angle C$.

Law of Detachment

<u>deductive reasoning</u> stating that if the hypothesis of a true conditional statement is true then the conclusion is also true



Example:

If m∠A > 90°, then ∠A is an obtuse angle

m∠A = 120°

Therefore, ∠A is an obtuse angle.

If $p \rightarrow q$ is a true conditional statement and p is true, then q is true.

Law of Syllogism

deductive reasoning that draws a new conclusion from two conditional statements when the conclusion of one is the hypothesis of the other

Example:

1. If a rectangle has four congruent sides, then it is a square.
2. If a polygon is a square, then it is a regular polygon.
3. If a rectangle has four congruent sides, then it is a regular polygon.

If $p \rightarrow q$ and $q \rightarrow r$ are true conditional statements, then $p \rightarrow r$ is true.

Counterexample

specific case for which a conjecture is false

Example:

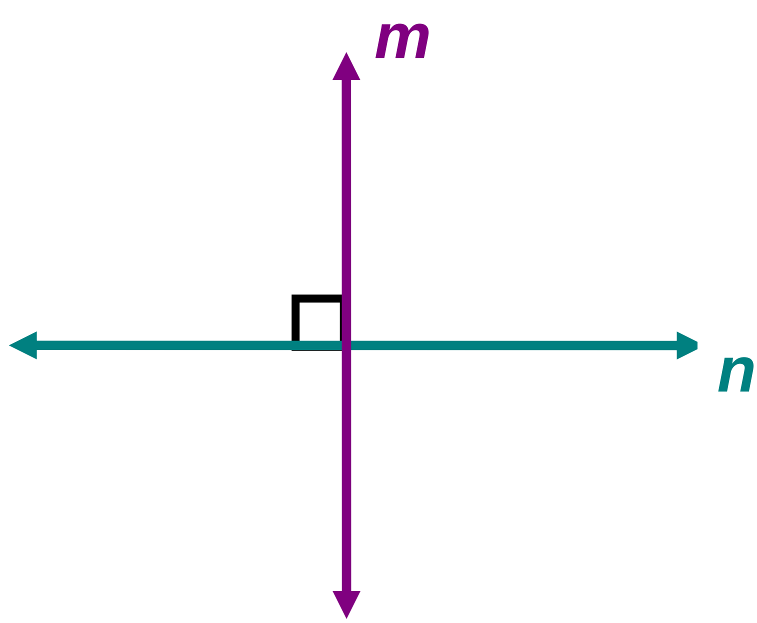
Conjecture: “The product of any two numbers is odd.”

Counterexample: **$2 \cdot 3 = 6$**

<u>One</u> counterexample proves a conjecture false.

Perpendicular Lines

two lines that intersect to form a right angle



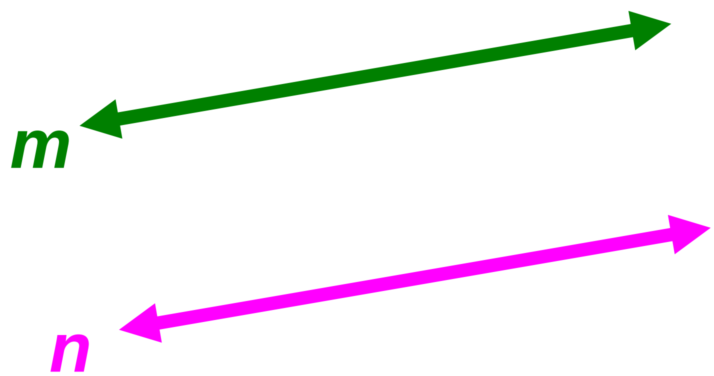
Line *m* is perpendicular to line *n.*

$m \perp n$

Perpendicular lines have slopes that are <u>negative</u> <u>reciprocals</u>.

Parallel Lines

coplanar lines that do not intersect



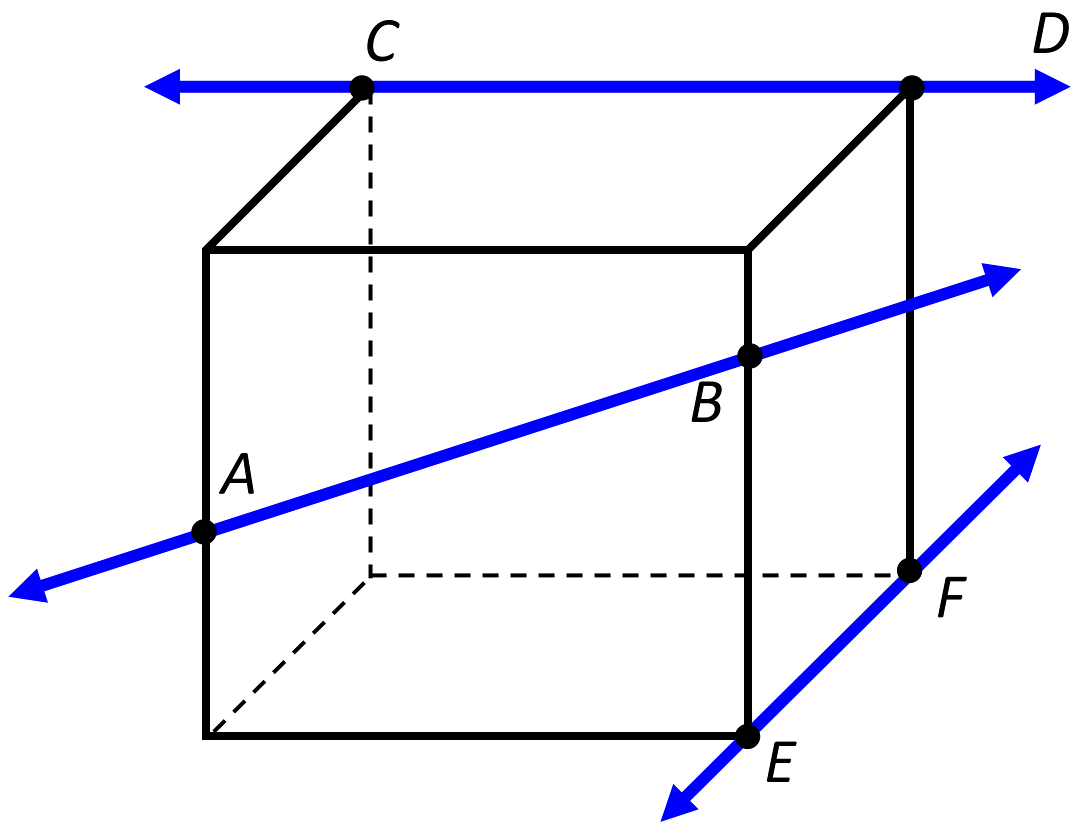
m || *n*

Line *m* is parallel to line *n.*

Parallel lines have the <u>same</u> <u>slope</u>.

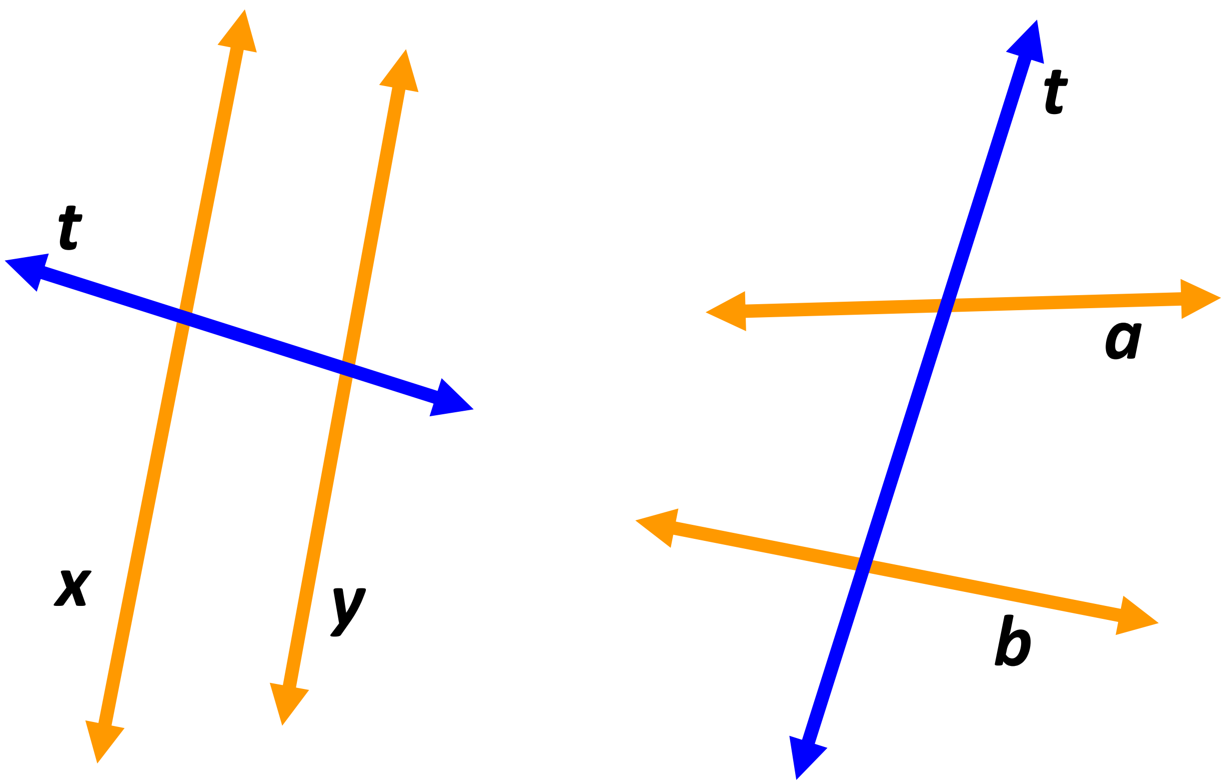
Skew Lines

lines that do not intersect and are not coplanar



Transversal

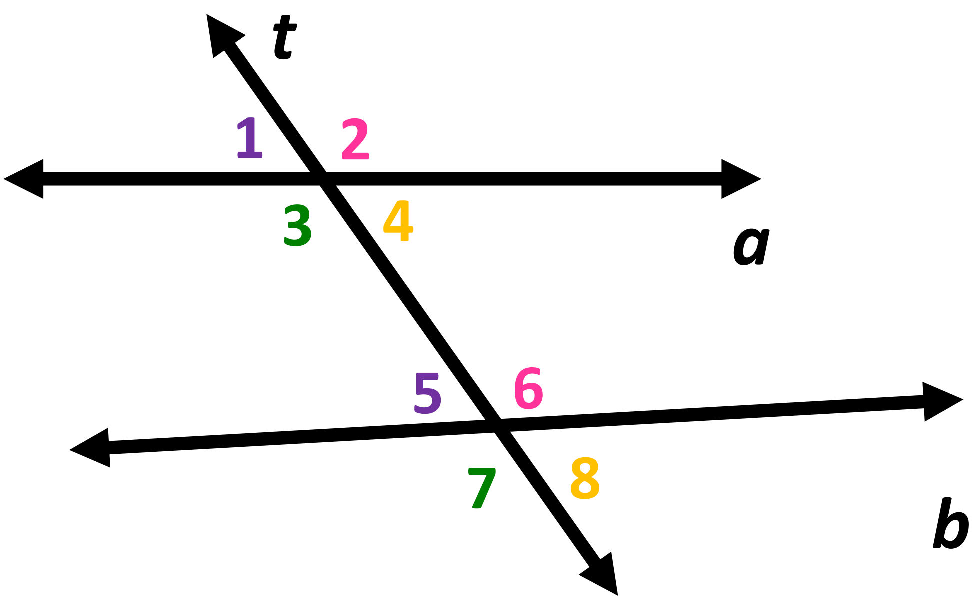
a line that intersects at least two other lines



Line *t* is a transversal.

Corresponding Angles

angles in matching positions when a transversal crosses at least two lines

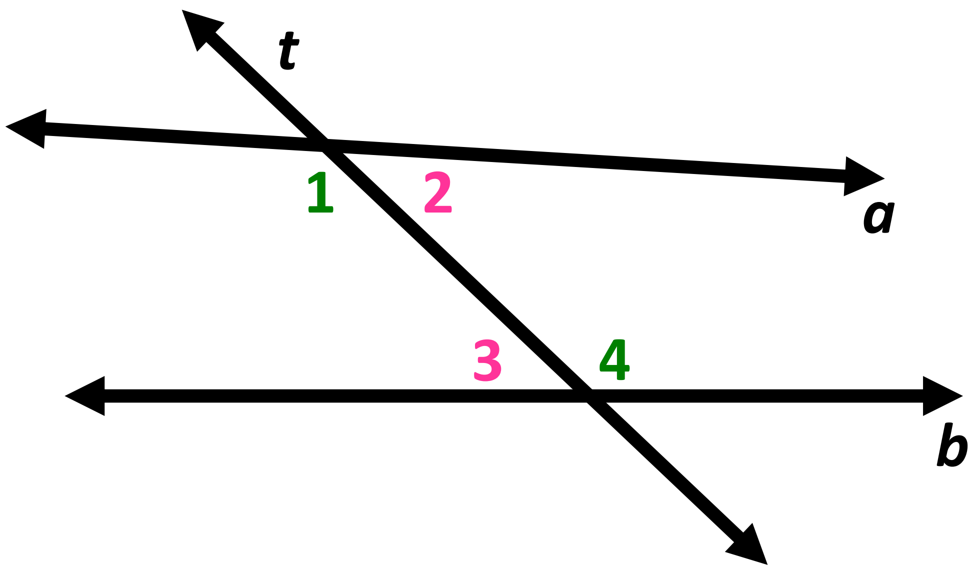


Examples:

1) ∠2 and ∠6
2) ∠3 and ∠7
3) ∠1 and ∠5
4) ∠4 and ∠8

Alternate Interior Angles

angles inside the lines and on opposite sides of the transversal

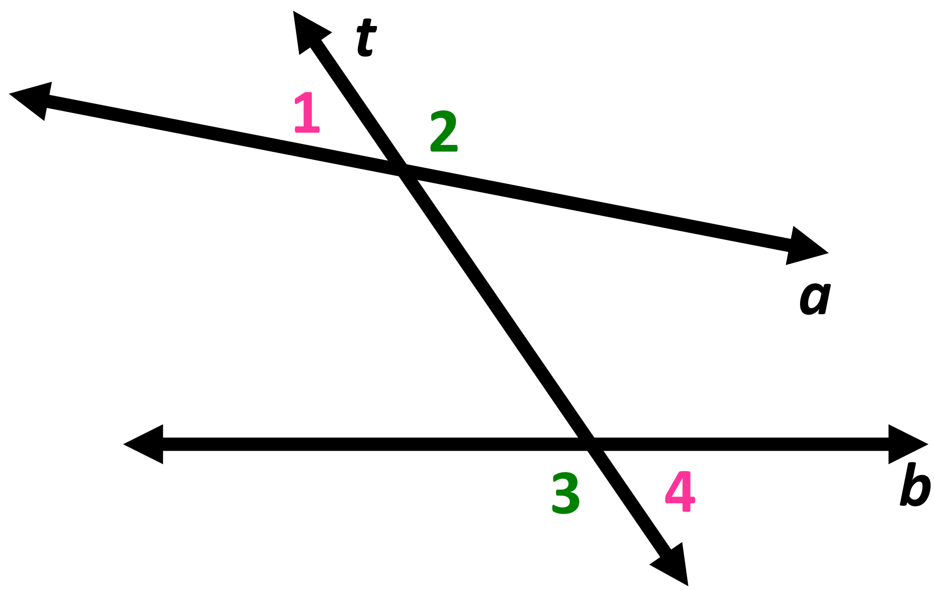


Examples:

1) $\angle 1$ and $\angle 4$
2) $\angle 2$ and $\angle 3$

Alternate Exterior Angles

angles outside the two lines and on opposite sides of the transversal

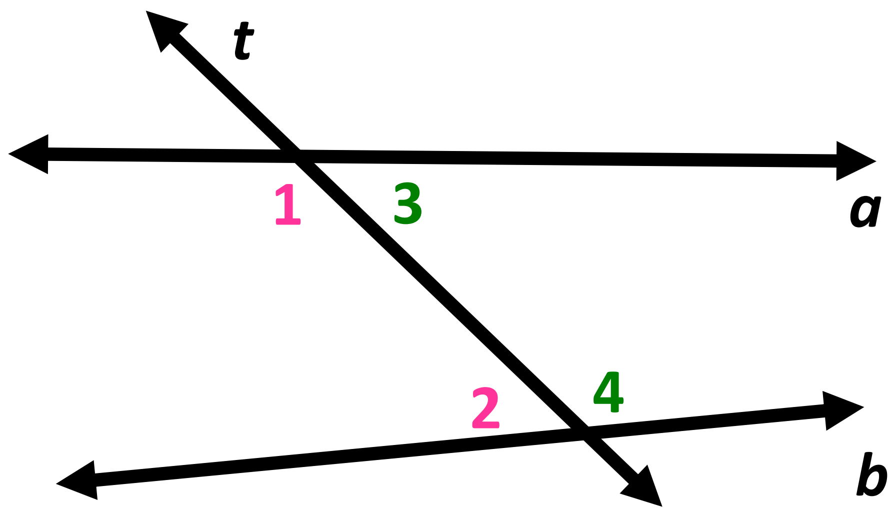


Examples:

1) $\angle 1$ and $\angle 4$
2) $\angle 2$ and $\angle 3$

Consecutive Interior Angles

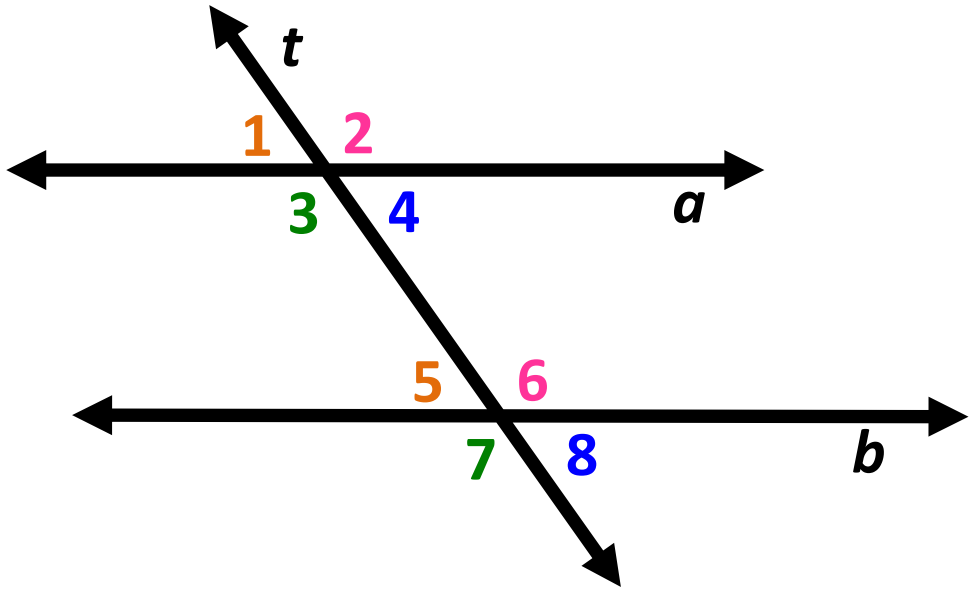
angles between the two lines and on the same side of the transversal



Examples:

1) ∠1 and ∠2
2) ∠3 and ∠4

Parallel Lines



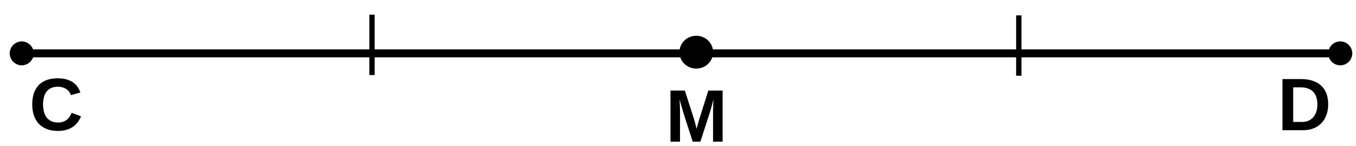
Line *a* is parallel to line *b* when

Corresponding angles are congruent	$\angle 1 \cong \angle 5$, $\angle 2 \cong \angle 6$, $\angle 3 \cong \angle 7$, $\angle 4 \cong \angle 8$
Alternate interior angles are congruent	$\angle 3 \cong \angle 6$ $\angle 4 \cong \angle 5$
Alternate exterior angles are congruent	$\angle 1 \cong \angle 8$ $\angle 2 \cong \angle 7$
Consecutive interior angles are supplementary	$m\angle 3 + m\angle 5 = 180°$ $m\angle 4 + m\angle 6 = 180°$

Midpoint

(Definition)

divides a segment into two congruent segments



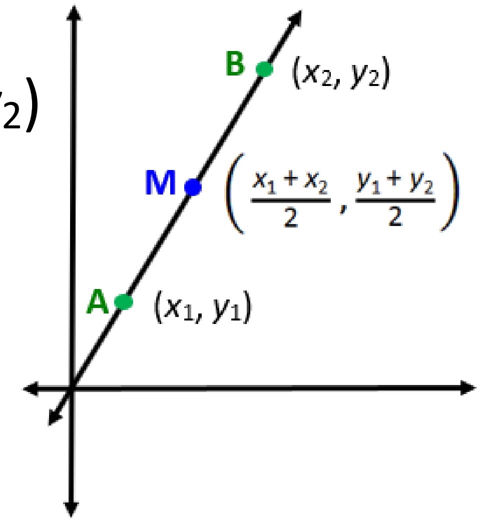
Example: M is the midpoint of $\overline{CD}$

$\overline{CM} \cong \overline{MD}$

$CM = MD$

Segment bisector may be a point, ray, line, line segment, or plane that intersects the segment at its midpoint.

Midpoint Formula



given points A(x_1, y_1) and B(x_2, y_2)

midpoint M = $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

Example:

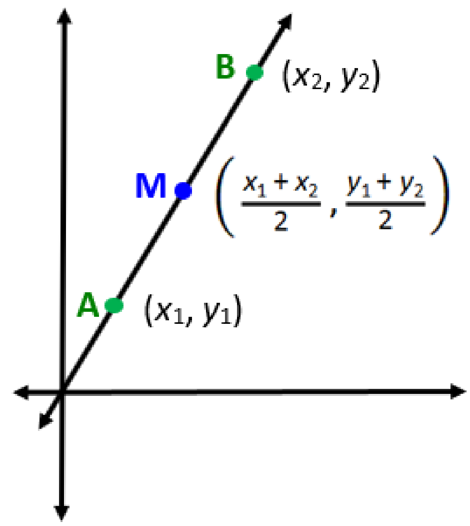
Find the midpoint, M, of the segment with endpoints A(4,1) and B(-2,5).

$$M = \left(\frac{4+-2}{2}, \frac{1+5}{2}\right) = \left(\frac{2}{2}, \frac{6}{2}\right) = (1,3)$$

Find a Missing Endpoint

given points A(x_1, y_1) and B(x_2, y_2)

midpoint M = $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$



Example:
Find the endpoint B(x,y) if A(-2,3) and M(3,8).

$$\left(\frac{-2+x}{2}, \frac{3+y}{2}\right) = (3,8)$$

$$\frac{-2+x}{2} = 3 \text{ and } \frac{3+y}{2} = 8$$

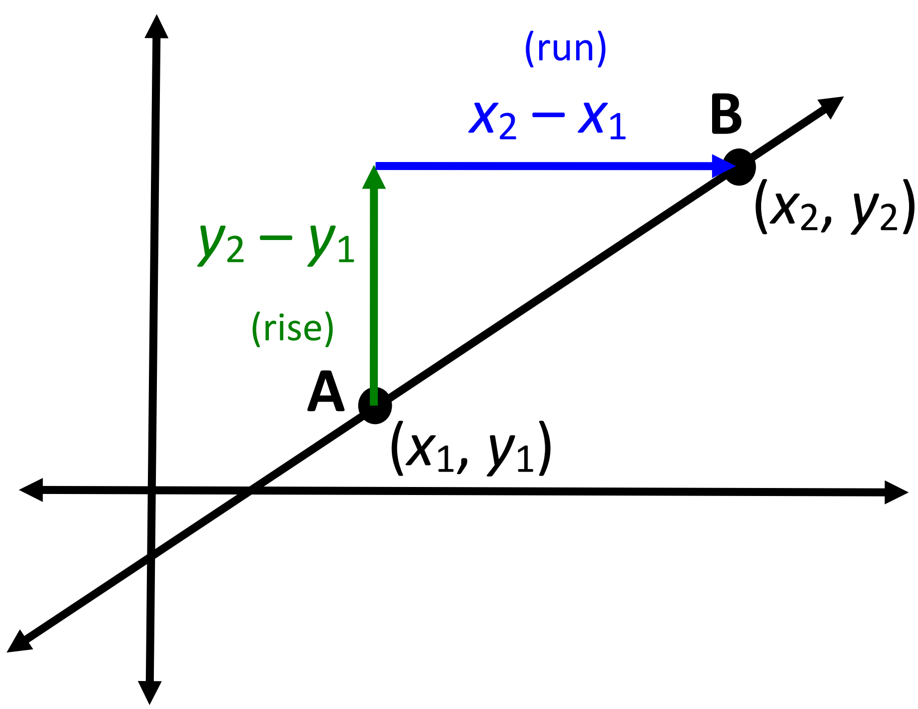
x = 8 and y = 13

B (8,13)

Slope Formula

ratio of vertical change to horizontal change

$$\text{slope} = m = \frac{\text{change in } y}{\text{change in } x} = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1}$$



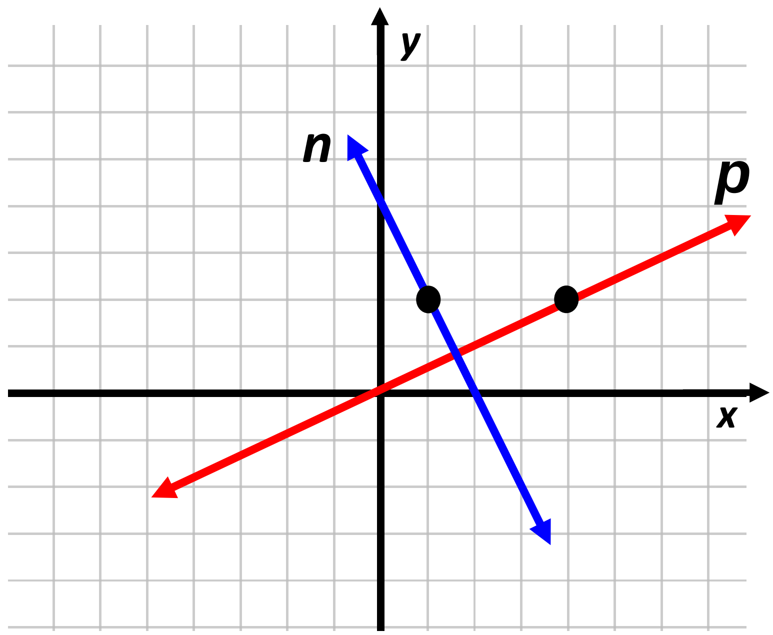
Slopes of Lines in Coordinate Plane

Parallel lines have the same slope.

