Going the Distance

Strand:	Measurement
Topic:	Applying formulas to measure attributes of shapes
Primary SOL:	6.7 The student will

        a) derive pi;

        b) solve problems, including practical problems involving circumference and area of a circle.

Related SOL:	6.7c

Materials

- Scissors
- Yarn
- Rulers
- Calculators
- Unit squares
- Bubbles
- Construction paper (card stock optional)
- Different-sized Circles activity sheet (attached)
- Circling Measures graphic organizer (attached)
- Bubbles Galore activity sheet (attached)
- Missing Measures and Practical Applications activity sheet (attached)

Vocabulary

area of rectangular figure, chord, circumference of a circle, diameter, perimeter of rectangular figure, volume of rectangular figure, radius (earlier grades)

pi (π), surface area (6.7)

Student/Teacher Actions: What should students be doing? What should teachers be doing?

Note: Per SOL 5.10, students in the fifth grade have had the opportunity to identify and describe the concepts of diameter, radius, chord, and circumference of a circle. Students have learned that circumference is the distance around, or "perimeter" of, a circle; an approximation for circumference is about three times the diameter of a circle; and, an approximation for circumference is about six times the radius of a circle. Students also learned to investigate and describe the relationship between (a) diameter and radius; (b) diameter and chord; (c) radius and circumference; and, (d) diameter and circumference. In the sixth grade, students are expected to derive pi (π) and connect and apply their understanding of circumference and area of a circle using the following formulas:

- $C = 2\pi r$ or $C = 2d$, where C is the circumference, d is the diameter, and r is the radius of the circle.
- $A = \pi r^2$, where A is the area and r is the radius of the circle.

1. Open the discussion by asking students for the definitions of *circumference, chord, diameter,* and *radius*. Tell students that they will now investigate how the circumference of a circle compares to the circle's diameter (and radius).

2. Distribute scissors and copies of the Different-sized Circles handout. Have students cut out the circles.

3. Give each student a ruler, a 3-foot length of yarn, and a copy of the Circling Measures graphic organizer. Direct students to use the yarn to measure the distance around each circle, cutting the exact length of yarn needed for each circle. Then, have students use the ruler to measure the length of each piece of yarn. Instruct them to record each measurement in the chart under "Length of Yarn." Emphasize that this is the circumference of each circle.

4. Have students fold each circle at some point, but *not* in half. Share with students that the line created is a chord, because two endpoints both lie on the circle. Next, have students fold each circle in half, crease it, unfold it, and draw a line along the crease. Direct students to use their rulers to measure the length of this line across the center of each circle and record each measurement in the chart under "Length of Line." Emphasize that this is the diameter of each circle and that a diameter is also a chord.

5. Have students divide the diameter of each circle in half and record each value under "Length of Line Divided by 2." Emphasize that this is the radius of each circle.

6. Have students divide Length of Yarn by Length of Line for each circle and record each value under $\frac{\text{Length of Yarn}}{\text{Length of Line}}$. Advise students that they will determine if there is a relationship between the length of yarn (circumference) and the length of line (diameter)—that is, the ratio of the circumference of a circle to its diameter. Ask them what they observe about the circumference divided by the diameter of each circle. They should notice that each ratio is the whole number 3 followed by different numbers in the decimal places. Point out that they have discovered that <u>the circumference of a circle is a little more than three times larger than the diameter of the same circle</u>.

7. Display the formula for circumference, $C = 2\pi r$, and explain each aspect of it as follows:
   - C = circumference (length of yarn)
   - π or pi = the ratio of the circumference of a circle to its diameter (ratio of length of yarn to length of line, or length of yarn divided by length of line)
   - $2r$ = radius multiplied by 2, which is the diameter (length of line)

8. Distribute calculators. In the sixth (blank) column on the Circling Measures graphic organizer, have students write, $C = 2\pi r$, in the heading box. Then, have them use calculators to find the exact circumference of each circle by substituting the known values into the formula and performing the indicated operation. Discuss with students how their results could differ slightly when using either the π button located on the calculator to arrive at their solutions or using the approximations for pi (3.14 or $\frac{22}{7}$).

9. Next, have students use unit squares to fill in each circle without going beyond the edges. This will enable them to estimate the area of each circle. Considering that a square does not accommodate rounded edges, point out to students that they will have to estimate
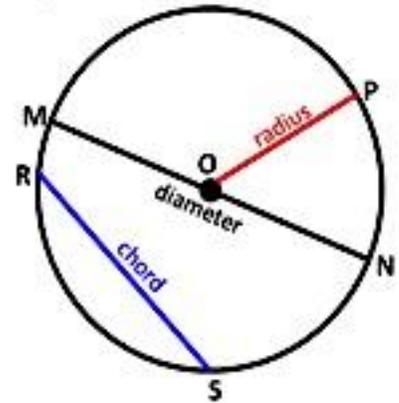
the amount of some of the squares being used. Share with students that the area of a closed curve is the number of non-overlapping square units required to fill the regions enclosed by the curve.

10. Suggestions would be to overlay graph paper over a given circle, use small linking cubes, or dynamic geometry software to simulate square units. Remember to make connections to how this process is similar when finding the area of squares, rectangles, and other shapes. Discuss the similarities between students' findings using the concrete and pictorial representations.

11. After students have completed their estimates of the area of each circle, introduce the formula for the area of a circle, $A = \pi r^2$. In the seventh (blank) column on the Circling Measures graphic organizer, have students write $A = \pi r^2$ in the heading box. Then, have them use calculators to find the exact area of each circle by substituting the known values into the formula and performing the indicated operations.

12. Have students share how their estimated areas, derived by filling the circles with unit squares, compare with the exact areas calculated by using the formula.