Perpendicular lines have slopes whose product is -1.

Vertical lines have undefined slope.

Horizontal lines have 0 slope.



Example:

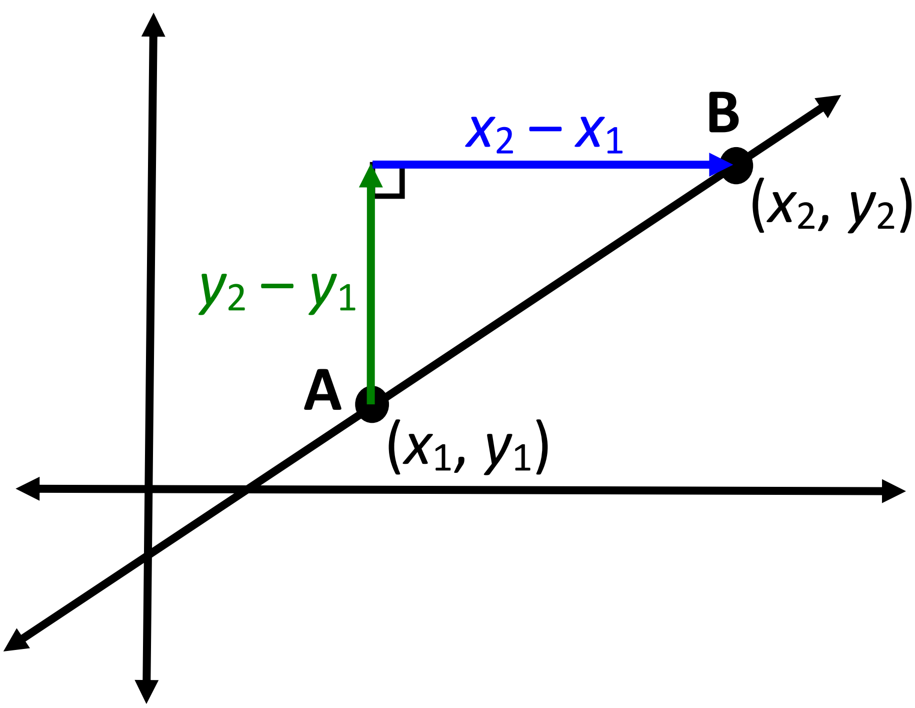
The slope of line n = -2. The slope of line $p = \frac{1}{2}$.

$-2 \cdot \frac{1}{2} = -1$, therefore, $n \perp p$.

Distance Formula

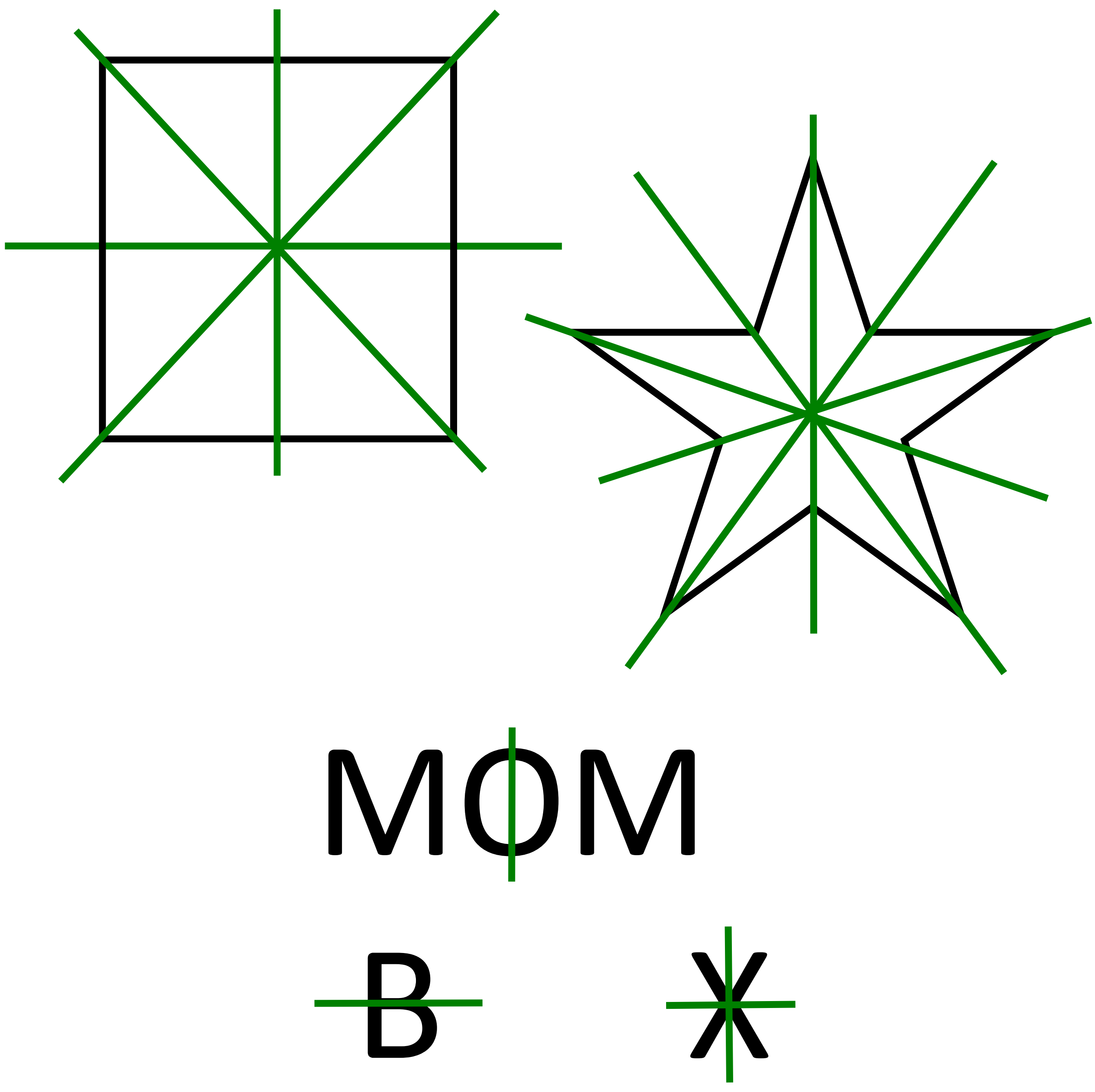
given points A (x_1, y_1) and B (x_2, y_2)

$$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

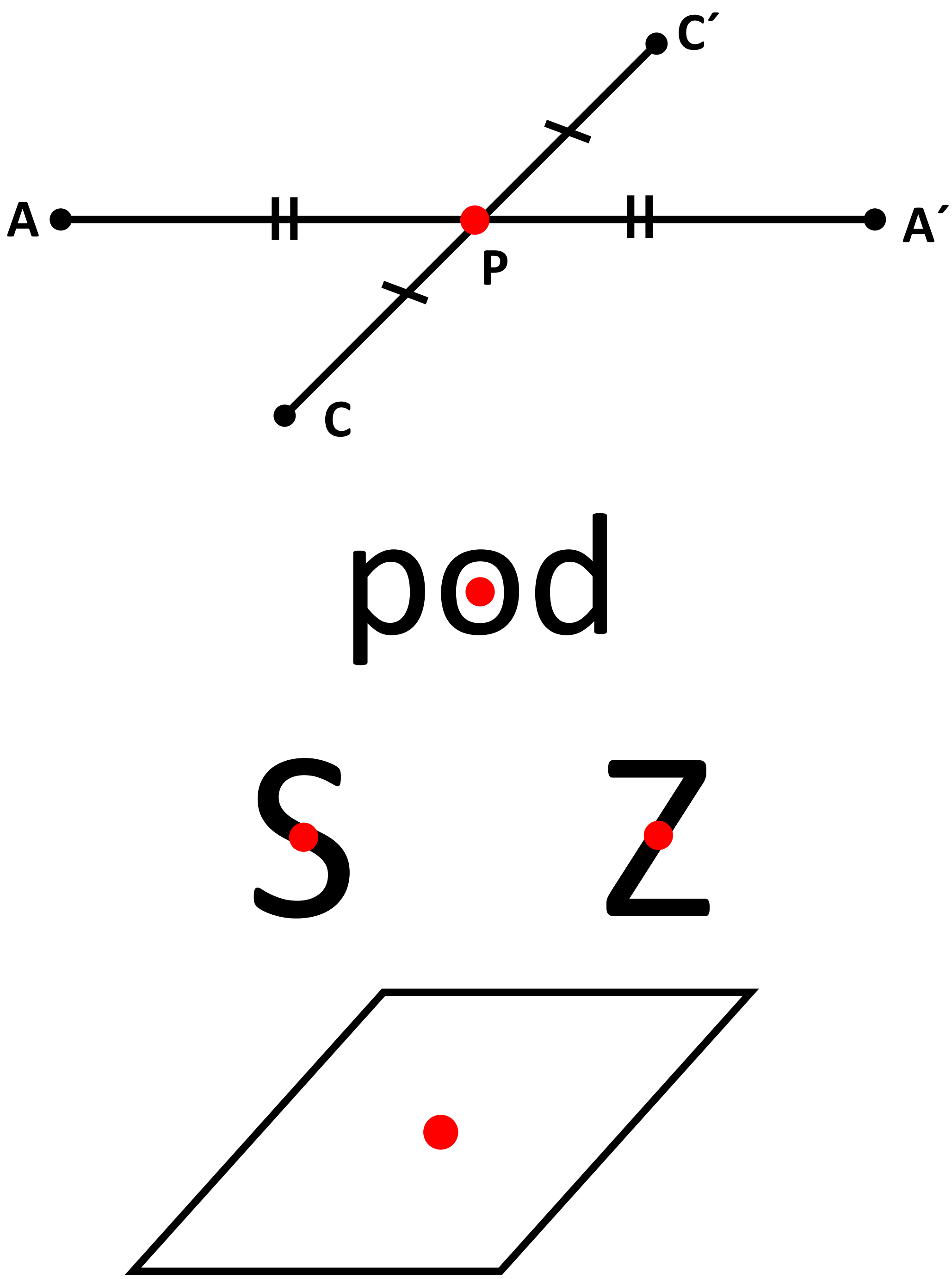


The distance formula is derived from the application of the Pythagorean Theorem.

Examples of Line Symmetry

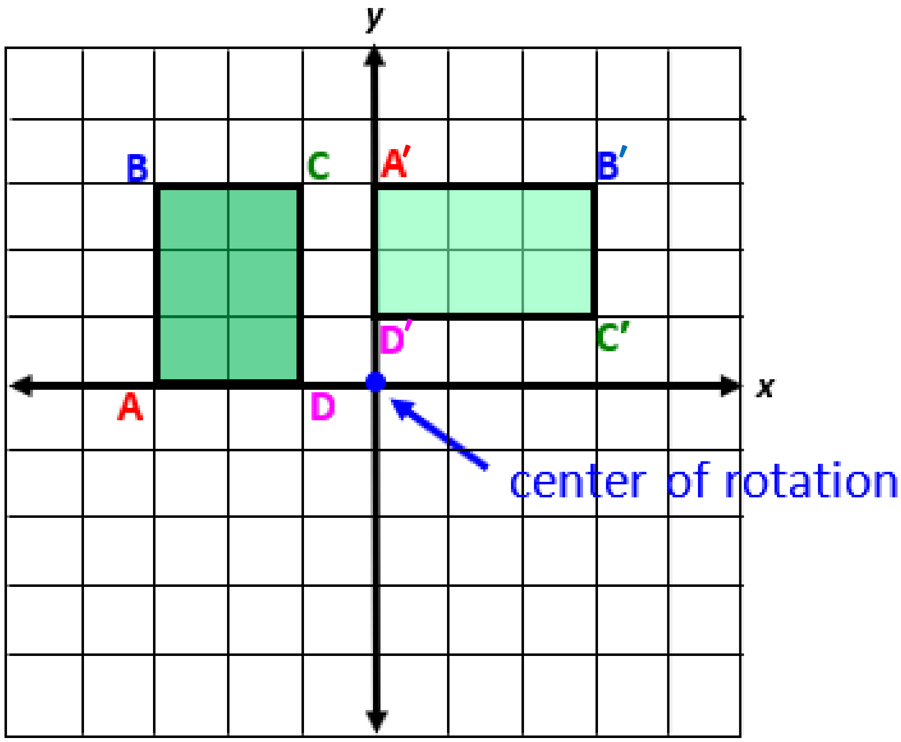


Examples of Point Symmetry



Rotation

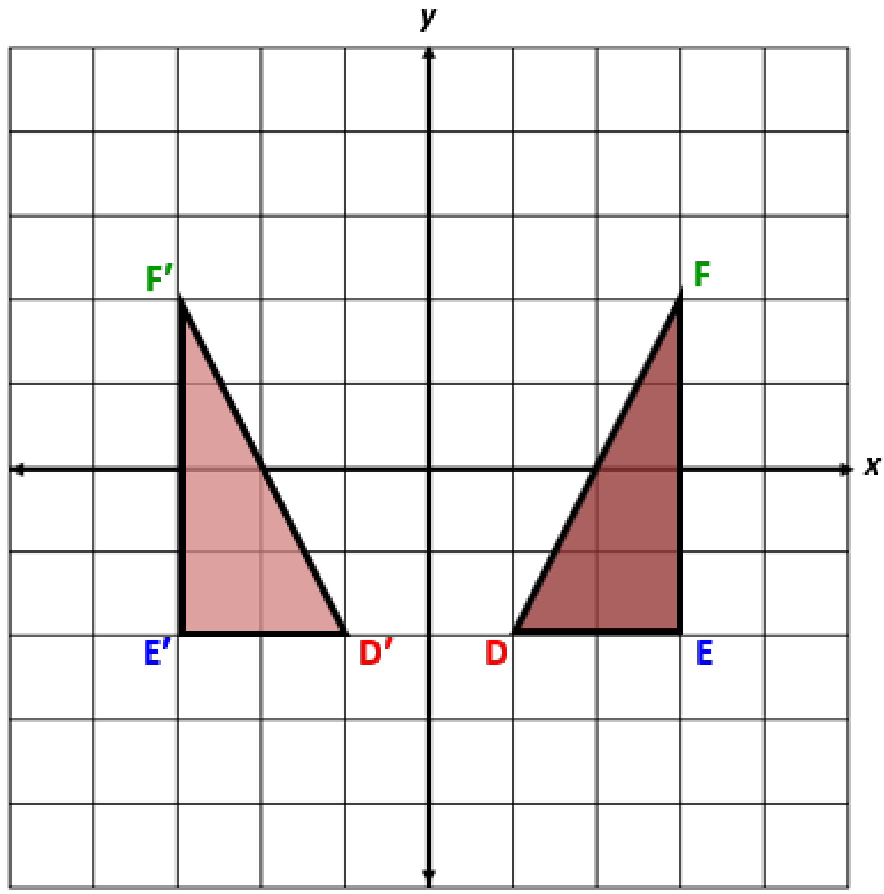
(Origin)



Preimage	Image
A(-3,0)	A′(0,3)
B(-3,3)	B′(3,3)
C(-1,3)	C′(3,1)
D(-1,0)	D′(0,1)

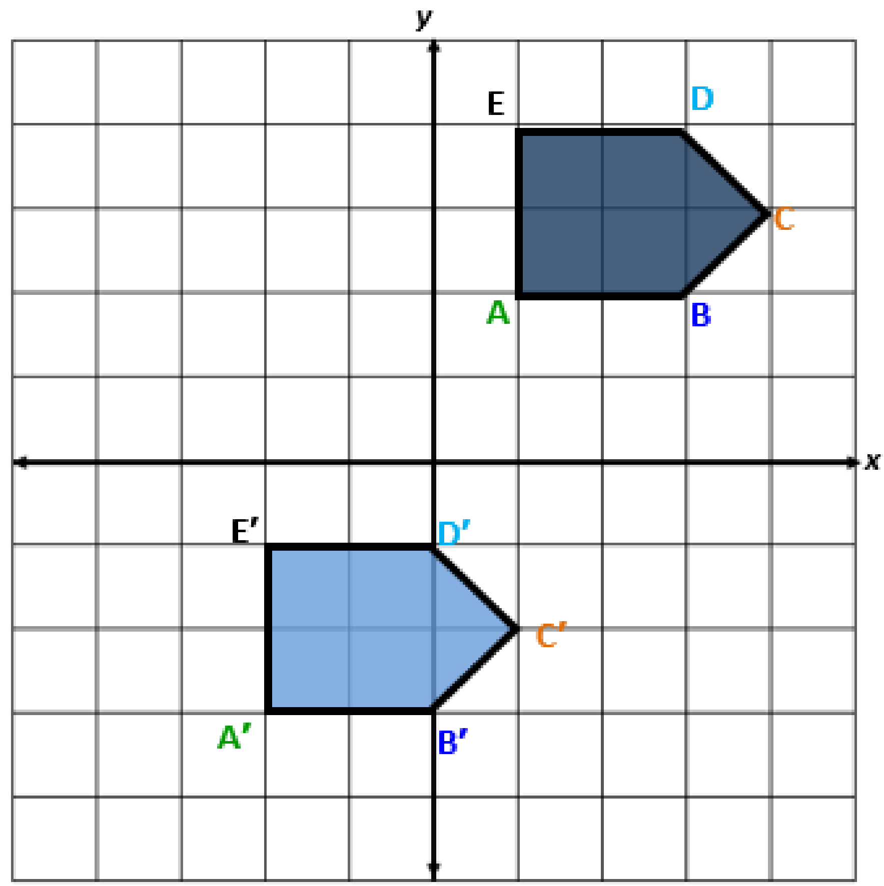
Pre-image has been transformed by a 90° clockwise rotation about the origin.

Reflection



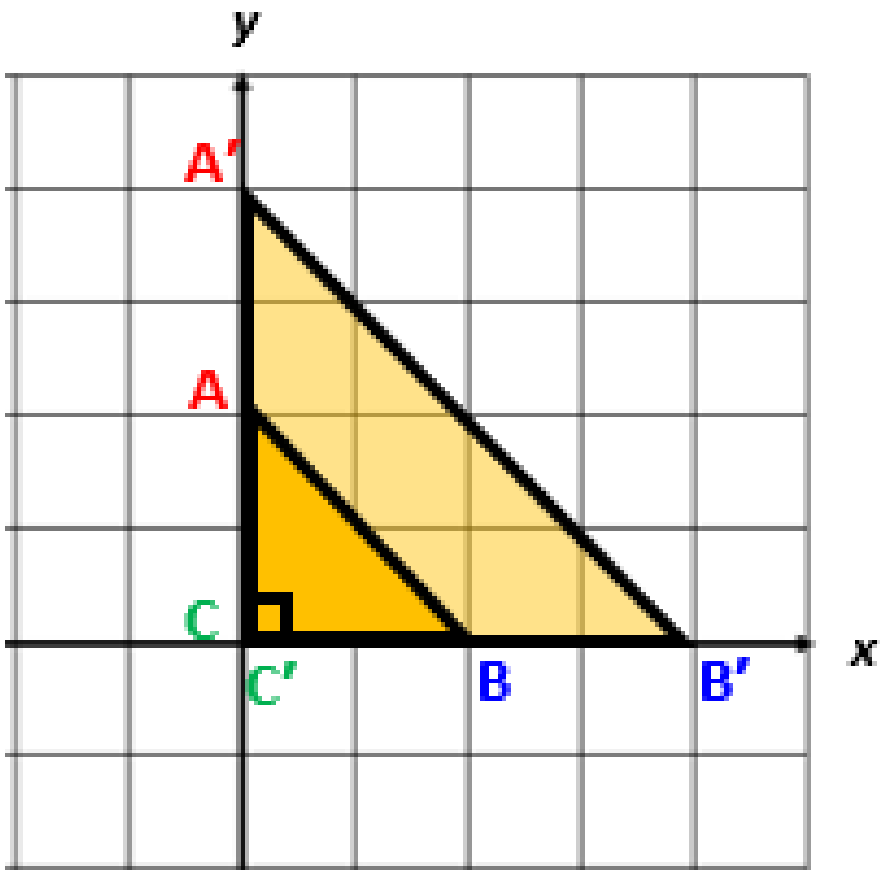
Preimage	Image
D(1,-2)	D′(-1,-2)
E(3,-2)	E′(-3,-2)
F(3,2)	F′(-3,2)

Translation



Preimage	Image
A(1,2)	A′(-2,-3)
B(3,2)	B′(0,-3)
C(4,3)	C′(1,-2)
D(3,4)	D′(0,-1)
E(1,4)	E′(-2,-1)

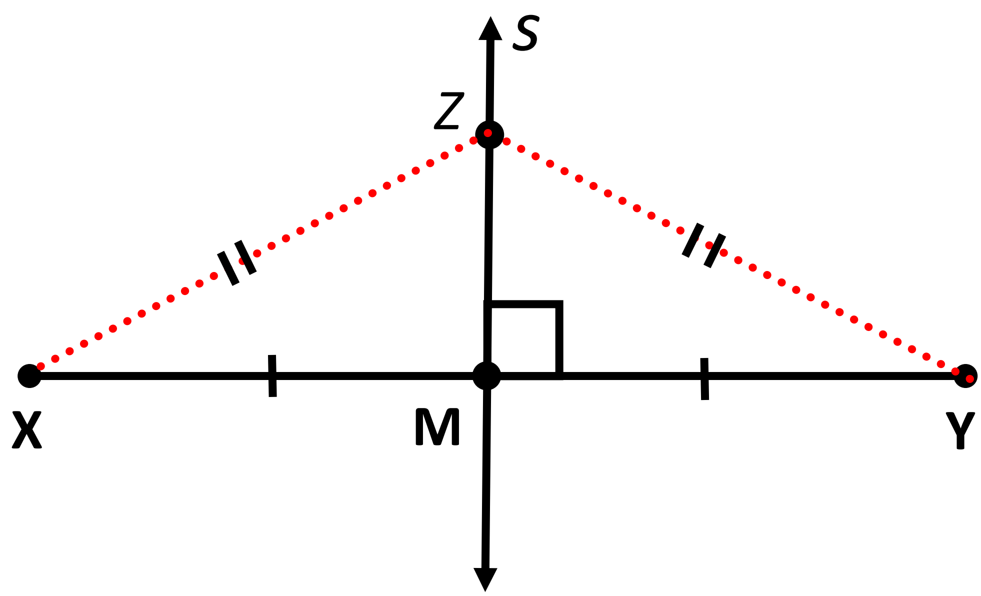
Dilation



Preimage	Image
A(0,2)	A′(0,4)
B(2,0)	B′(4,0)
C(0,0)	C′(0,0)

Perpendicular Bisector

a segment, ray, line, or plane that is perpendicular to a segment at its midpoint



Example:

Line *s* is perpendicular to $\overline{XY}$.

M is the midpoint, therefore $\overline{XM} \cong \overline{MY}$.

Z lies on line *s* and is equidistant from X and Y.

Constructions

Traditional constructions involving a compass and straightedge reinforce students' understanding of geometric concepts. Constructions help students visualize Geometry.

There are multiple methods to most geometric constructions. These cards illustrate only one method. Students would benefit from experiences with more than one method, including dynamic geometry software, and should be able to justify each step of geometric constructions.

Construct

segment *CD* congruent to segment *AB*

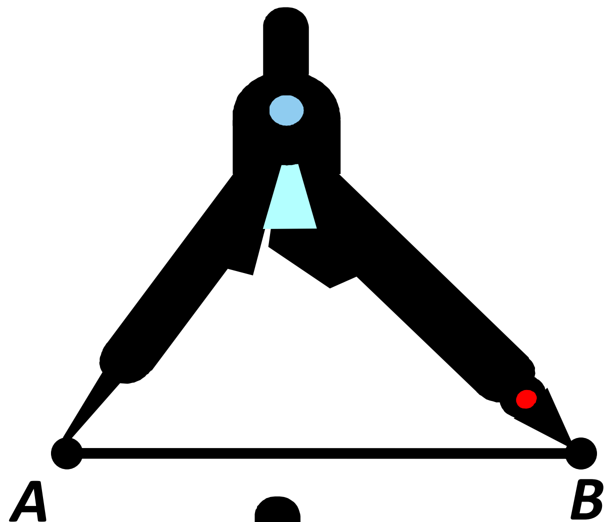


Fig. 1

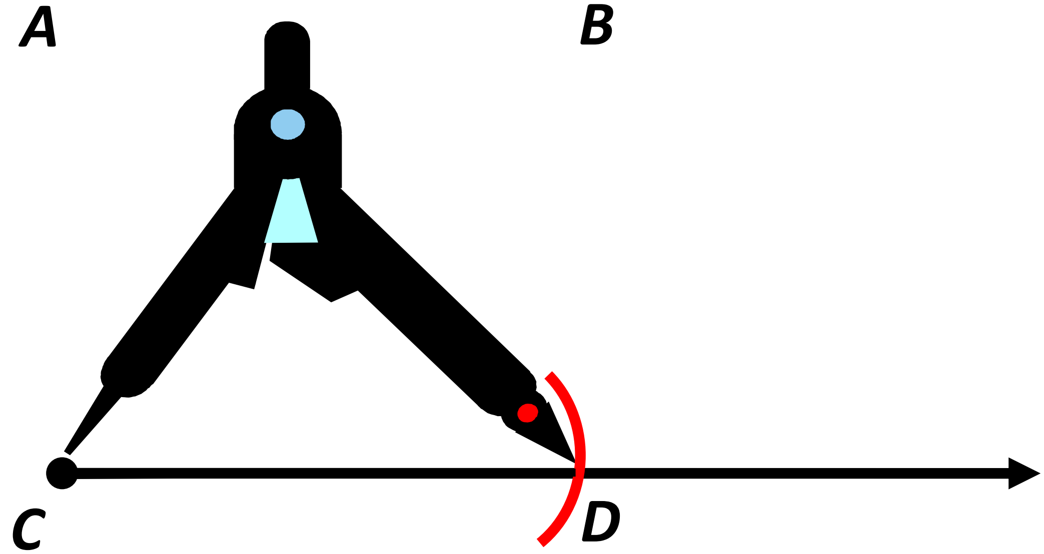


Fig. 2

Construct

a perpendicular bisector of segment *AB*

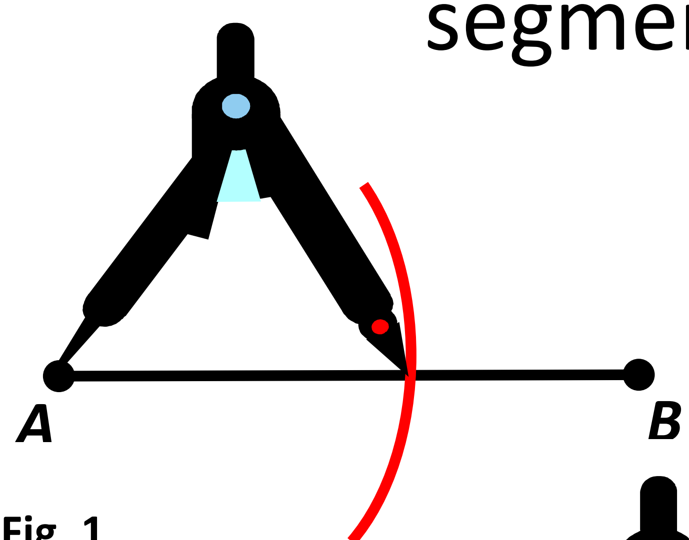


Fig. 1

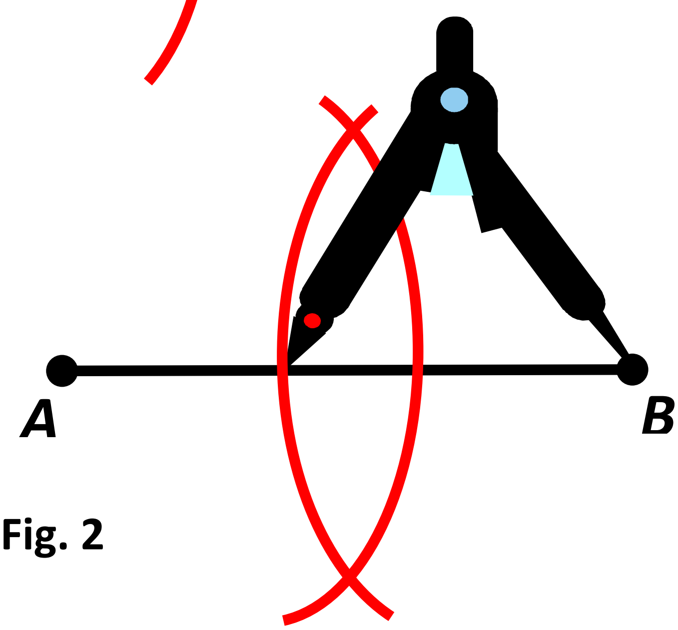


Fig. 2

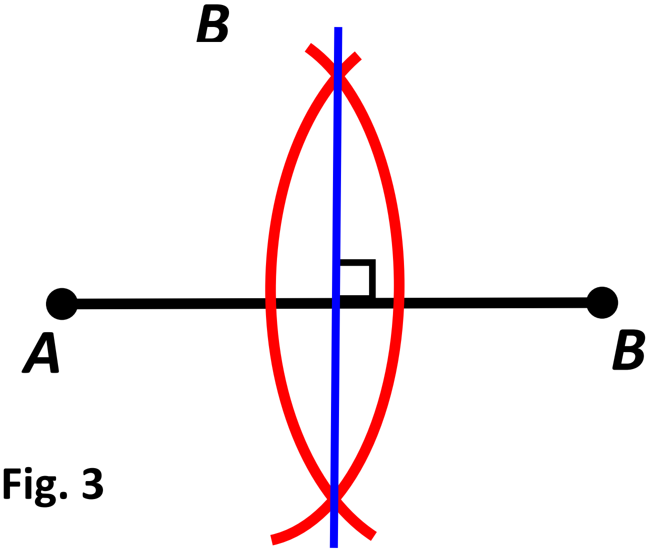
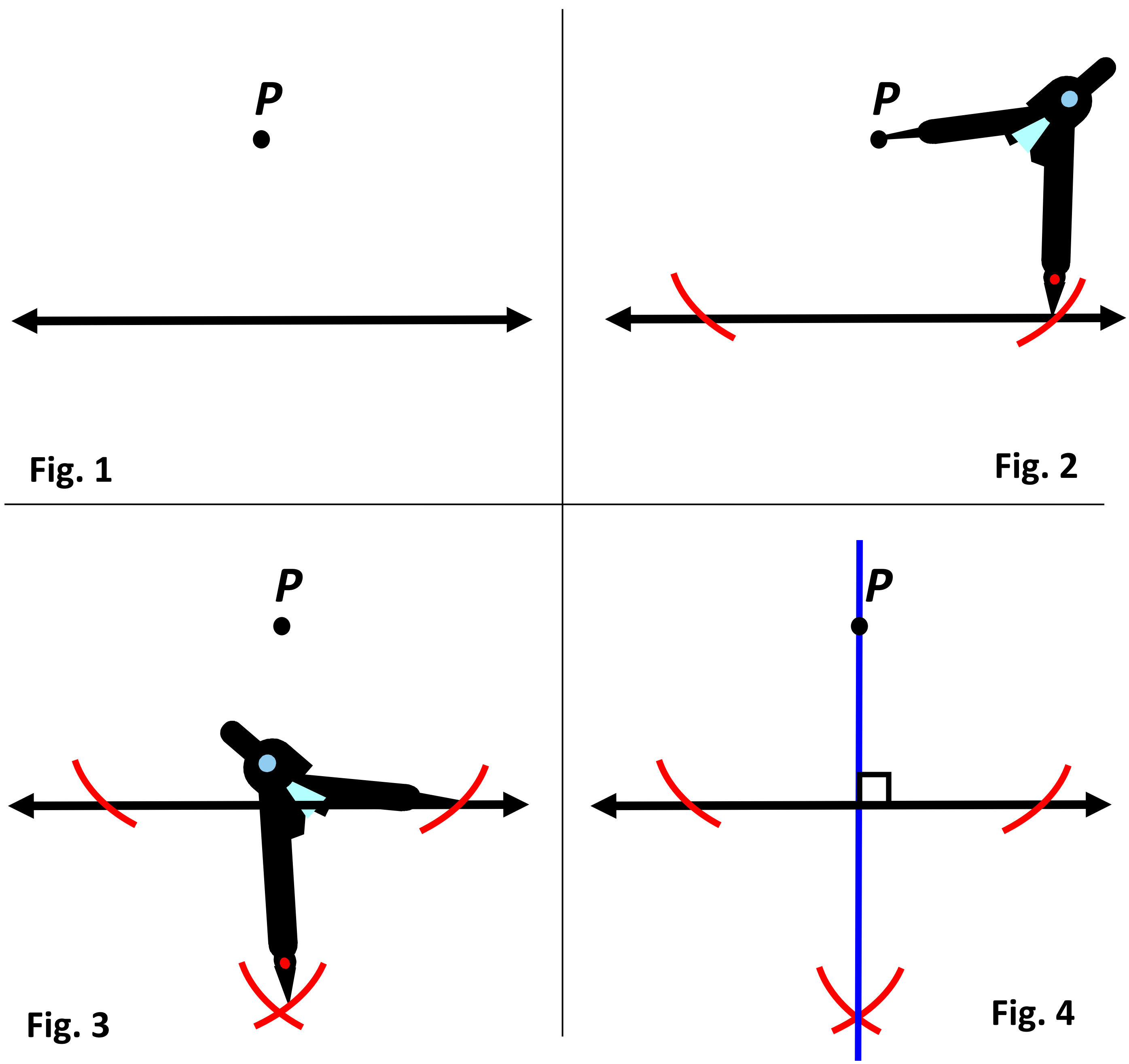


Fig. 3

Construct

a perpendicular to a line from point P not on the line



Construct

a perpendicular to a line from point P on the line

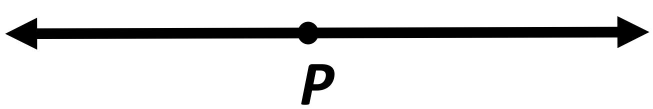


Fig. 1

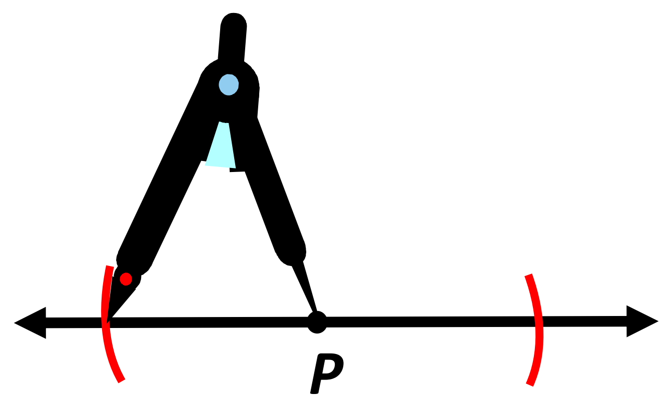


Fig. 2

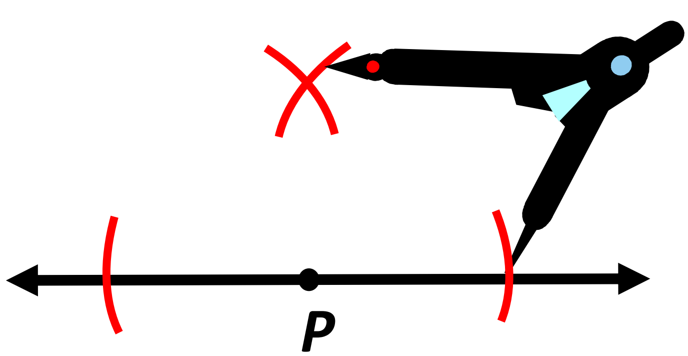


Fig. 3

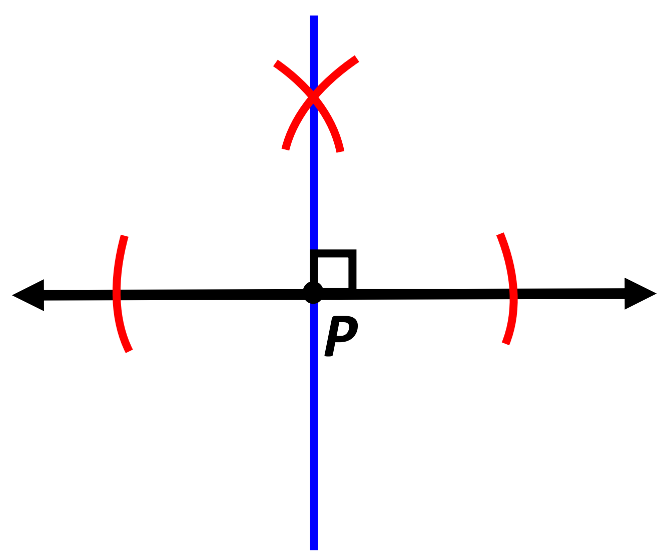


Fig. 4

Construct

a bisector of ∠*A*

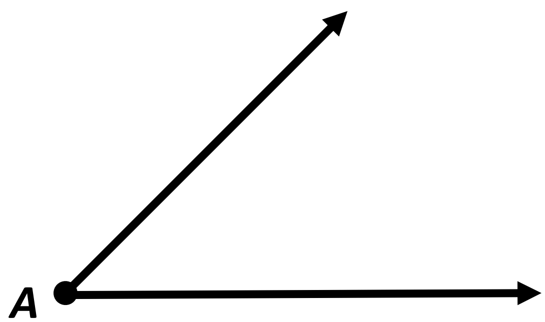


Fig. 1

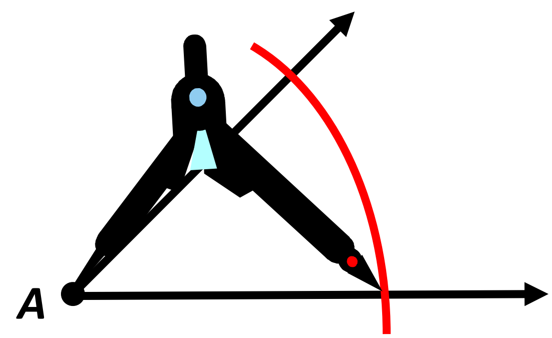


Fig. 2

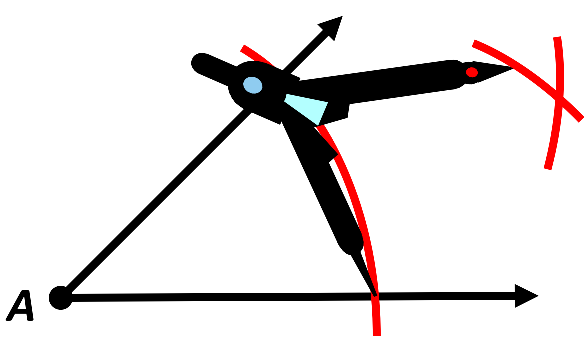


Fig. 3

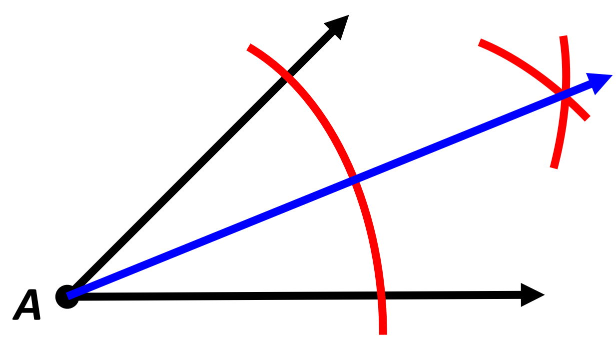


Fig. 4

Construct

$\angle Y$ congruent to $\angle A$

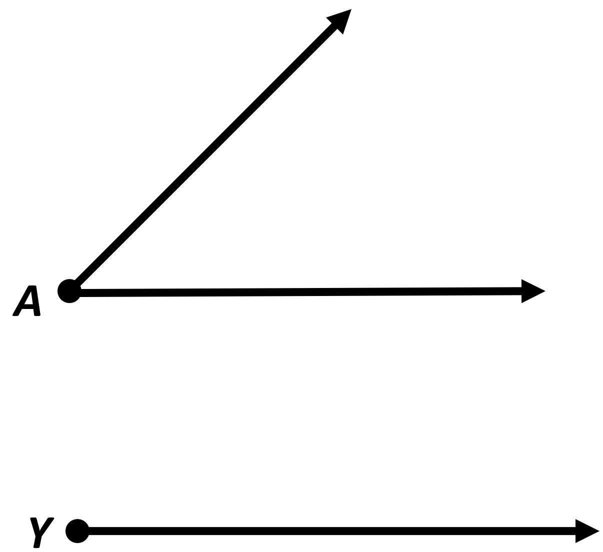


Fig. 1

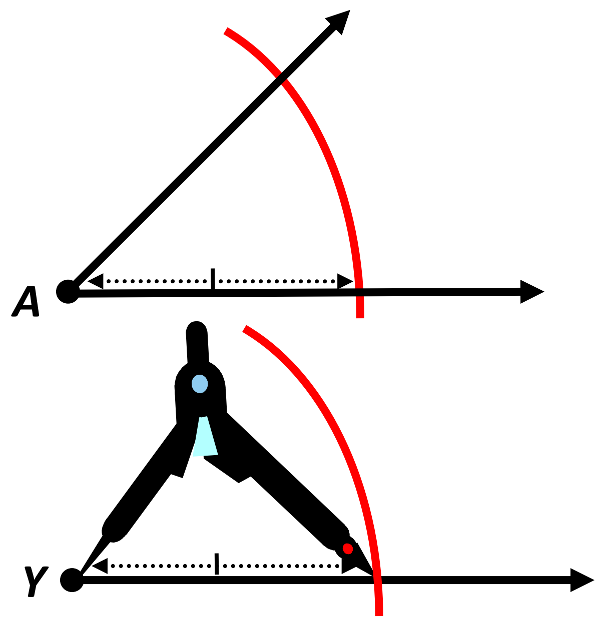


Fig. 2

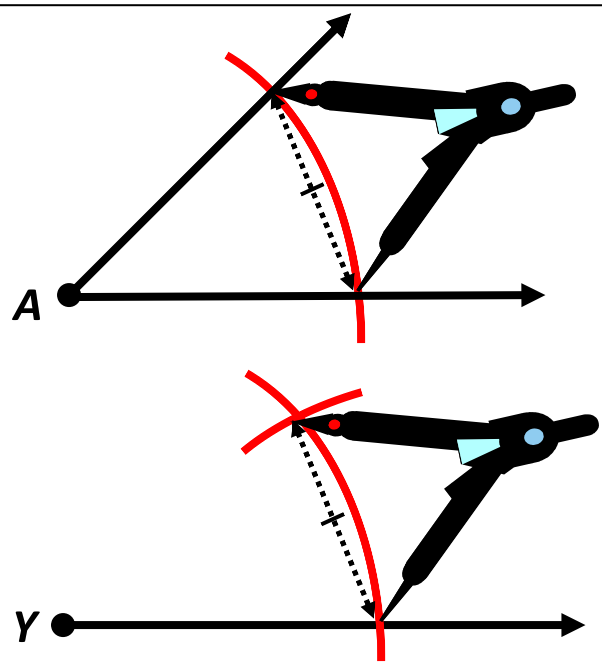


Fig. 3

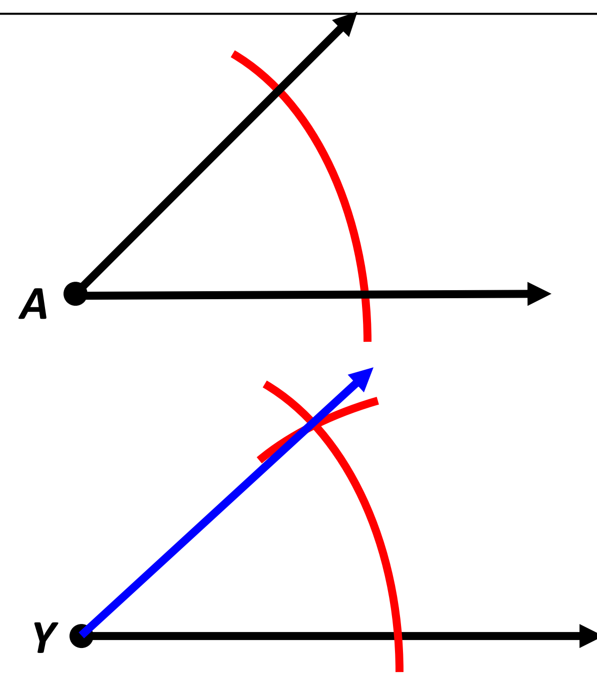
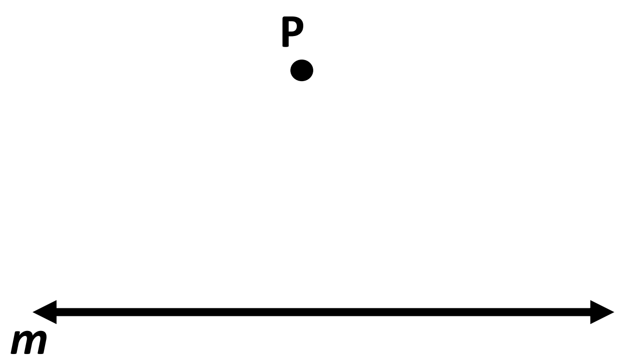


Fig. 4

Construct

line *n* parallel to line *m* through point *P* not on the line



Draw a line through point P intersecting line m.

Fig. 1

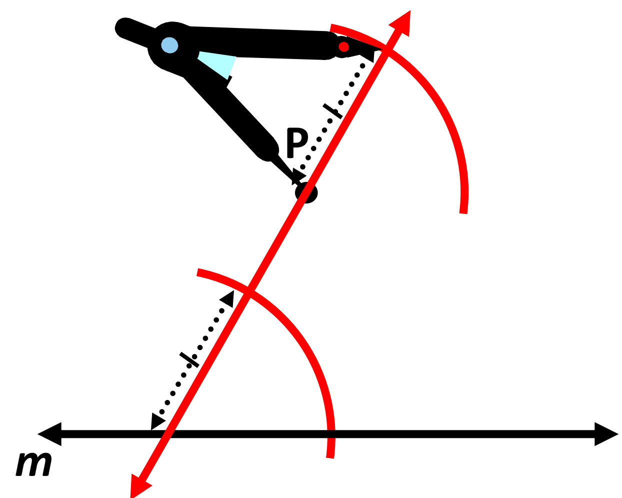


Fig. 2

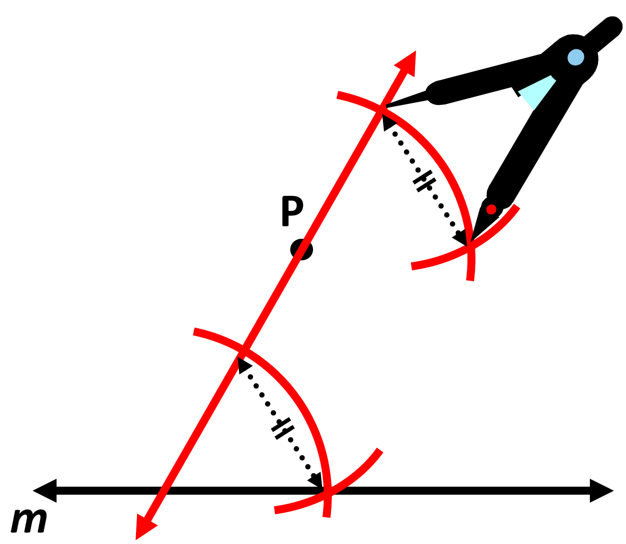


Fig. 3

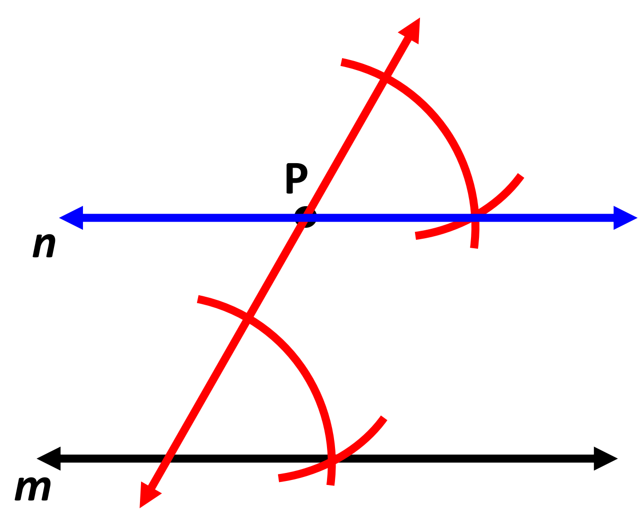


Fig. 4

Construct

an equilateral triangle inscribed in a circle

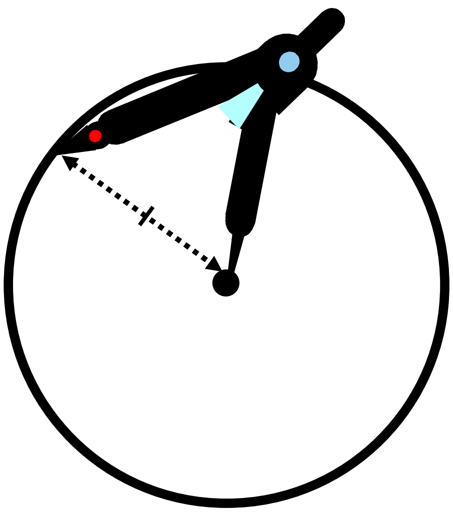


Fig. 1

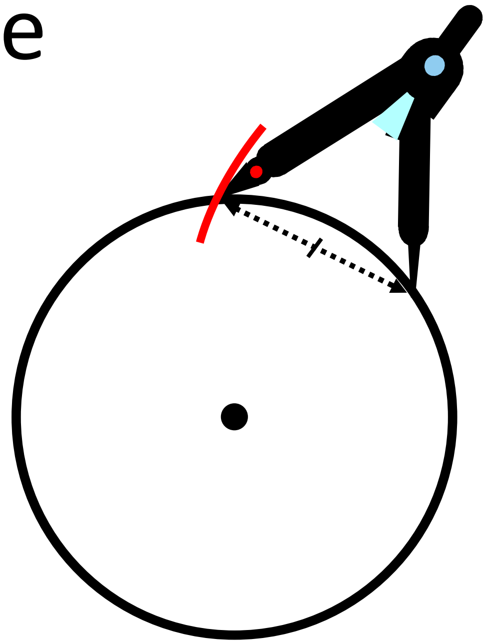


Fig. 2

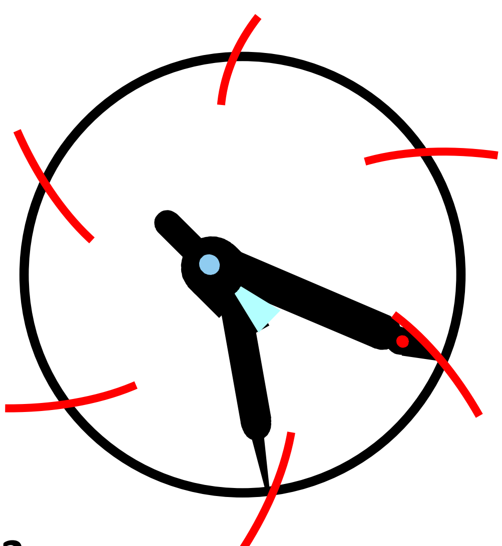


Fig. 3

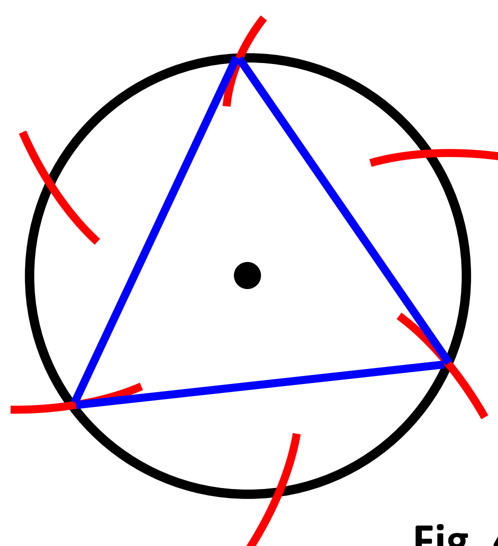


Fig. 4

Construct

a square inscribed in a circle

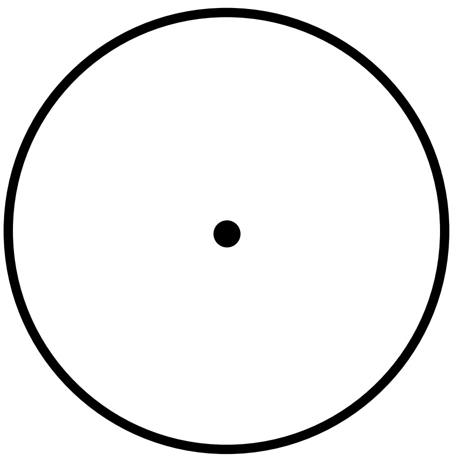


Fig. 1

Draw a diameter.

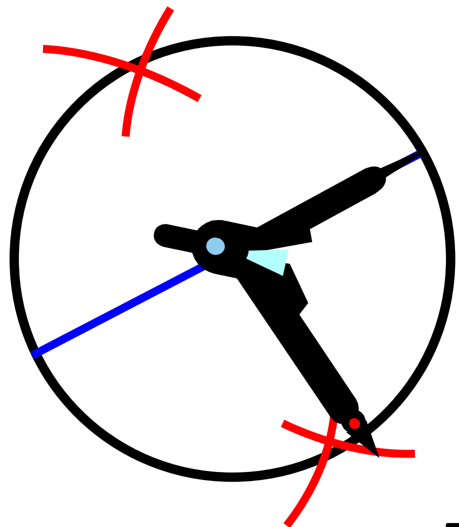


Fig. 2

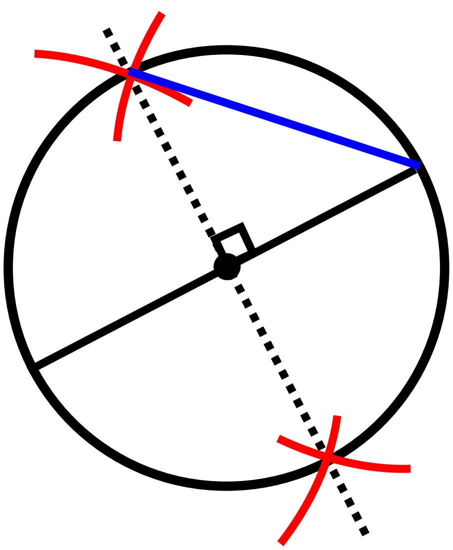


Fig. 3

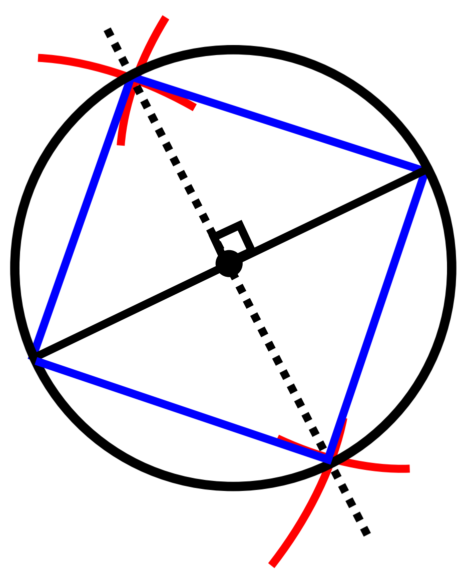


Fig. 4

Construct

a regular hexagon inscribed in a circle

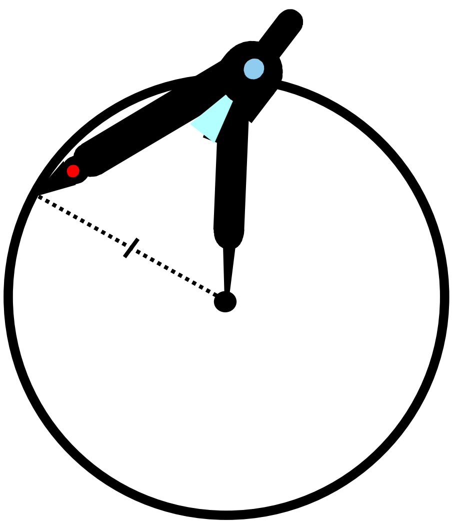


Fig. 1

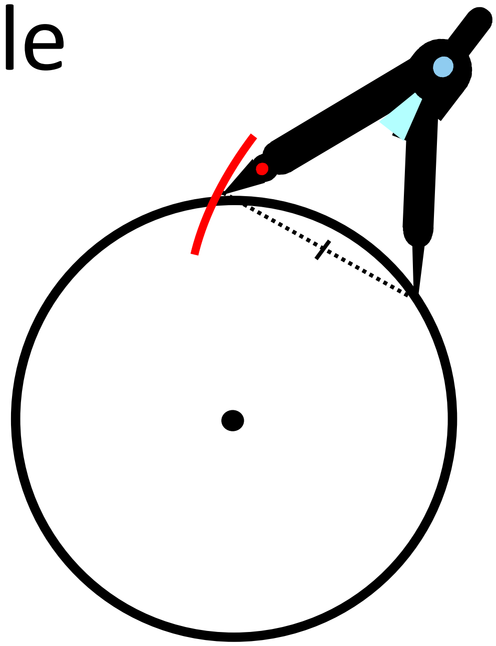


Fig. 2

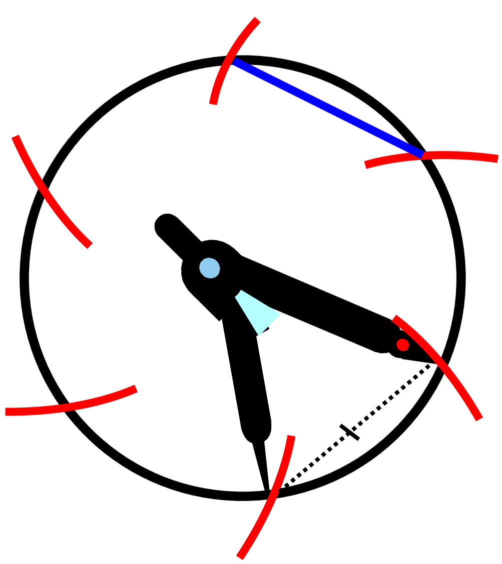


Fig. 3

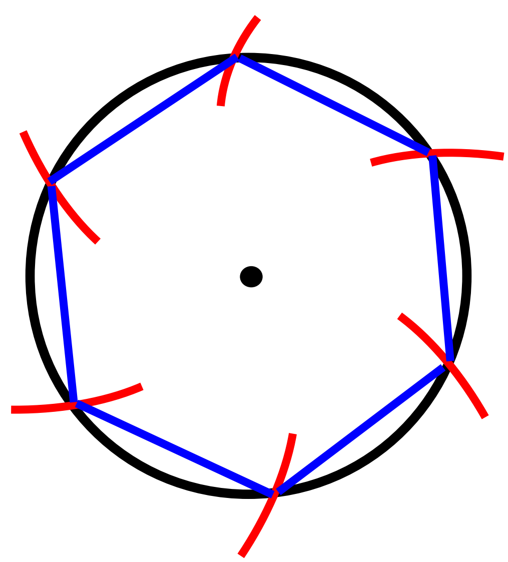
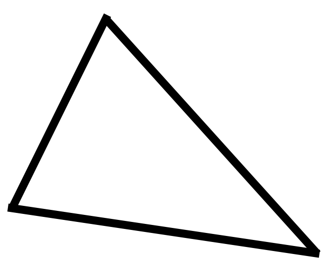
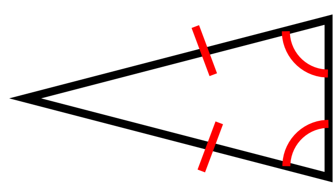
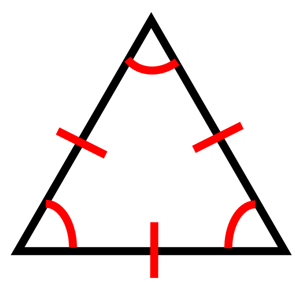


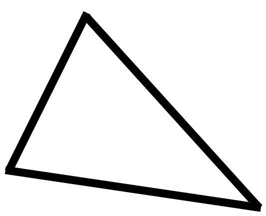
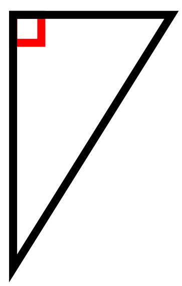
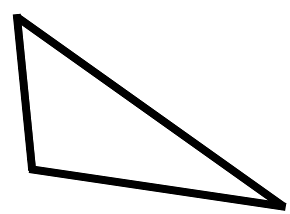
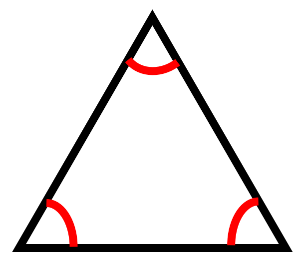
Fig. 4

Classifying Triangles by Sides

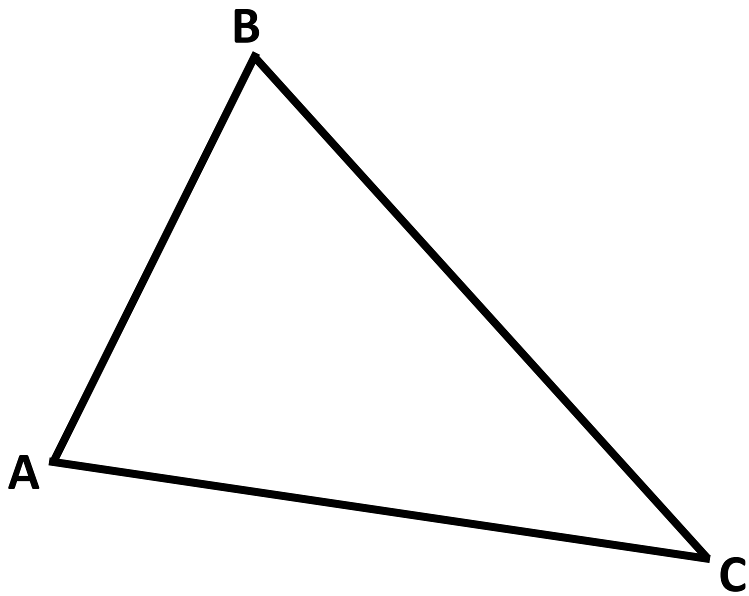
Scalene	***Isosceles***	***Equilateral***
		
No congruent sides	At least 2 congruent sides	3 congruent sides
No congruent angles	2 or 3 congruent angles	3 congruent angles

All equilateral triangles are isosceles.

Classifying Triangles by Angles

Acute	*Right*	*Obtuse*	*Equiangular*
			
3 acute angles	1 right angle	1 obtuse angle	3 congruent angles
3 angles, each less than 90°	1 angle equals 90°	1 angle greater than 90°	3 angles, each measures 60°

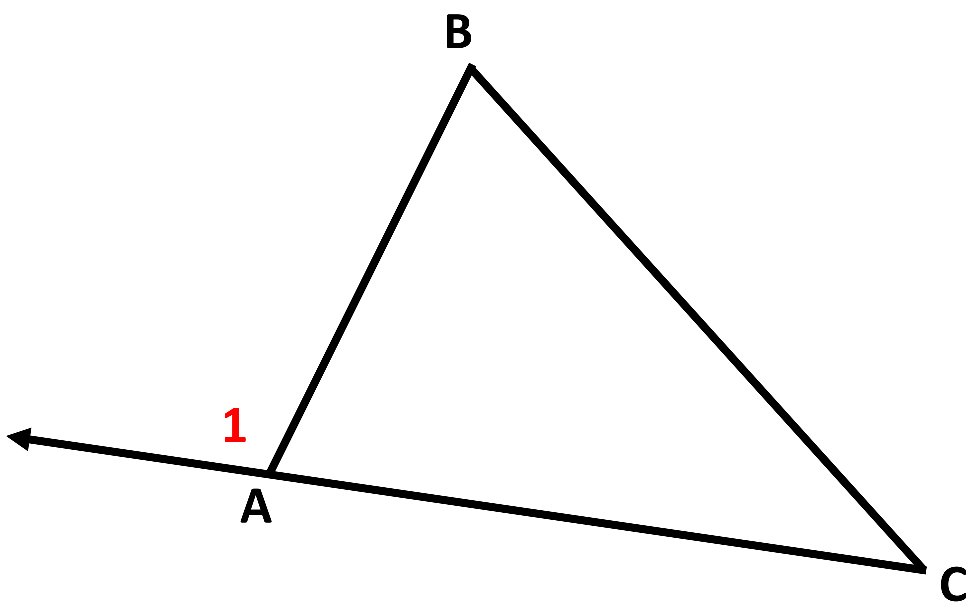
Triangle Sum Theorem



measures of the interior angles of a triangle = 180°

$$m\angle A + m\angle B + m\angle C = 180^\circ$$

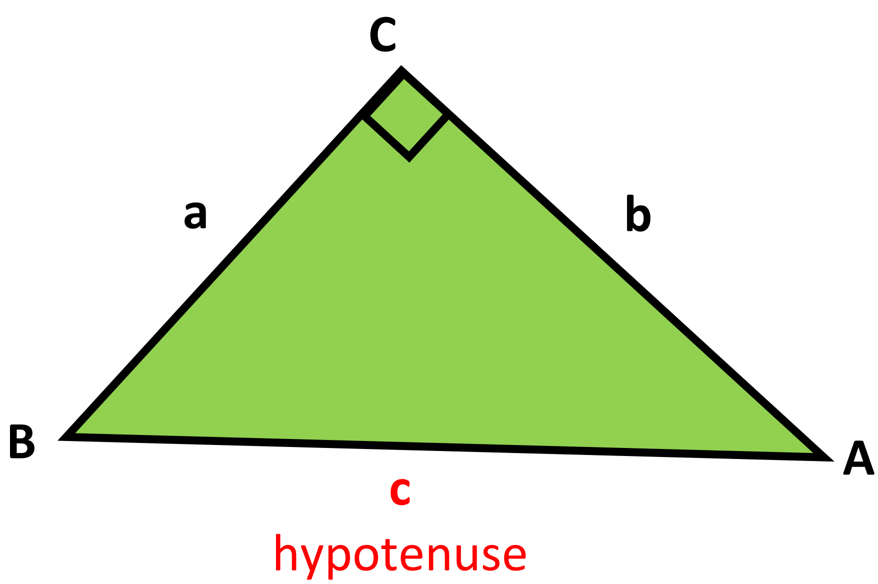
Exterior Angle Theorem



Exterior angle, $m\angle 1$, is equal to the sum of the measures of the two nonadjacent interior angles.

$$m\angle 1 = m\angle B + m\angle C$$

Pythagorean Theorem

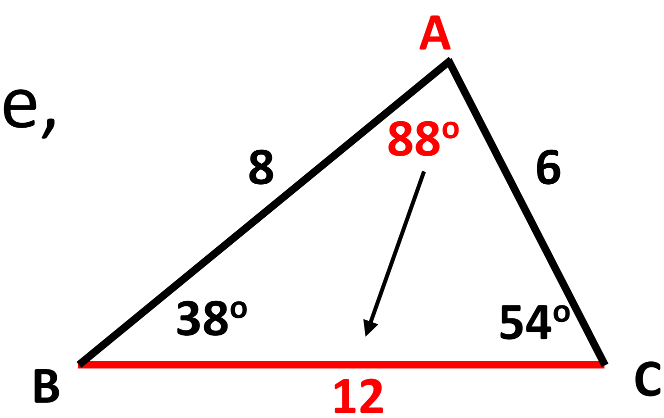


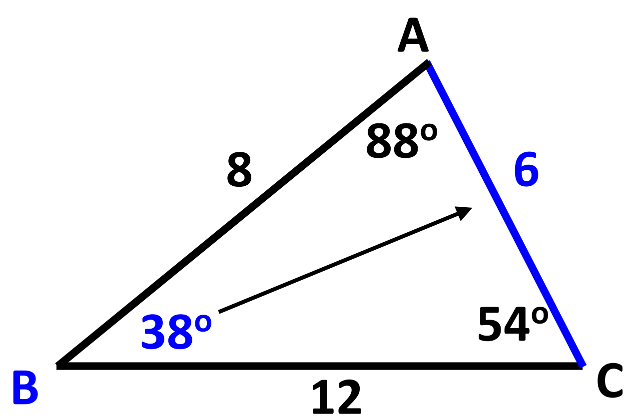
If ΔABC is a right triangle, then $a^2 + b^2 = c^2$.

Conversely, if $a^2 + b^2 = c^2$, then ΔABC is a right triangle.

Angle and Side Relationships

$\angle A$ is the largest angle, therefore $\overline{BC}$ is the longest side.

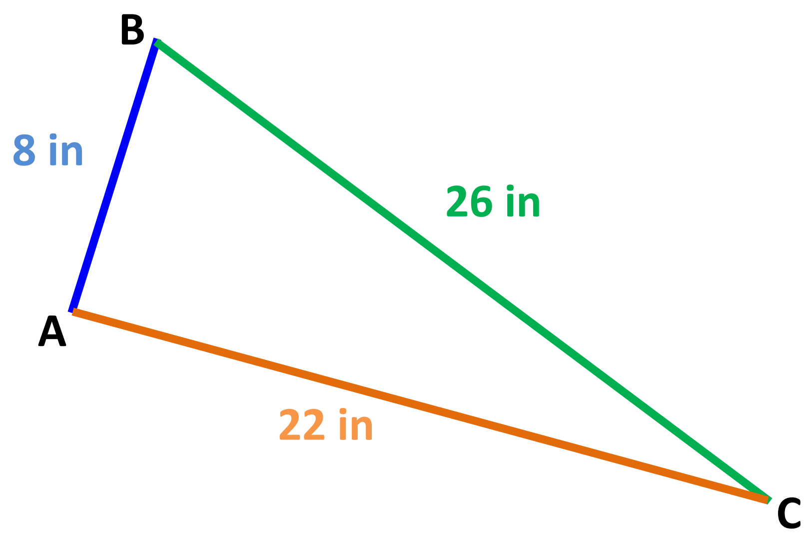




$\angle B$ is the smallest angle, therefore $\overline{AC}$ is the shortest side.

Triangle Inequality Theorem

The sum of the lengths of any two sides of a triangle is greater than the length of the third side.



Example:

$AB + BC > AC$

$8 + 26 > 22$

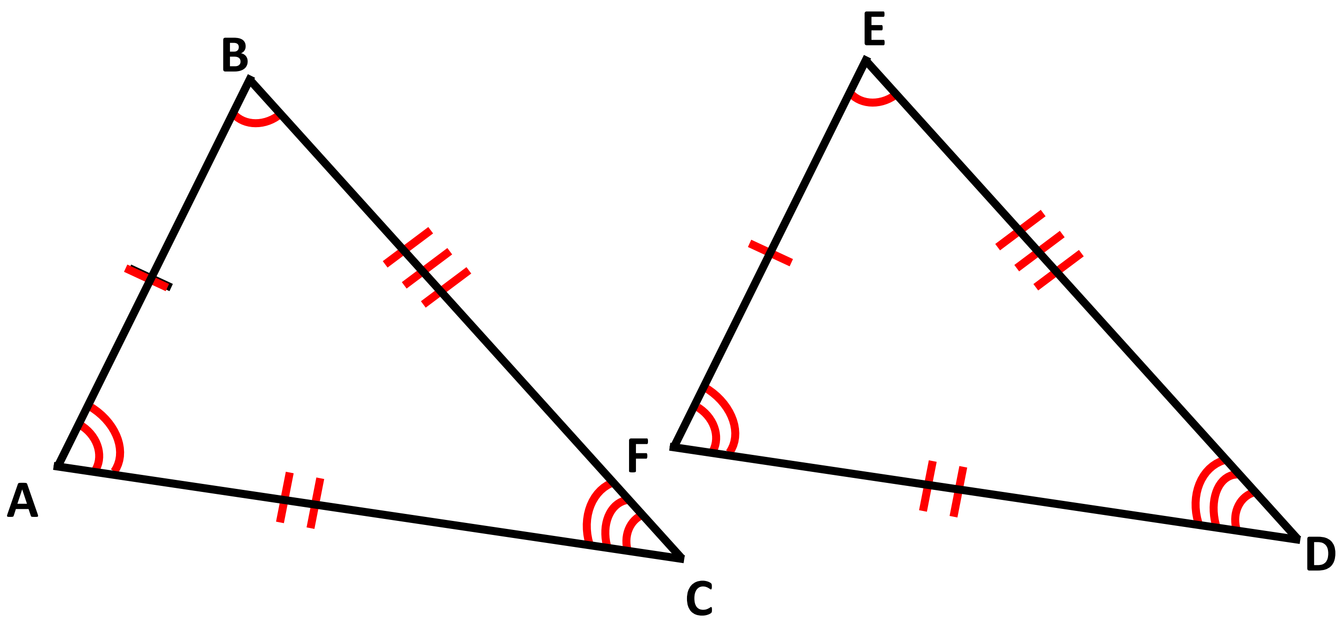
$AC + BC > AB$

$22 + 26 > 8$

$AB + AC > BC$

$8 + 22 > 26$

Congruent Triangles



Two possible congruence statements:

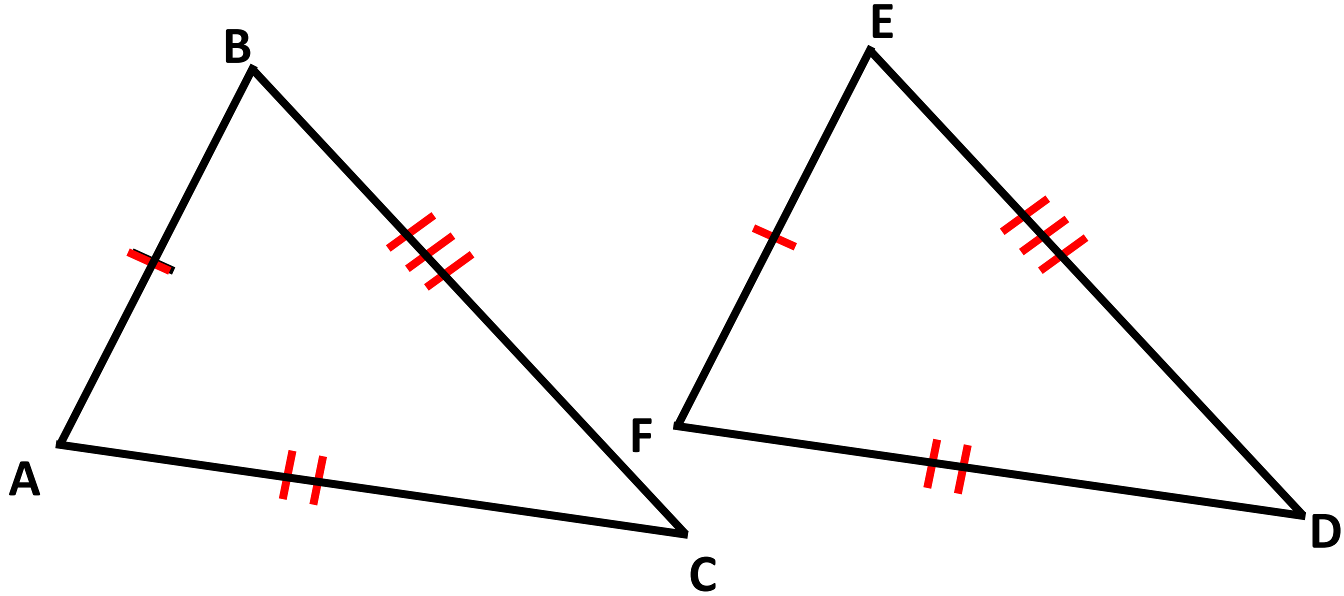
$\Delta ABC \cong \Delta FED$

$\Delta BCA \cong \Delta EDF$

Corresponding Parts of Congruent Figures

$\angle A \cong \angle F$	$\overline{AB} \cong \overline{FE}$
$\angle B \cong \angle E$	$\overline{BC} \cong \overline{ED}$
$\angle C \cong \angle D$	$\overline{CA} \cong \overline{DF}$

SSS Triangle Congruence Postulate



Example:

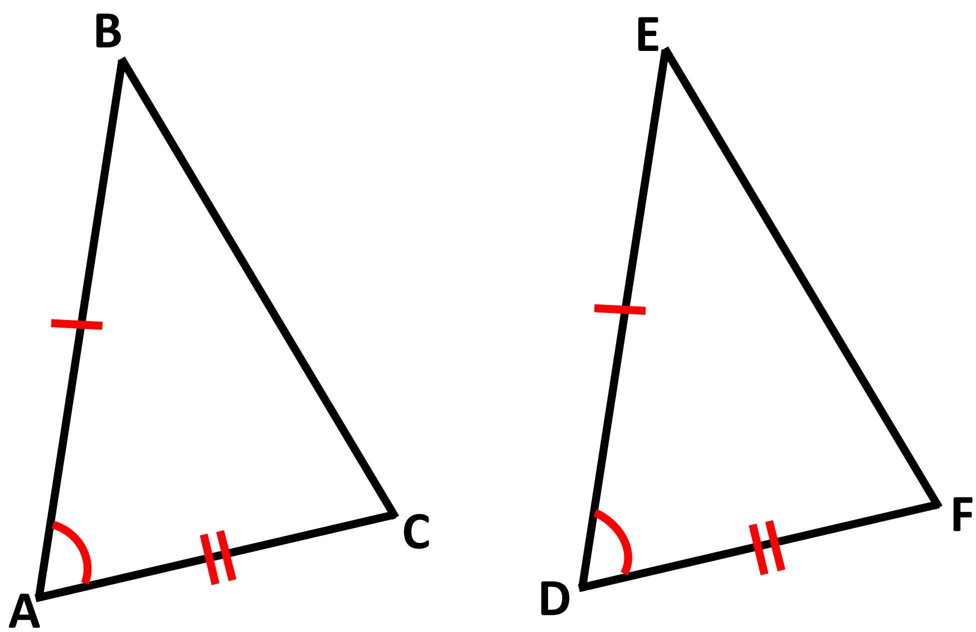
If Side $\overline{AB} \cong \overline{FE}$,

Side $\overline{AC} \cong \overline{FD}$, and

Side $\overline{BC} \cong \overline{ED}$,

then Δ ABC $\cong$ ΔFED.

SAS Triangle Congruence Postulate



Example:

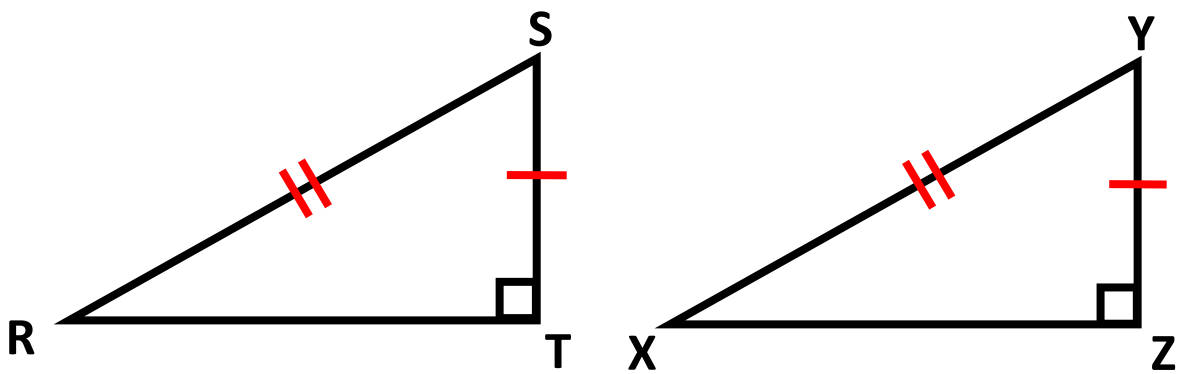
If Side $\overline{AB} \cong \overline{DE}$,

Angle $\angle A \cong \angle D$, and

Side $\overline{AC} \cong \overline{DF}$,

then $\Delta\, ABC \cong \Delta DEF$.

HL Right Triangle Congruence



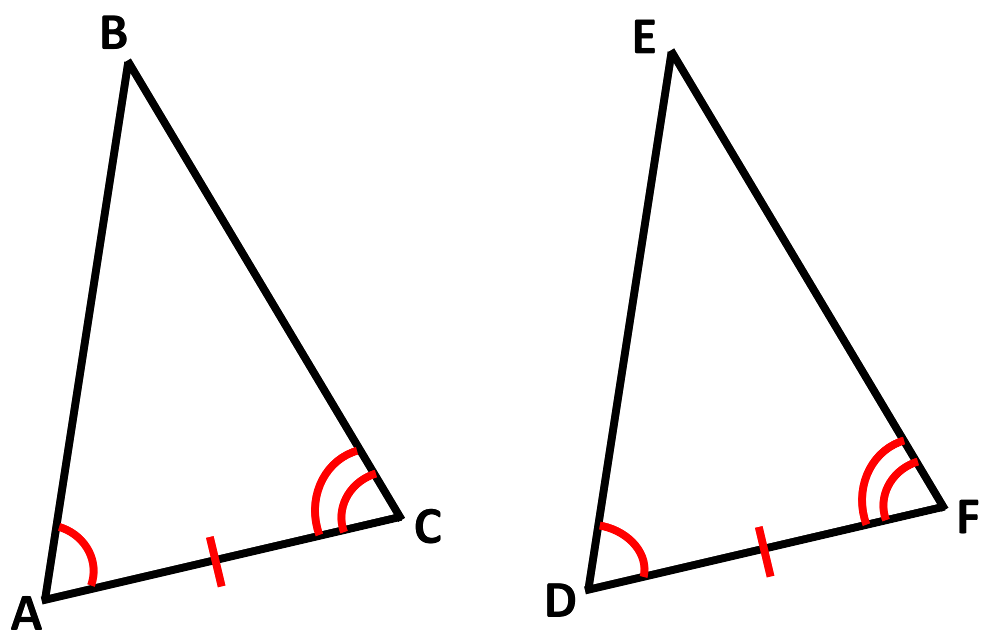
Example:

If Hypotenuse $\overline{RS} \cong \overline{XY}$, and

Leg $\overline{ST} \cong \overline{YZ}$,

then Δ RST $\cong$ ΔXYZ.

ASA Triangle Congruence Postulate



Example:

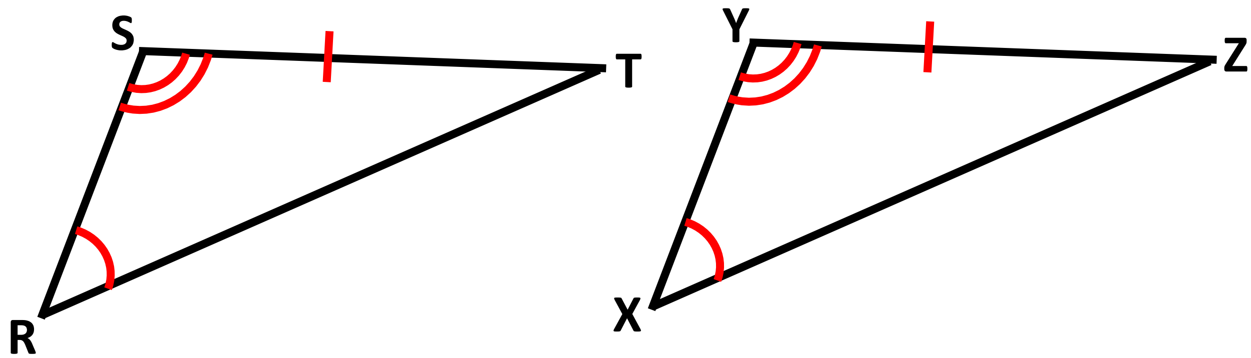
If Angle $\angle A \cong \angle D$,

Side $\overline{AC} \cong \overline{DF}$, and

Angle $\angle C \cong \angle F$

then $\Delta ABC \cong \Delta DEF$.

AAS Triangle Congruence Theorem



Example:

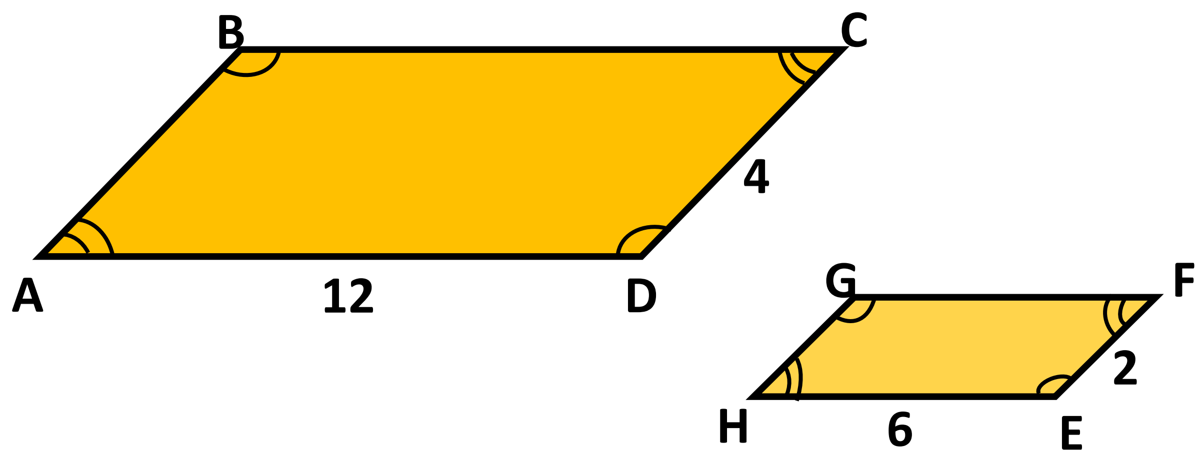
If Angle $\angle R \cong \angle X$,

Angle $\angle S \cong \angle Y$, and

Side $\overline{ST} \cong \overline{YZ}$

then $\Delta RST \cong \Delta XYZ$.

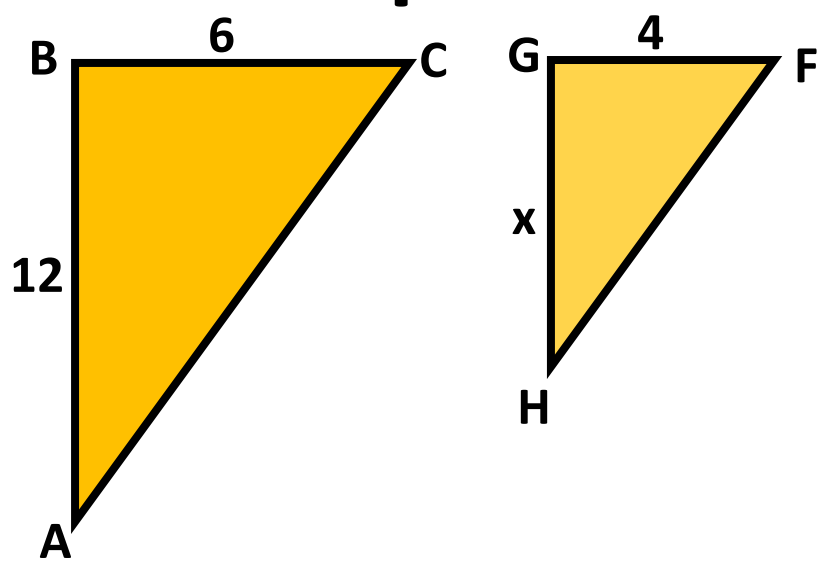
Similar Polygons



ABCD ~ HGFE	
Angles	**Sides**
∠A corresponds to ∠H	$\overline{AB}$ corresponds to $\overline{HG}$
∠B corresponds to ∠G	$\overline{BC}$ corresponds to $\overline{GF}$
∠C corresponds to ∠F	$\overline{CD}$ corresponds to $\overline{FE}$
∠D corresponds to ∠E	$\overline{DA}$ corresponds to $\overline{EH}$

Corresponding angles are congruent.
Corresponding sides are proportional.

Similar Polygons and Proportions



Corresponding vertices are listed in the same order.

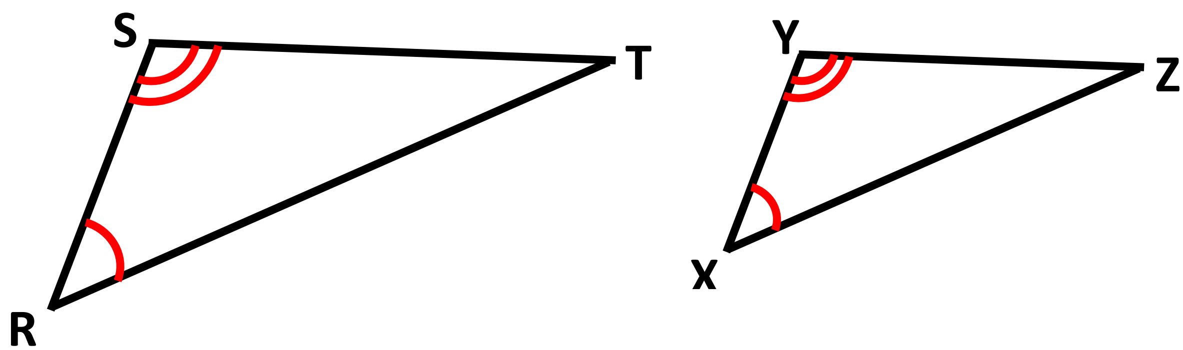
Example: $\Delta ABC \sim \Delta HGF$

$$\frac{AB}{HG} = \frac{BC}{GF}$$

$$\frac{12}{x} = \frac{6}{4}$$

The perimeters of the polygons are also proportional.

AA Triangle Similarity Postulate



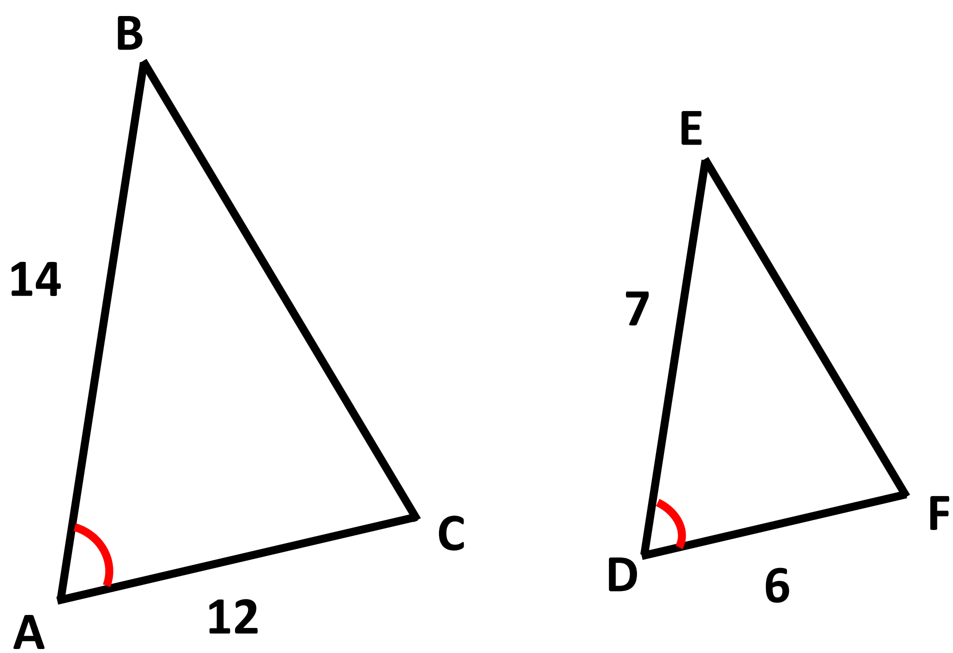
Example:

If Angle $\angle R \cong \angle X$ and

Angle $\angle S \cong \angle Y$,

then $\Delta RST \sim \Delta XYZ$.

SAS Triangle Similarity Theorem



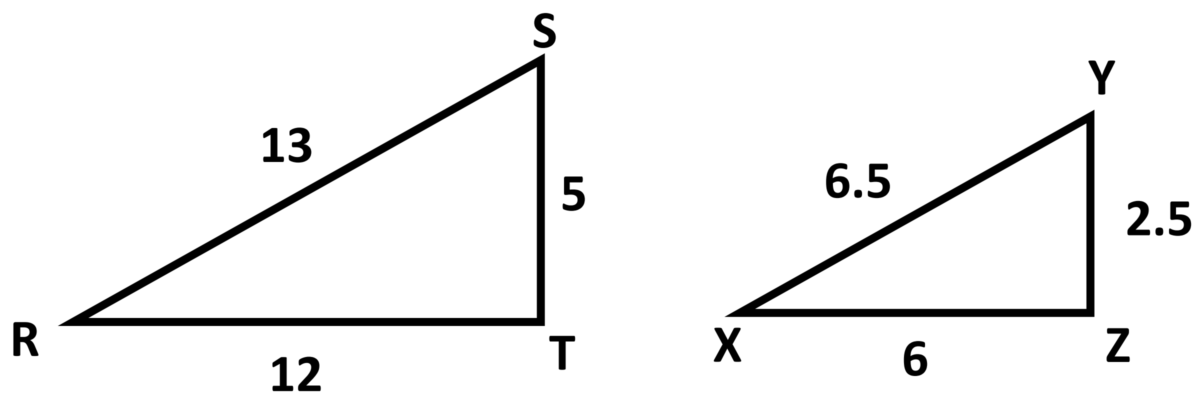
Example:

If $\angle A \cong \angle D$ and

$$\frac{AB}{DE} = \frac{AC}{DF}$$

then $\Delta ABC \sim \Delta DEF$.

SSS Triangle Similarity Theorem



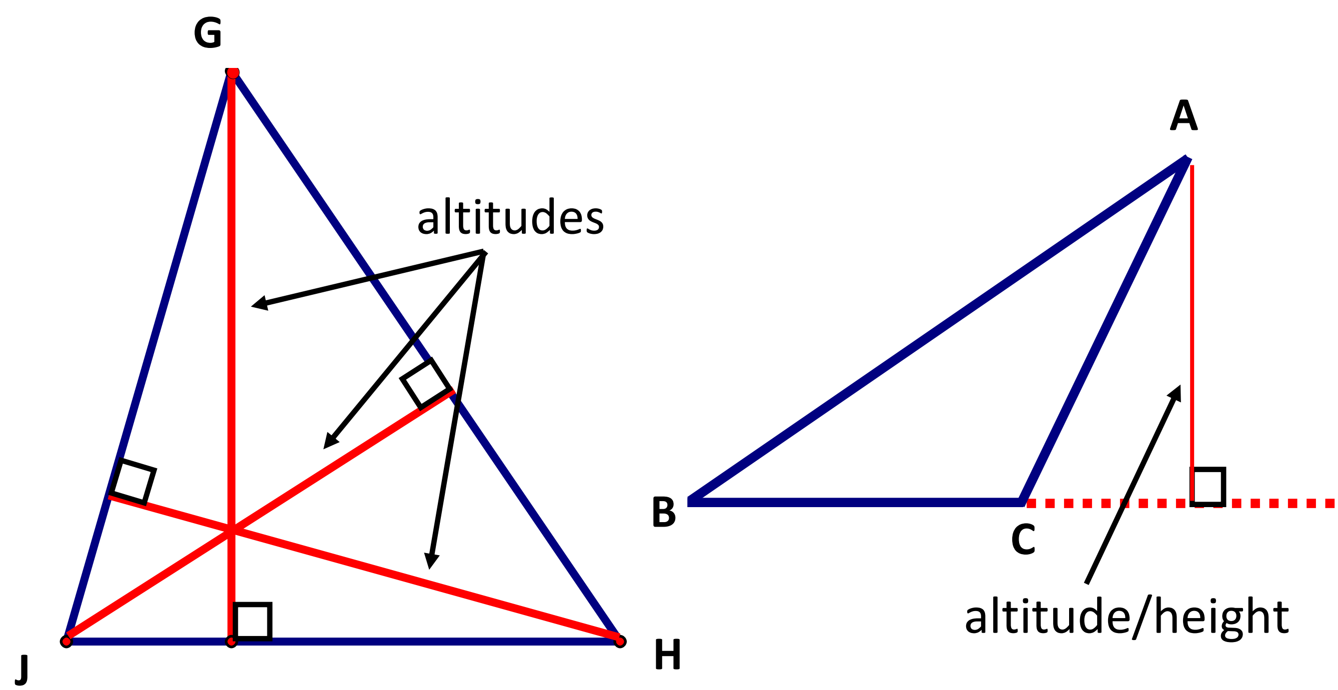
Example:

$$\text{If } \frac{RT}{XZ} = \frac{RS}{XY} = \frac{ST}{YZ}$$

then ∆RST ~ ∆XYZ.

Altitude of a Triangle

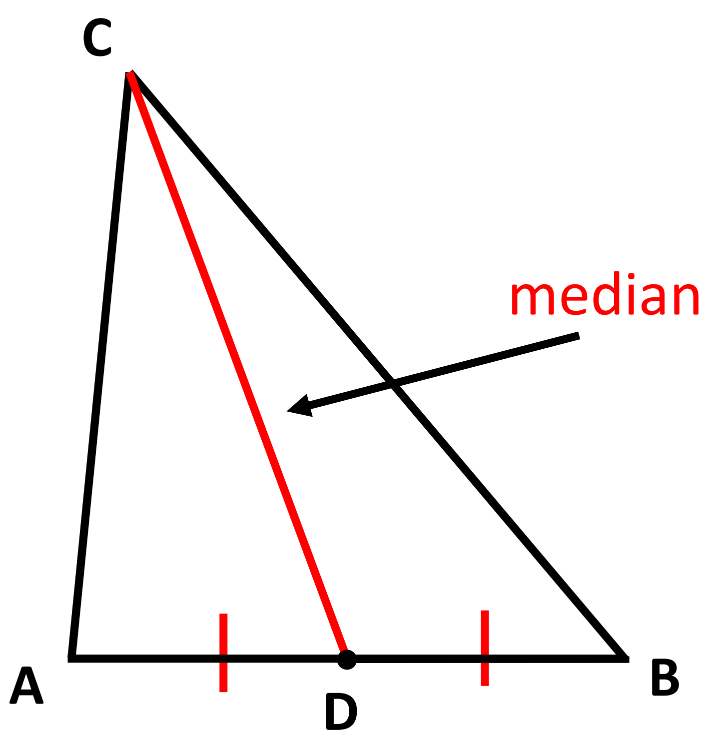
a segment from a vertex perpendicular to the line containing the opposite side



Every triangle has 3 altitudes.

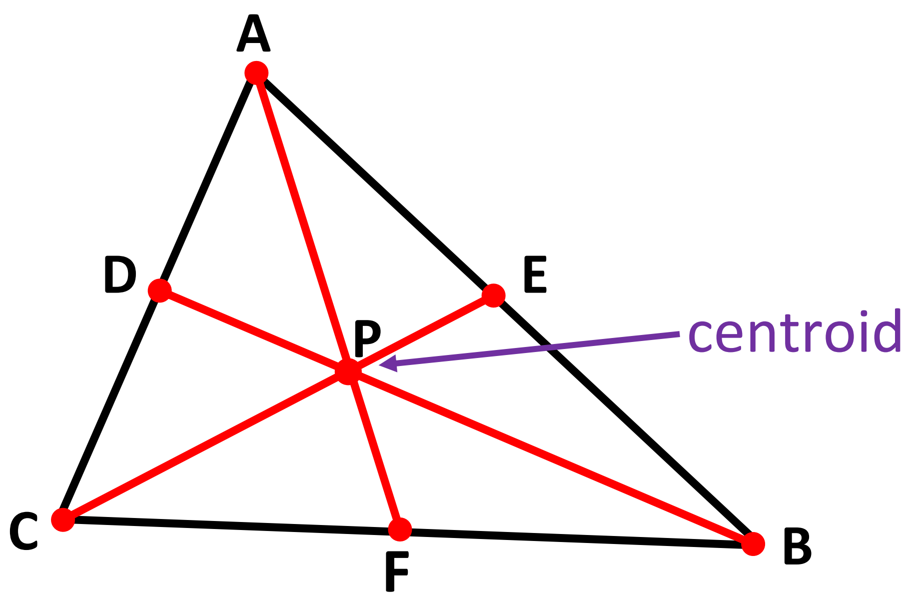
Median of a Triangle

A line segment from a vertex to the midpoint of the opposite side



D is the midpoint of $\overline{AB}$; therefore,
$\overline{CD}$ is a median of ΔABC.
Every triangle has 3 medians.

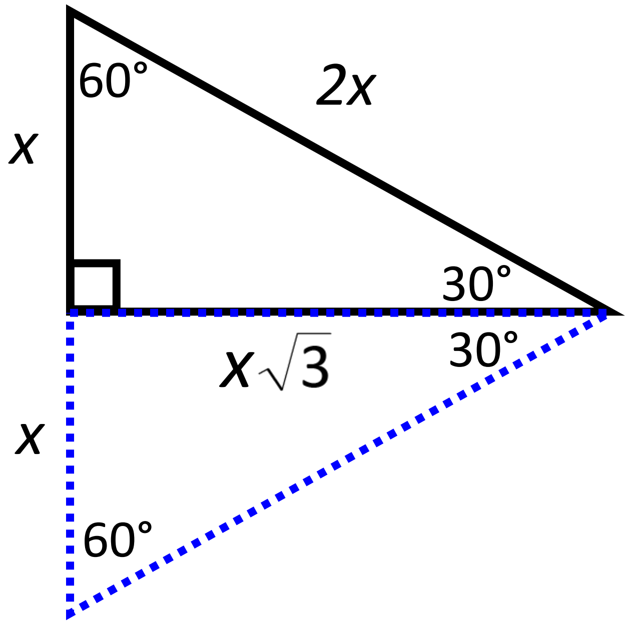
Concurrency of Medians of a Triangle



Medians of ΔABC intersect at P (centroid) and

$$AP = \frac{2}{3}AF, \quad CP = \frac{2}{3}CE, \quad BP = \frac{2}{3}BD.$$

30°-60°-90° Triangle Theorem



Given: short leg = x

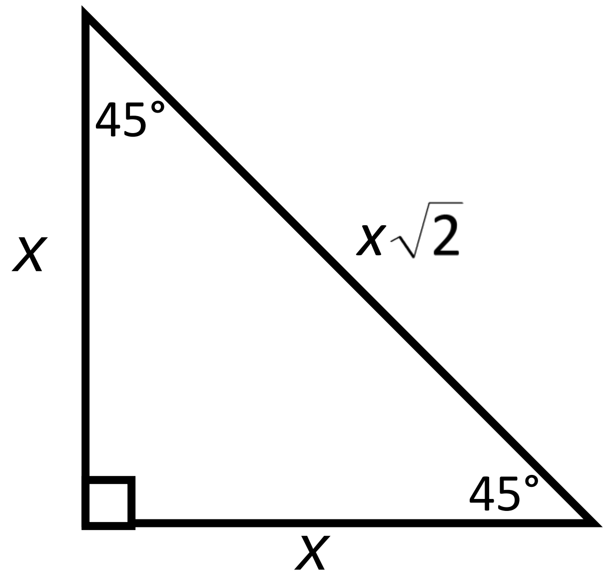
Using equilateral triangle,

hypotenuse = $2 \cdot x$

Applying the Pythagorean Theorem,

longer leg = $x \cdot \sqrt{3}$

45°-45°-90° Triangle Theorem



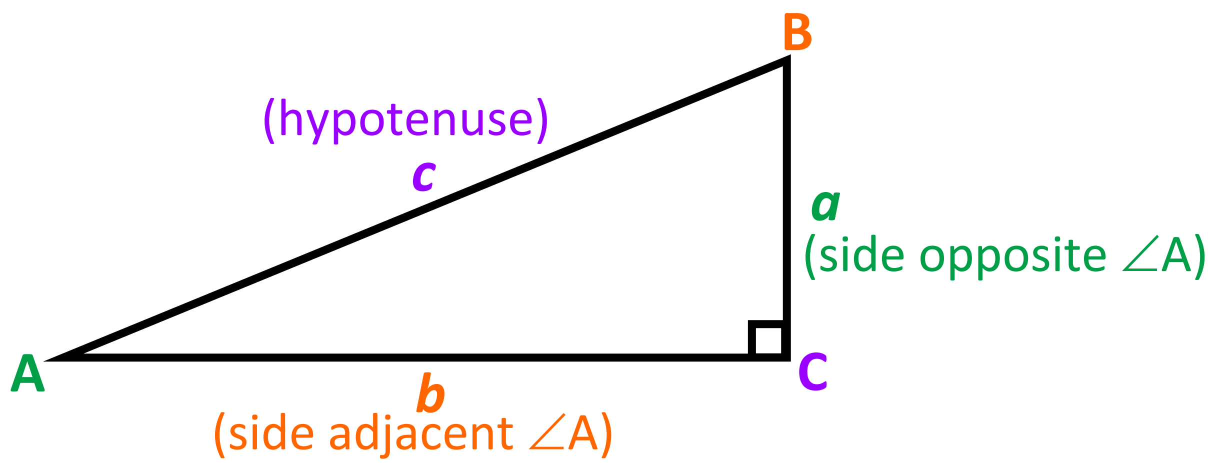
Given: leg = *x*,

then applying the Pythagorean Theorem;

$$\text{hypotenuse}^2 = x^2 + x^2$$

$$\text{hypotenuse} = x\sqrt{2}$$

Trigonometric Ratios

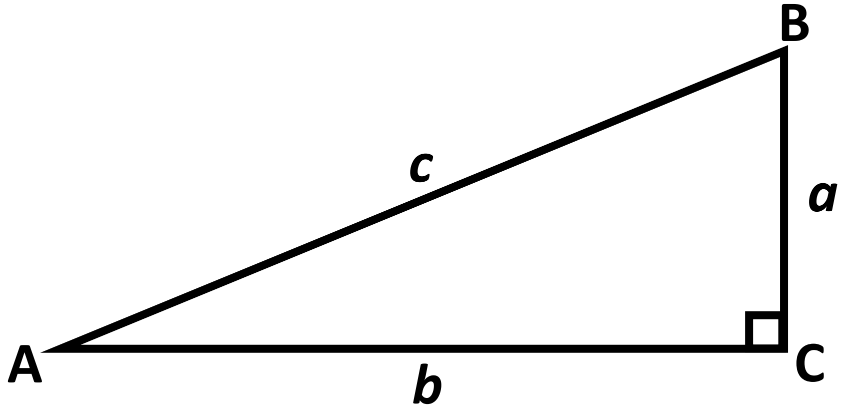


$$\sin A = \frac{\text{side opposite } \angle A}{\text{hypotenuse}} = \frac{a}{c}$$

$$\cos A = \frac{\text{side adjacent } \angle A}{\text{hypotenuse}} = \frac{b}{c}$$

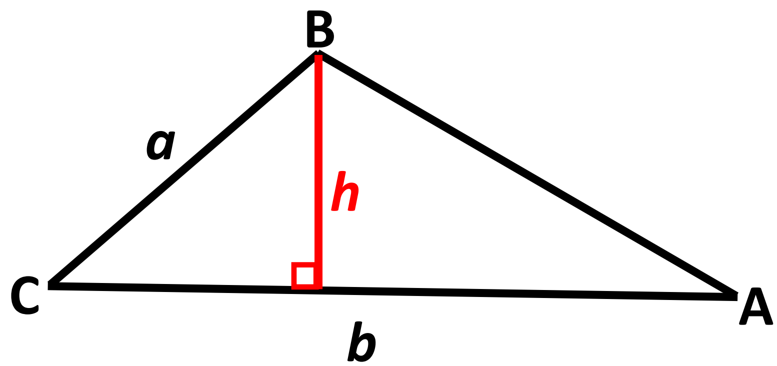
$$\tan A = \frac{\text{side opposite } \angle A}{\text{side adjacent to } \angle A} = \frac{a}{b}$$

Inverse Trigonometric Ratios



Definition	Example
If tan A = x, then $\tan^{-1} x = m\angle A$.	$\tan^{-1} \frac{a}{b} = m\angle A$
If sin A = y, then $\sin^{-1} y = m\angle A$.	$\sin^{-1} \frac{a}{c} = m\angle A$
If cos A = z, then $\cos^{-1} z = m\angle A$.	$\cos^{-1} \frac{b}{c} = m\angle A$

Area of a Triangle



$$\sin C = \frac{h}{a}$$

$$h = a \cdot \sin C$$

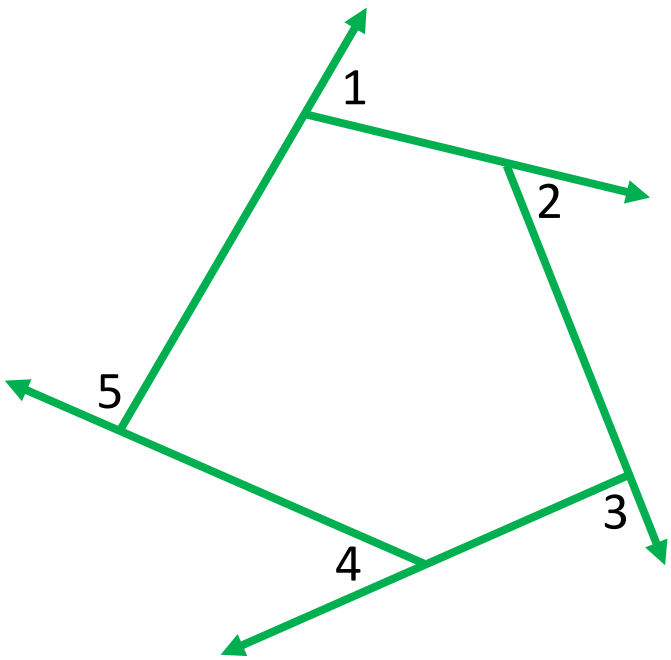
$$A = \frac{1}{2}bh$$ (area of a triangle formula)

By substitution, $A = \frac{1}{2}b(a \cdot \sin C)$

$$A = \frac{1}{2}ab \cdot \sin C$$

Polygon Exterior Angle Sum Theorem

The sum of the measures of the exterior angles of a convex polygon is 360°.



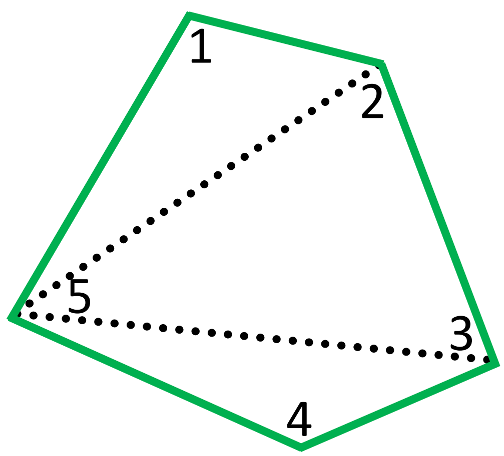
Example:

$m\angle 1 + m\angle 2 + m\angle 3 + m\angle 4 + m\angle 5 = 360^\circ$

Polygon Interior Angle Sum Theorem

The sum of the measures of the interior angles of a convex *n*-gon is $(n - 2)\cdot 180°$.

$$S = m\angle 1 + m\angle 2 + \ldots + m\angle n = (n - 2)\cdot 180°$$



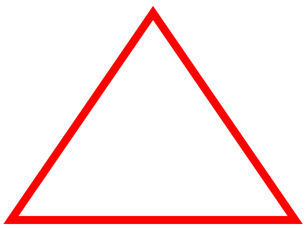
Example:

If $n = 5$, then $S = (5 - 2)\cdot 180°$

$S = 3 \cdot 180° = 540°$

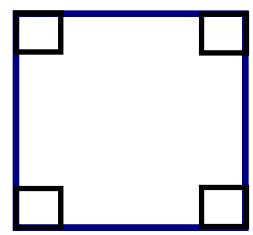
Regular Polygon

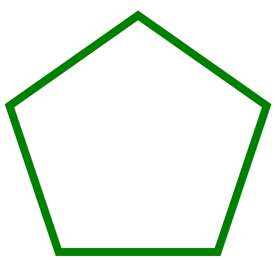
a convex polygon that is both equiangular and equilateral



Equilateral Triangle
Each angle measures 60°.

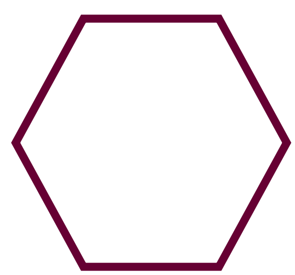
Square
Each angle measures 90°.

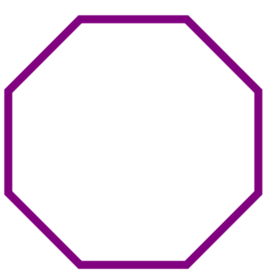




Regular Pentagon
Each angle measures 108°.

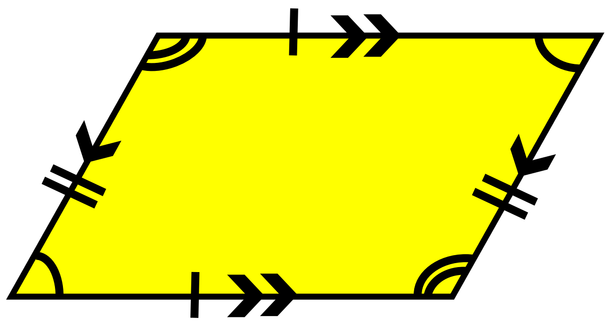
Regular Hexagon
Each angle measures 120°.



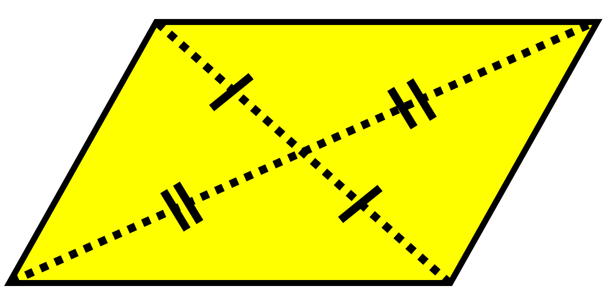


Regular Octagon
Each angle measures 135°.

Properties of Parallelograms

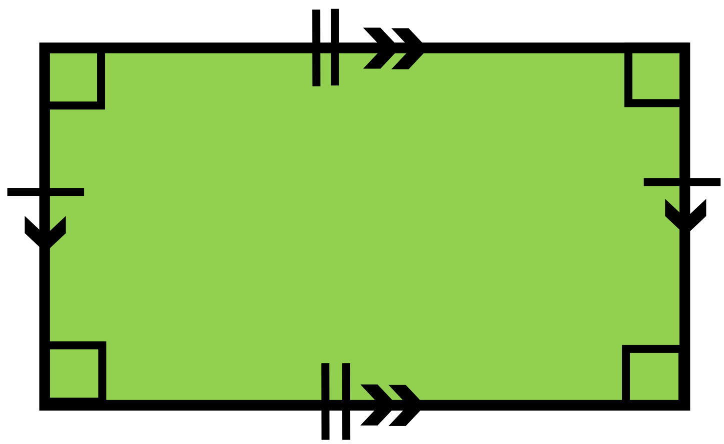


- Opposite sides are parallel.
- Opposite sides are congruent.
- Opposite angles are congruent.
- Consecutive angles are supplementary.
- The diagonals bisect each other.

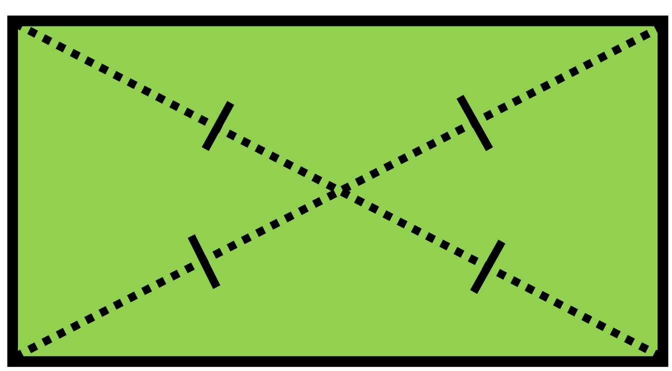


Rectangle

A parallelogram with four right angles

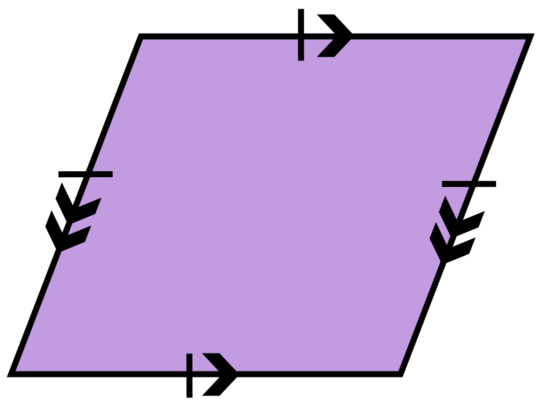


- Diagonals are congruent.
- Diagonals bisect each other.

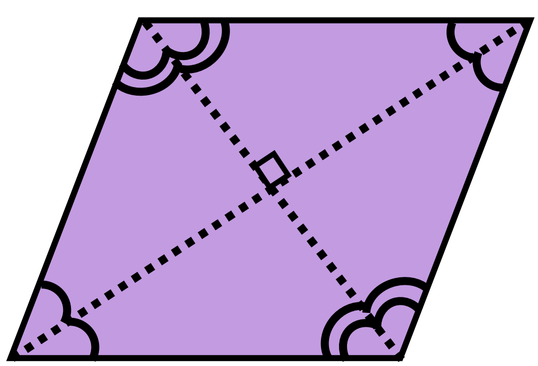


Rhombus

A parallelogram with four congruent sides

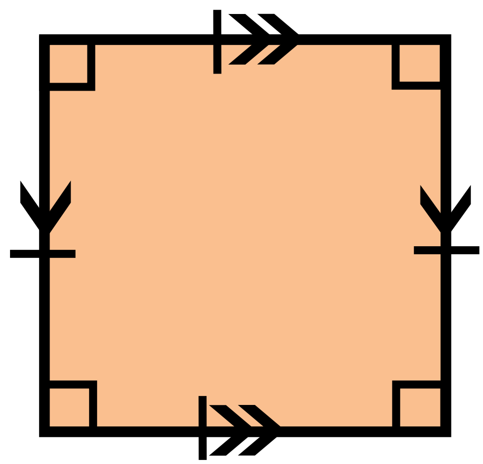


- Diagonals are perpendicular.
- Each diagonal bisects a pair of opposite angles.

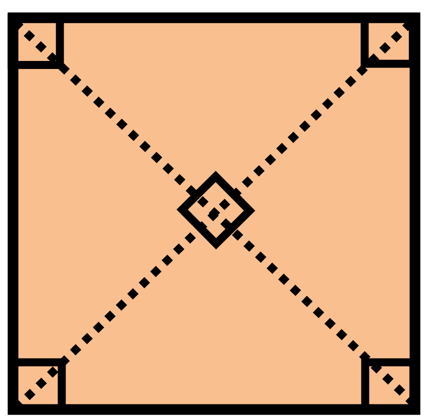


Square

A parallelogram and a rectangle with four congruent sides

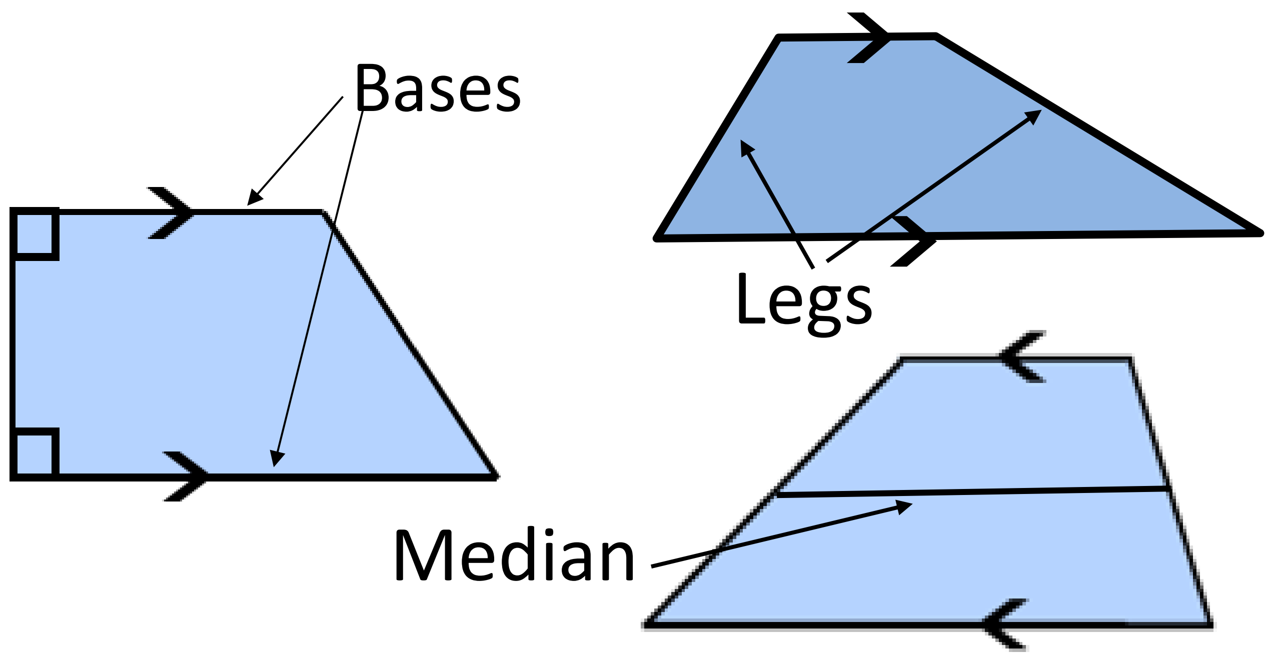


- Diagonals are perpendicular.
- Every square is a rhombus.



Trapezoid

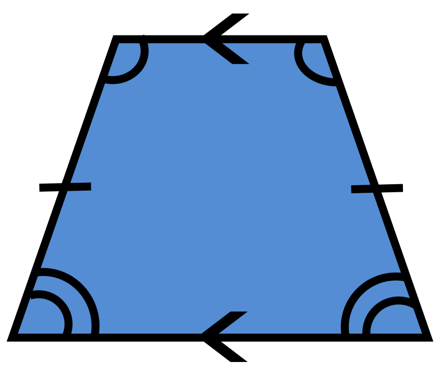
A quadrilateral with exactly one pair of parallel sides



- Two pairs of supplementary angles
- Median joins the midpoints of the nonparallel sides (legs)
- Length of median is half the sum of the lengths of the parallel sides (bases)

Isosceles Trapezoid

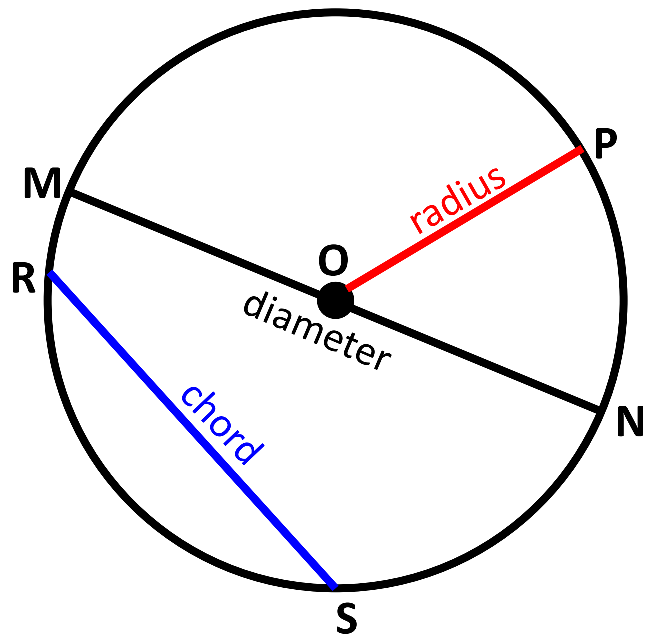
A quadrilateral where the two base angles are equal and therefore the sides opposite the base angles are also equal



- Legs are congruent
- Diagonals are congruent

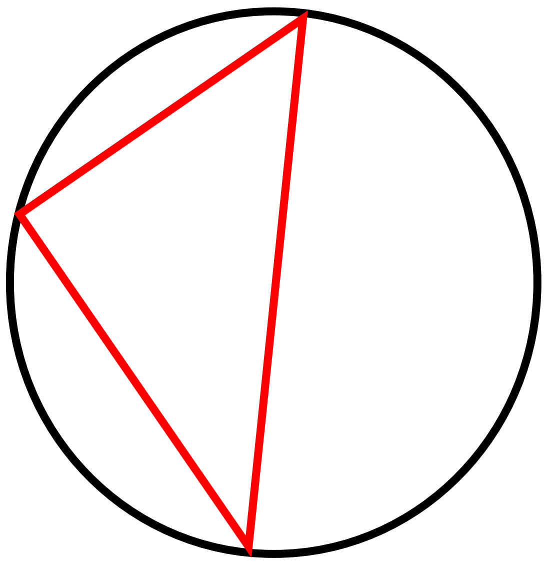
Circle

all points in a plane equidistant from a given point called the center



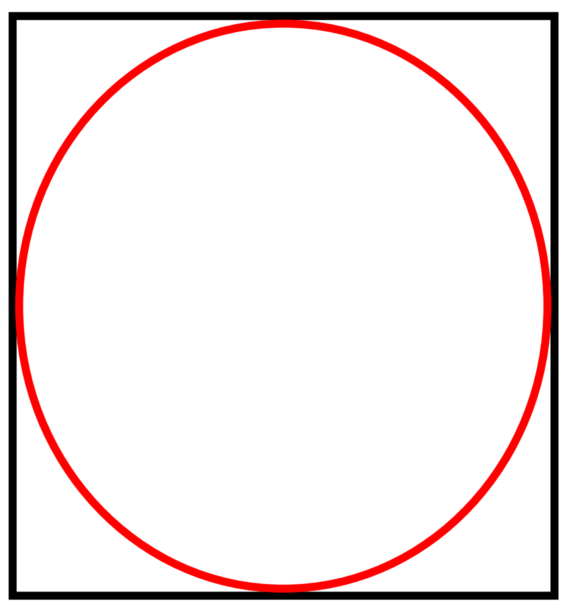
- Point O is the center.
- $\overline{MN}$ passes through the center O and therefore, $\overline{MN}$ is a diameter.
- $\overline{OP}$, $\overline{OM}$, and $\overline{ON}$ are radii and $\overline{OP} \cong \overline{OM} \cong \overline{ON}$.
- $\overline{RS}$ and $\overline{MN}$ are chords.

Circles

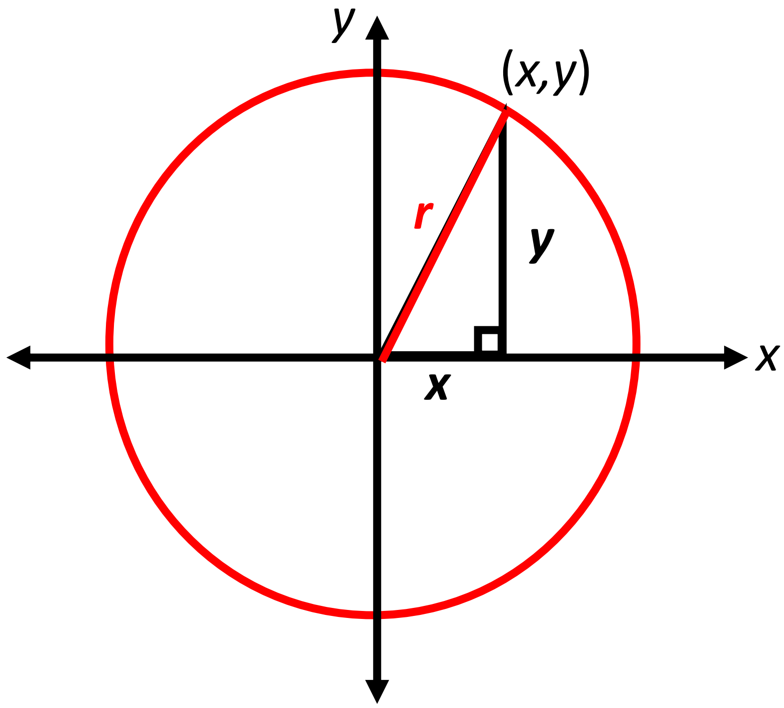


A polygon is an inscribed polygon if all of its vertices lie on a circle.

A circle is considered "inscribed" if it is tangent to each side of the polygon.



Circle Equation



$$x^2 + y^2 = r^2$$

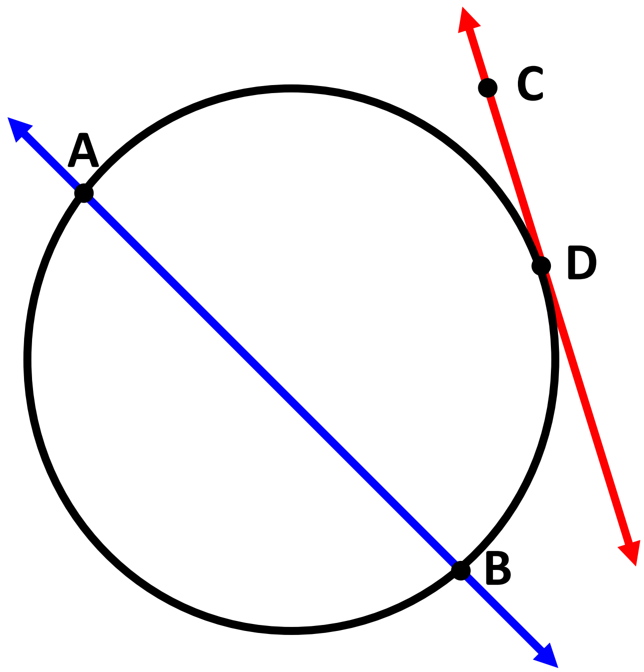
circle with radius r and center at the origin

standard equation of a circle

$$(x - h)^2 + (y - k)^2 = r^2$$

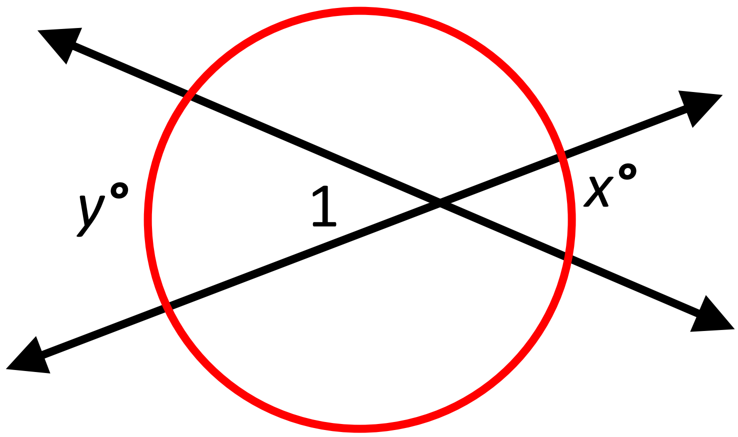
with center (h,k) and radius r

Lines and Circles



- Secant ($\overleftrightarrow{AB}$) – a line that intersects a circle in two points.
- Tangent ($\overleftrightarrow{CD}$) – a line (or ray or segment) that intersects a circle in exactly one point, the point of tangency, D.

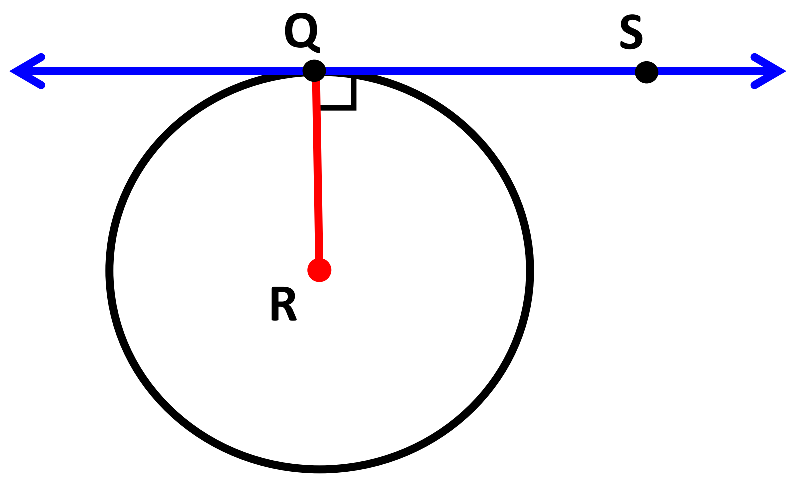
Secant



If two lines intersect in the interior of a circle, then the measure of the angle formed is one-half the sum of the measures of the intercepted arcs.

$$m\angle 1 = \frac{1}{2}(x^\circ + y^\circ)$$

Tangent

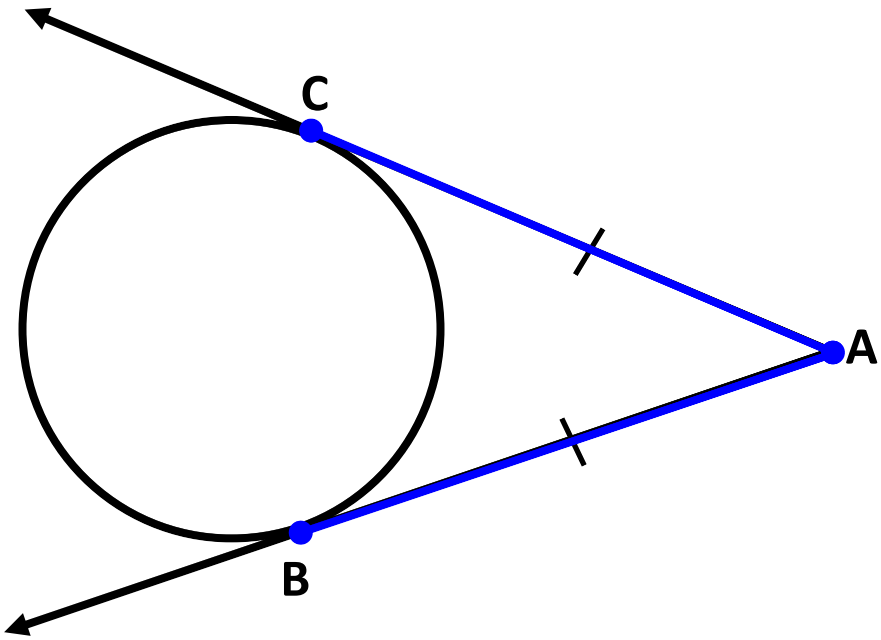


A line is tangent to a circle if and only if the line is perpendicular to a radius drawn to the point of tangency.

$\overleftrightarrow{QS}$ is tangent to circle R at point Q.

Radius $\overline{RQ} \perp \overleftrightarrow{QS}$

Tangent



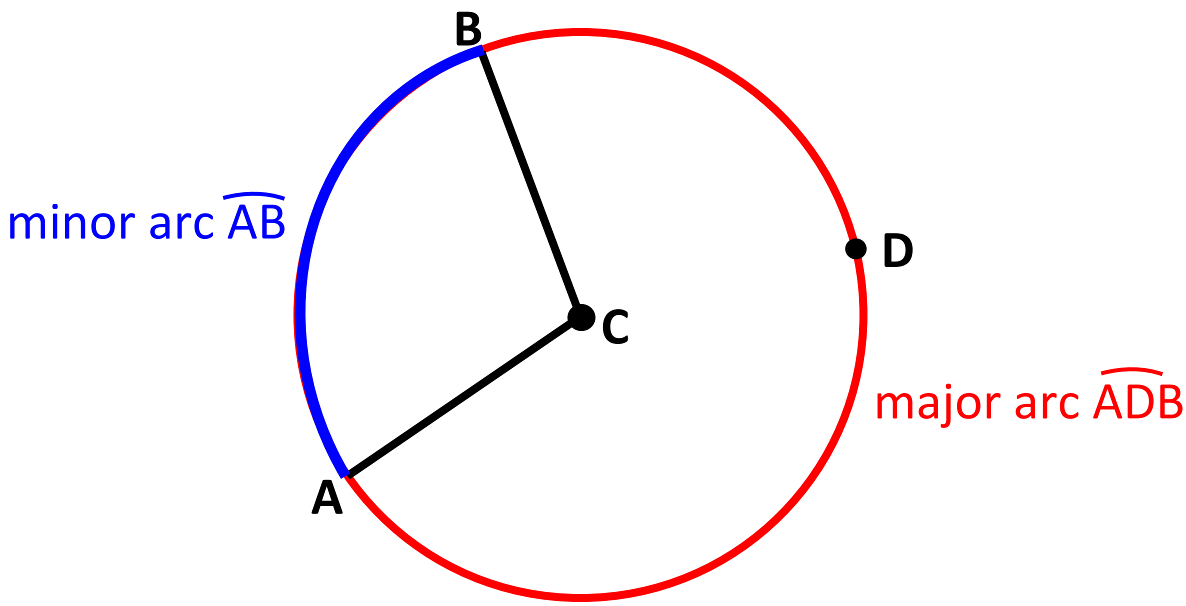
If two segments from the same exterior point are tangent to a circle, then they are congruent.

$\overline{AB}$ and $\overline{AC}$ are tangent to the circle at points B and C.

Therefore, $\overline{AB} \cong \overline{AC}$ and $AC = AB$.

Central Angle

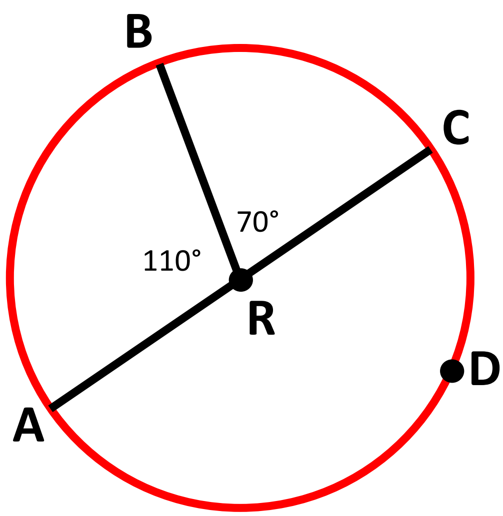
an angle whose vertex is the center of the circle



∠ACB is a central angle of circle C.

Minor arc – corresponding central angle is less than 180°
Major arc – corresponding central angle is greater than 180°

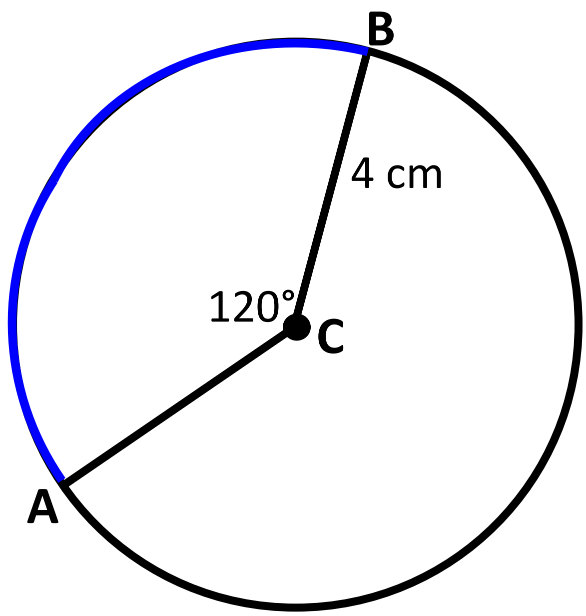
Measuring Arcs



Minor arcs	Major arcs	Semicircles
$m\overset{\frown}{AB} = 110°$	$m\overset{\frown}{BDA} = 250°$	$m\overset{\frown}{ADC} = 180°$
$m\overset{\frown}{BC} = 70°$	$m\overset{\frown}{BAC} = 290°$	$m\overset{\frown}{ABC} = 180°$

The measure of the entire circle is 360°.
The measure of a minor arc is equal to its central angle.
The measure of a major arc is the difference between 360° and the measure of the related minor arc.

Arc Length



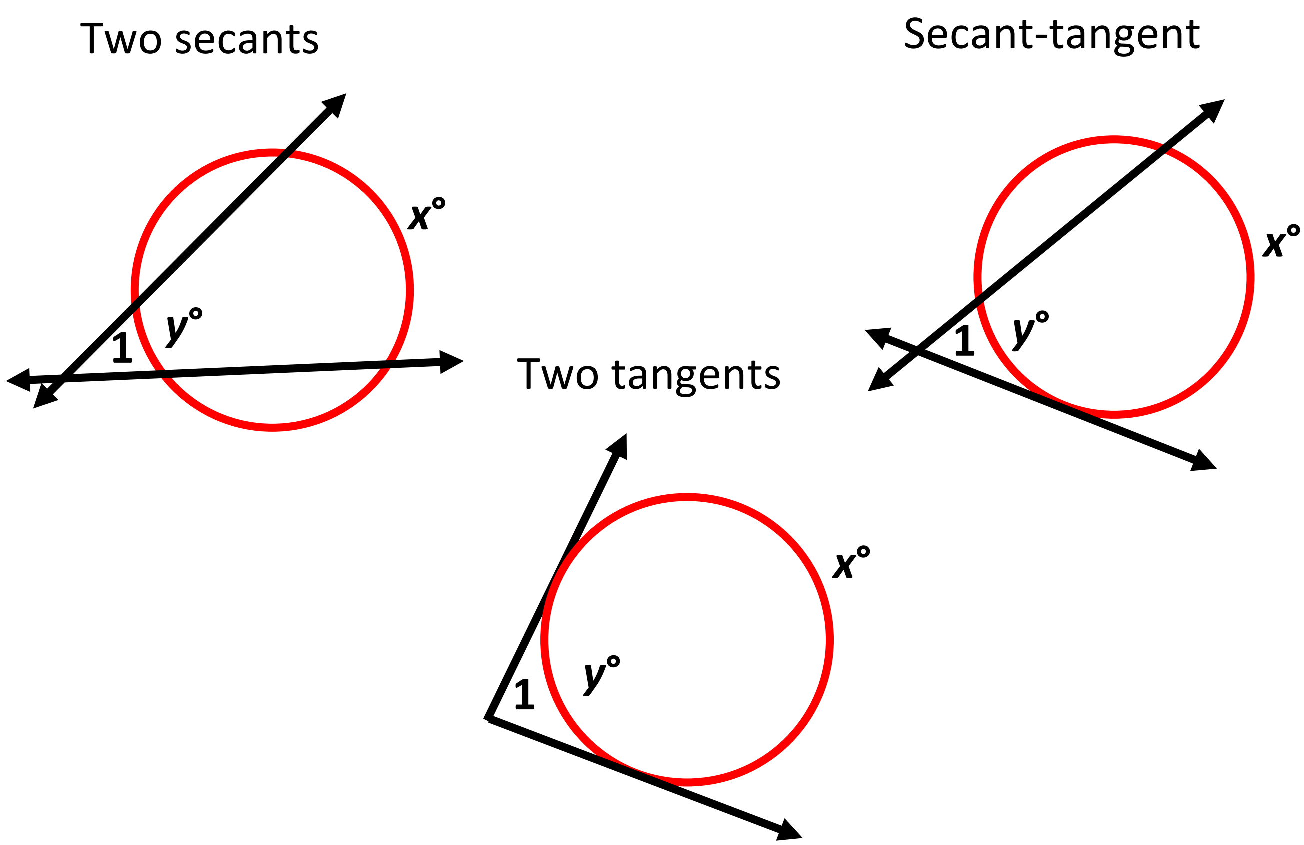
$$\frac{\text{arc length}}{2\pi r} = \frac{\text{central angle}}{360°}$$

Example:

$$\frac{\text{arc length of } \overset{\frown}{AB}}{2\pi \cdot 4} = \frac{120°}{360°}$$

$$\text{arc length of } \overset{\frown}{AB} = \frac{8}{3}\pi \text{ cm}$$

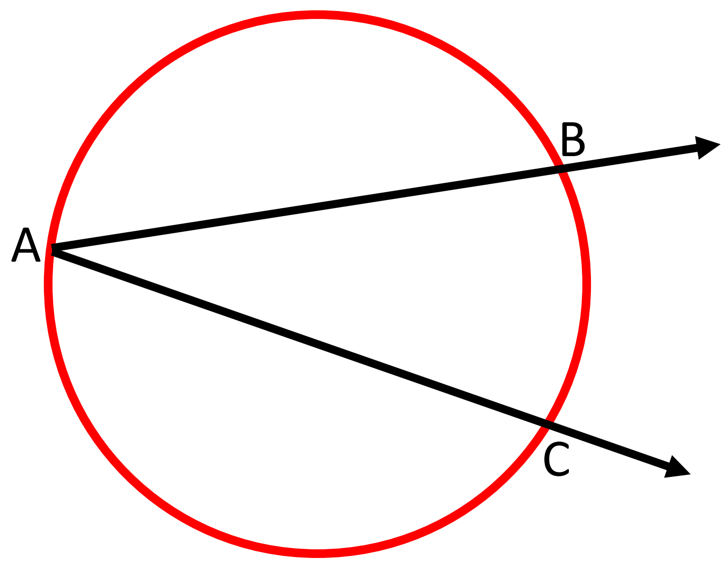
Secants and Tangents



$$m\angle 1 = \frac{1}{2}(x° - y°)$$

Inscribed Angle

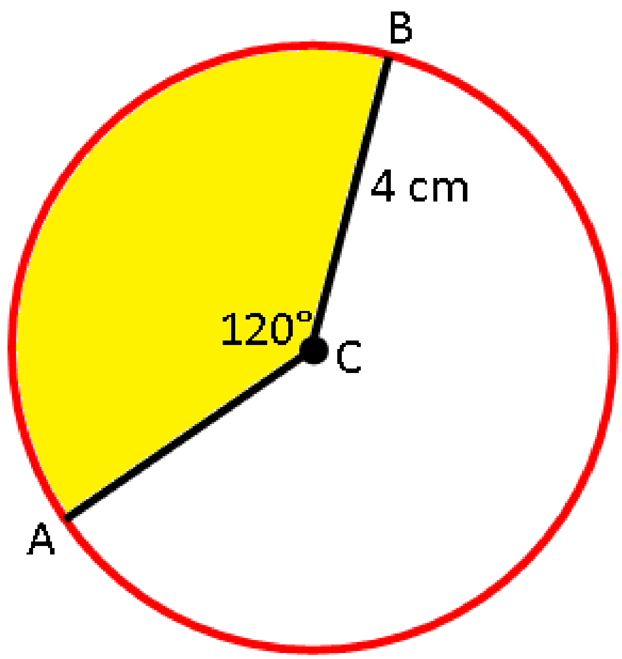
angle whose vertex is a point on the circle and whose sides contain chords of the circle



$$m\angle BAC = \frac{1}{2} m\overset{\frown}{BC}$$

Area of a Sector

region bounded by two radii and their intercepted arc



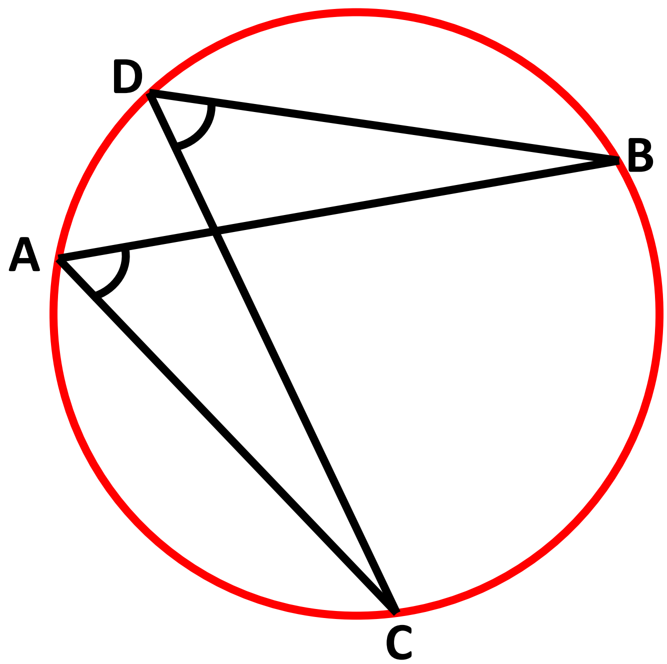
$$\frac{\text{area of sector}}{\pi r^2} = \frac{\text{measure of intercepted arc}}{360°}$$

Example:

$$\frac{\text{area of sector ACB}}{\pi \cdot 4^2} = \frac{120°}{360°}$$

$$\text{area of sector ACB} = \frac{16}{3}\pi \text{ cm}$$

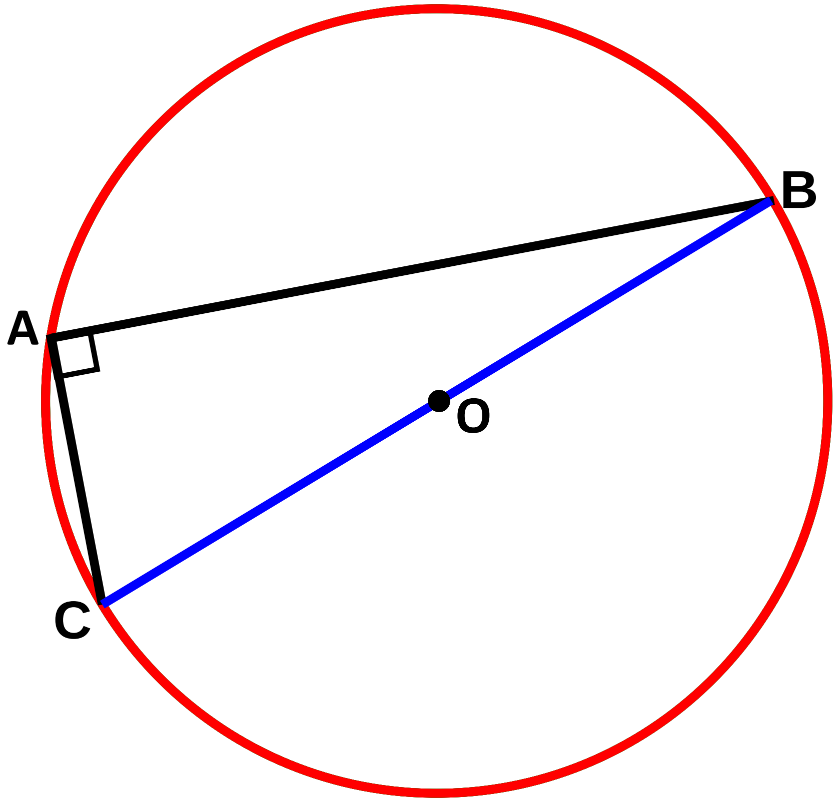
Inscribed Angle Theorem 1



If two inscribed angles of a circle intercept the same arc, then the angles are congruent.

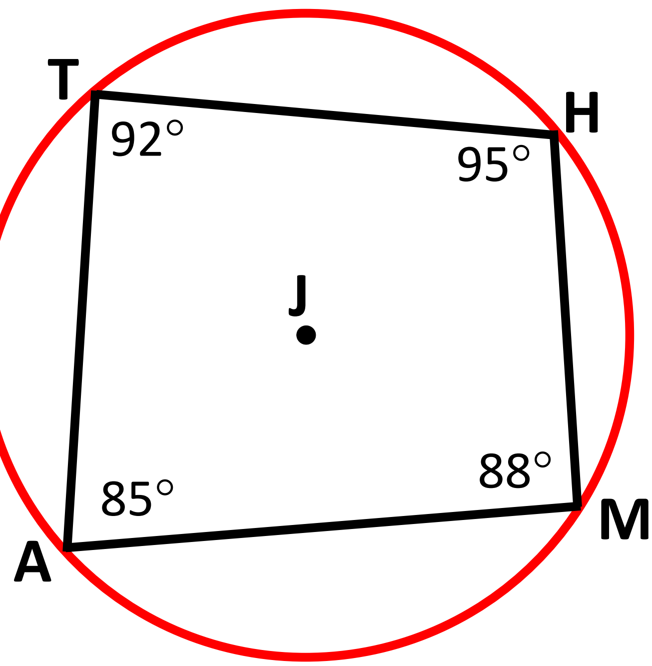
$\angle BDC \cong \angle BAC$

Inscribed Angle Theorem 2



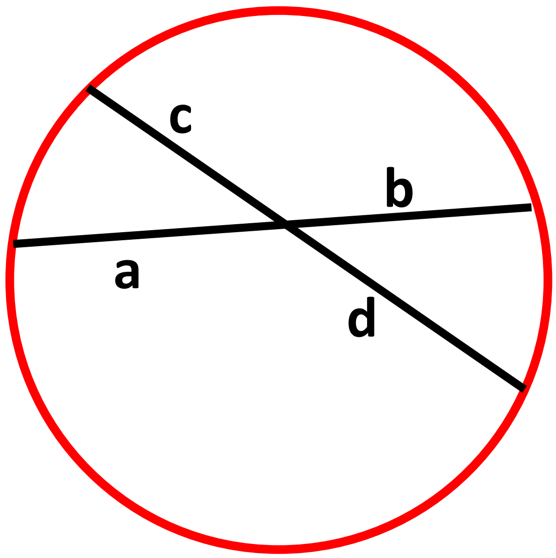
$m\angle BAC = 90°$ if and only if $\overline{BC}$ is a diameter of the circle.

Inscribed Angle Theorem 3



M, A, T, and H lie on circle J if and only if
$m\angle A + m\angle H = 180°$ and
$m\angle T + m\angle M = 180°$.
(opposite angles are supplementary)

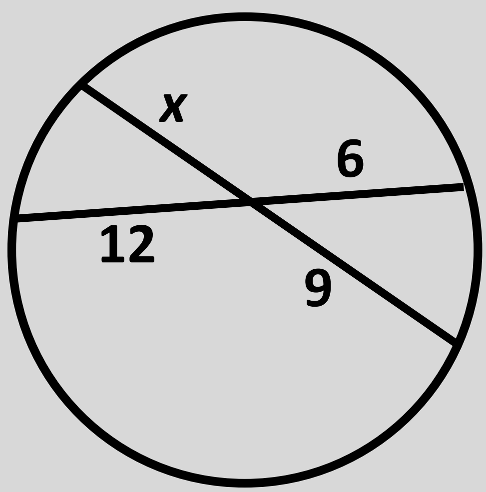
Segments in a Circle



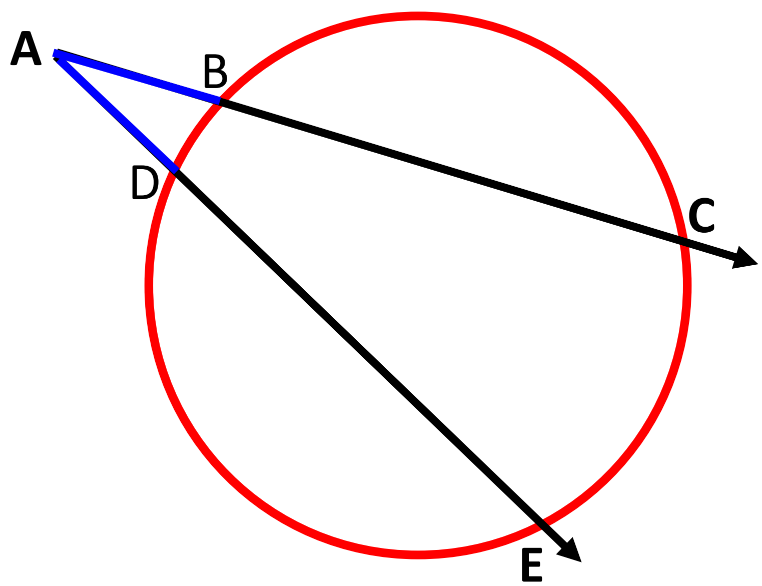
If two chords intersect in a circle, then a·b = c·d.

Example:

$$12(6) = 9x$$
$$72 = 9x$$
$$8 = x$$

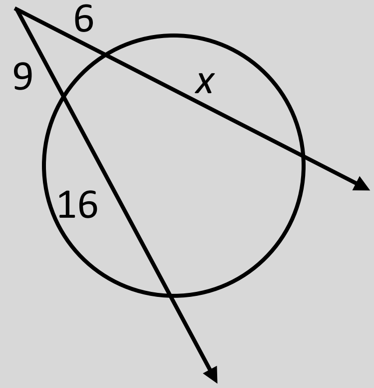


Segments of Secants Theorem



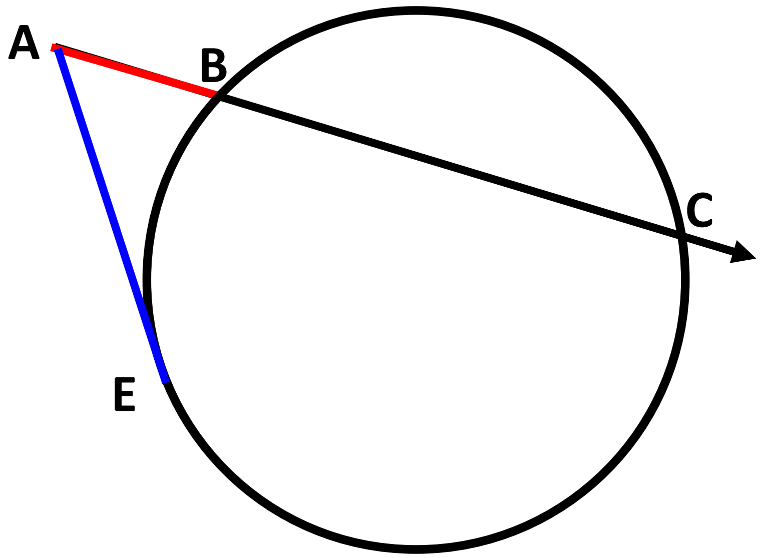
$$AB \cdot AC = AD \cdot AE$$

Example:



$$6(6 + x) = 9(9 + 16)$$
$$36 + 6x = 225$$
$$x = 31.5$$

Segments of Secants and Tangents Theorem



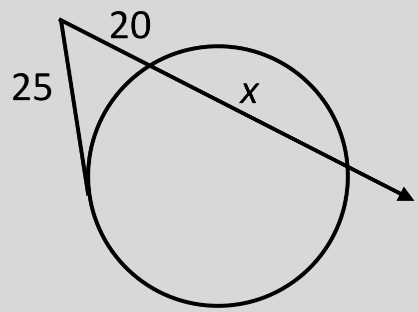
$AE^2 = AB \cdot AC$

Example:

$25^2 = 20(20 + x)$

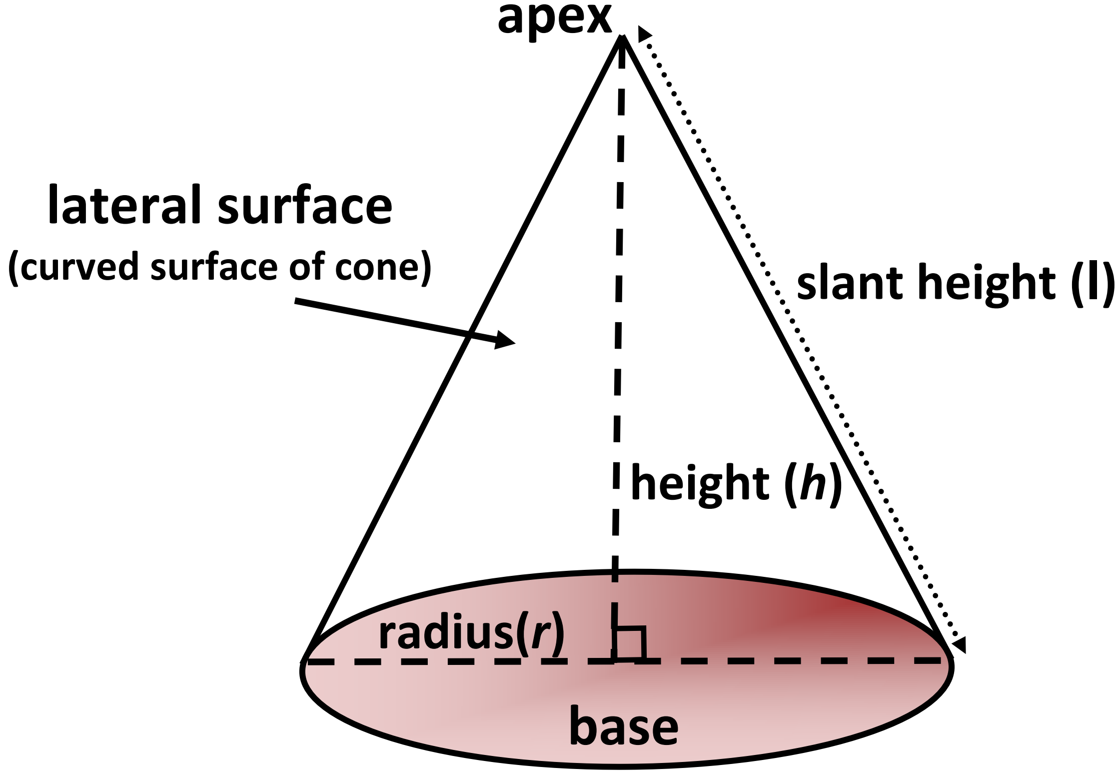
$625 = 400 + 20x$

$x = 11.25$



Cone

solid that has one circular base, an apex, and a lateral surface



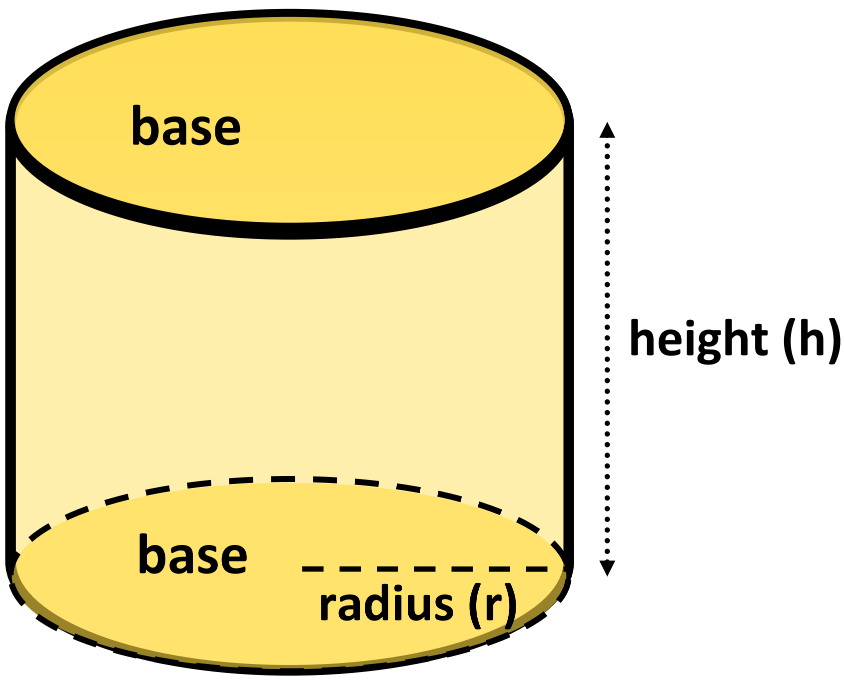
$$V = \frac{1}{3}\pi r^2 h$$

L.A. (lateral surface area) = $\pi r l$

S.A. (surface area) = $\pi r^2 + \pi r l$

Cylinder

solid figure with two congruent circular bases that lie in parallel planes



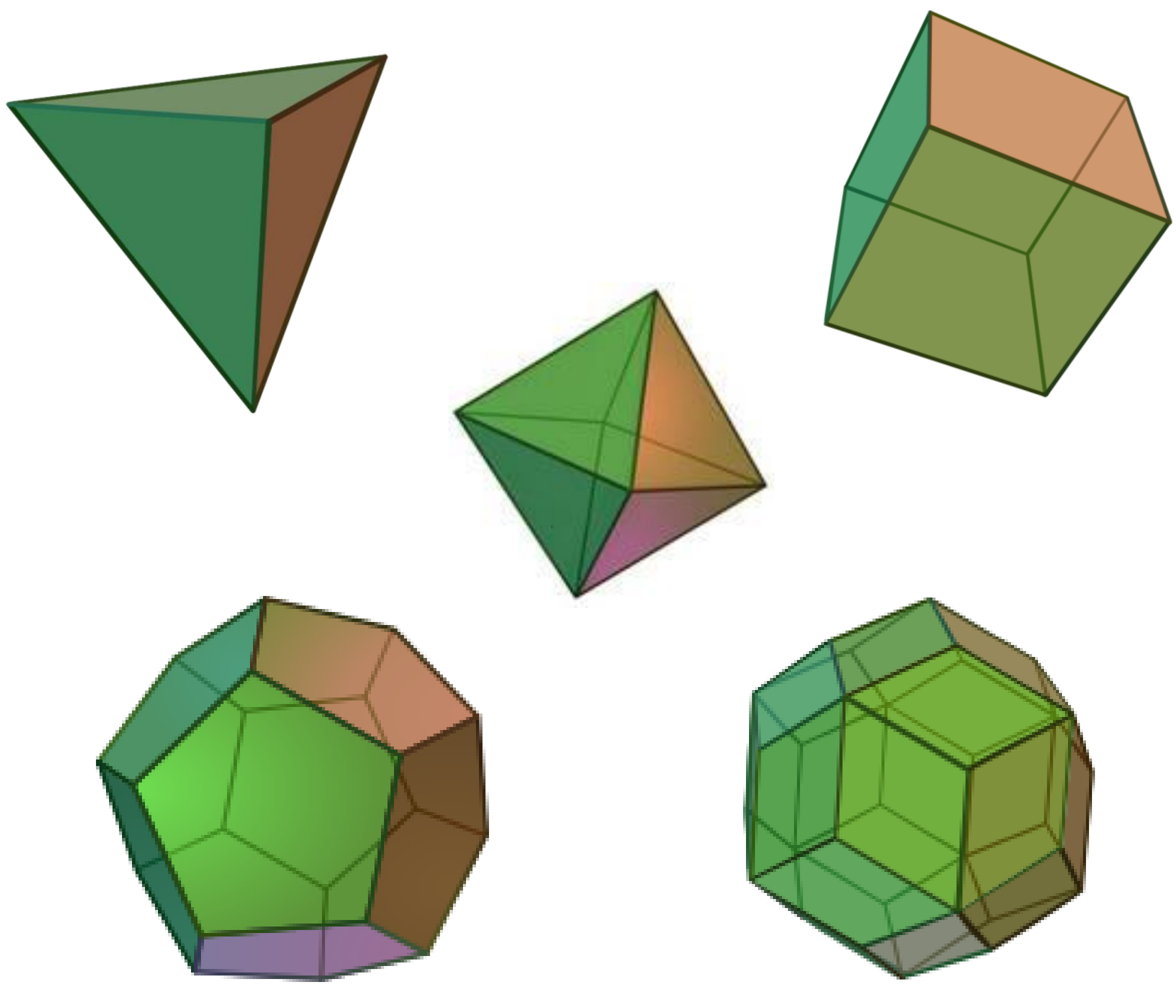
$$V = \pi r^2 h$$

L.A. (lateral surface area) $= 2\pi rh$

S.A. (surface area) $= 2\pi r^2 + 2\pi rh$

Polyhedron

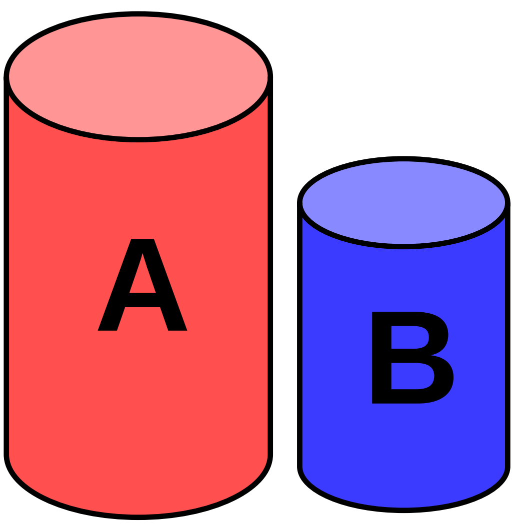
solid that is bounded by polygons, called faces



Similar Solids Theorem

If two similar solids have a scale factor of a:b, then their corresponding surface areas have a ratio of $a^2 : b^2$, and their corresponding volumes have a ratio of $a^3 : b^3$.

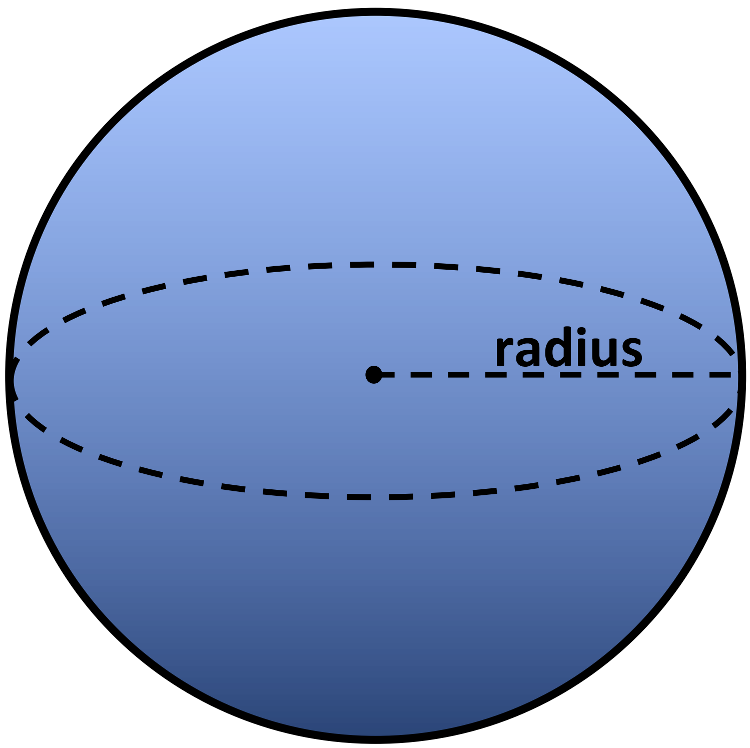
cylinder A ~ cylinder B



Example		
scale factor	a : b	3:2
ratio of surface areas	$a^2 : b^2$	9:4
ratio of volumes	$a^3 : b^3$	27:8

Sphere

a three-dimensional surface of which all points are equidistant from a fixed point

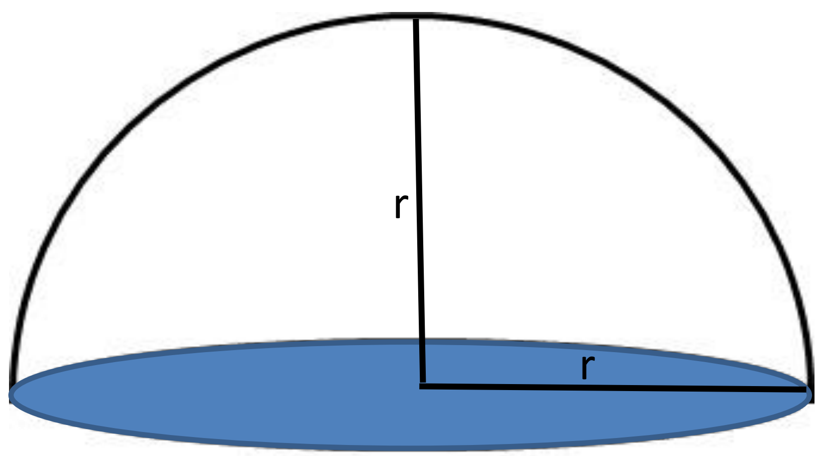


$$V = \frac{4}{3}\pi r^3$$

S.A. (surface area) $= 4\pi r^2$

Hemisphere

a solid that is half of a sphere with one flat, circular side

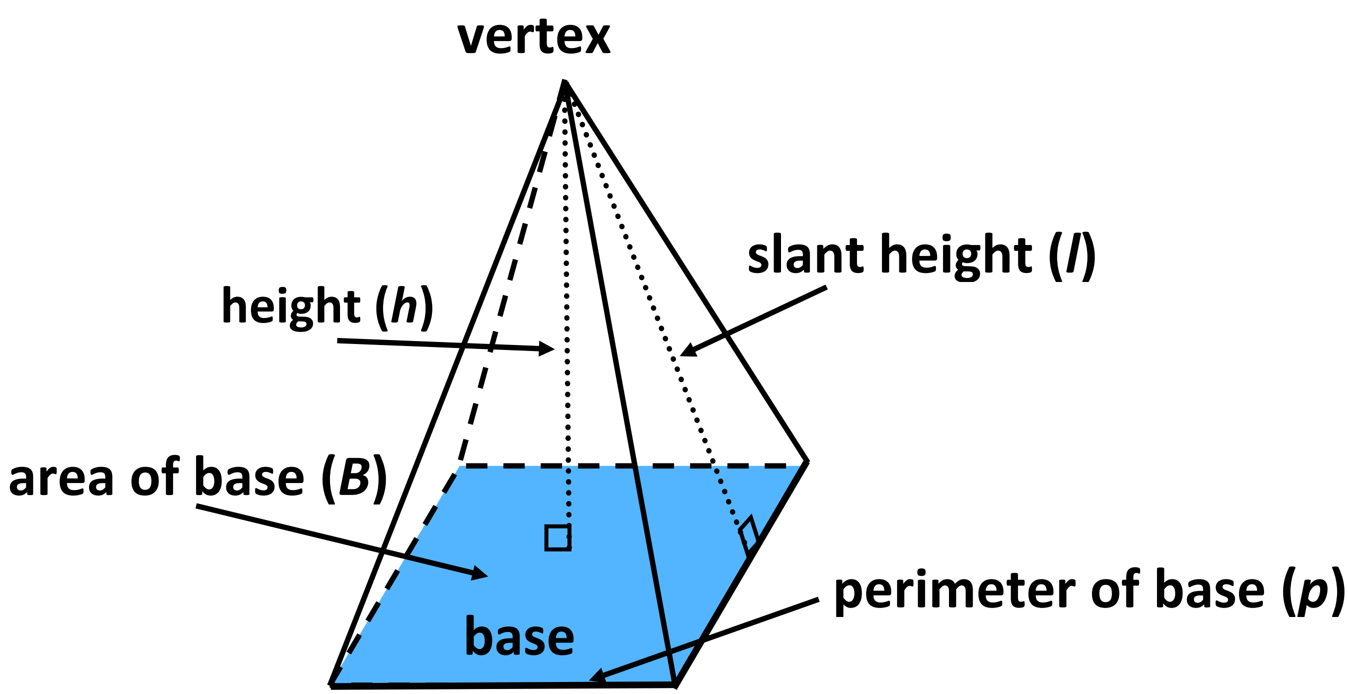


$$V = \frac{2}{3}\pi r^3$$

S.A. (surface area) = $3\pi r^2$

Pyramid

polyhedron with a polygonal base and triangular faces meeting in a common vertex



$$V \text{ (volume)} = \frac{1}{3}Bh$$

$$\text{L.A. (lateral surface area)} = \frac{1}{2}lp$$

$$\text{S.A. (surface area)} = \frac{1}{2}lp + B$$