13. Circumference Focus [Part I]: Distribute the Bubbles Galore activity sheet. Students will blow bubbles and catch one on their construction paper (or card stock). When the bubble bursts, it will leave a dark circle from the liquid. Only direct students to answer the questions on the handout from Part I. Engage the students in a discussion of their findings.

14. Area Focus [Part II]: Return to the Bubbles Galore activity sheet. Now have students calculate the area of the same circle from which they determined the circumference. Engage students in a discussion of their findings.

15. Summarize [Part III]: Allow students to discuss the relationship between the area and the circumference of their bubbles. Use this time to also find out who had the least and the greatest circumference and area.

16. Place students with a partner or in small groups to complete the Missing Measures and Practical Applications activity sheet. Allow partners or small groups to report their findings, justify their reasoning, and create their own practical application problems using the values that they found for either diameter, radius, circumference, or area.

Assessment

- **Questions**
  - What is the term for the distance around a circle?
  - What is the relationship between the diameter and the radius of a circle?
  - What is the relationship between the circumference of a circle and its diameter or radius?
  - How can the approximation for pi (π) be derived?

- **Journal/writing prompts**
  - Explain how the yarn was used to measure the circumference of circles.
  - Given the proportional relationship between circumference and diameter, explain why $\frac{C}{d} = \pi$. Would the results of this proportional relationship be the same as $\frac{C}{2r} = \pi$? Explain why or why not.

- **Other Assessments**
    - Have students to bring in circular objects and measure the distance around (circumference) and across (diameter) of each object. Engage in a discussion regarding who has the largest circumference and area (or smallest circumference and area). Allow students to work in small groups to derive pi (π) as well as to compare the circumference to the diameter. Allow the students to exchange their items to verify each other's work.
    - Ask students to create lengths of radii such that the area will be larger than the circumference or area will be smaller than the circumference.

Extensions and Connections

- Neuschwander, C. (1999). *Sir Cumference and the Dragon of Pi: A Math Adventure.* Watertown, MA: Charlesbridge Publishing Inc.

Follow the adventures of Radius as he seeks to discover the dosage of the potion that can turn his father, Sir Cumference, from a dragon into his human form. Use "The Circle's Measure" poem during guided instruction:

> Measure the middle and circle around,
> Divide so a number can be found,
> Every circle, great and small –
> The number is the same for all.
> It's also the dose, so be clever,
> Or a dragon he will stay … forever. (p. 13)

Have students use the given examples of an onion slice, a basket, a bowl, a round cheese, and their respective measurements to find correct the dosage (derive pi [π]) (p. 21).

- After students complete the Bubbles Galore handout, allow them to find the mean, median, and mode of the derivation of pi (π), the circumference, and the area values of data collected from the class.
- Ask students to create lengths of radii such that the area will be larger than the circumference or the area will be smaller than the circumference.

Strategies for Differentiation

- Provide students with pieces of yarn already cut to the length of the circumference of each circle.
- Demonstrate the process of using yarn to measure the circumference of one of the circles.
- Demonstrate the calculations for one of the circles.
- Clue students to recognize the relationships between the various values in the chart.
- Assign each student a partner for collaboration during all learning activities.
- Review relevant vocabulary with some students as necessary before introducing the lesson.

Note: The following pages are intended for classroom use for students as a visual aid to learning.

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Different-sized Circles

Circle 1

Circle 2
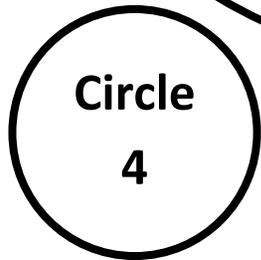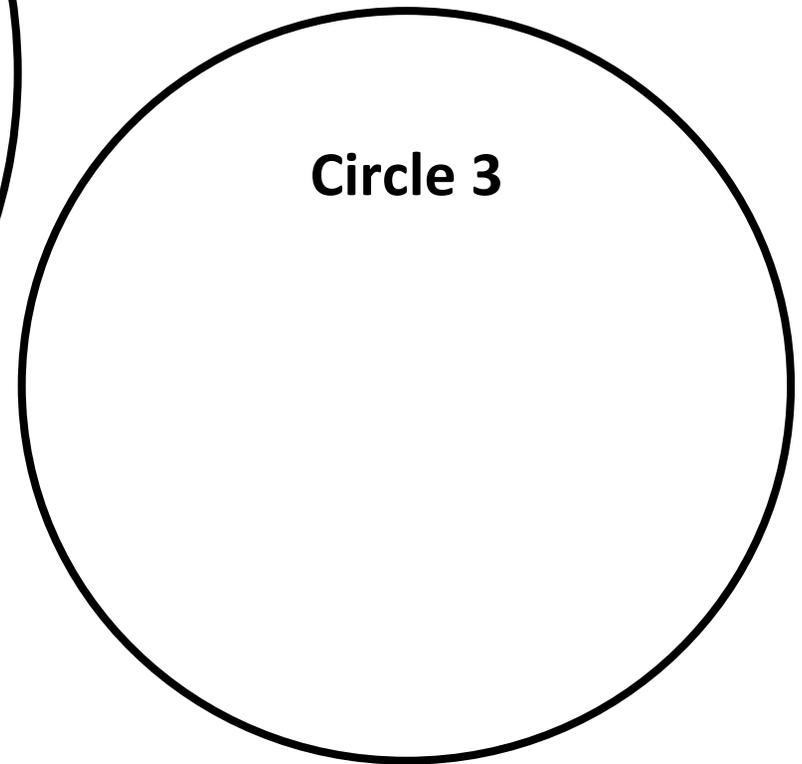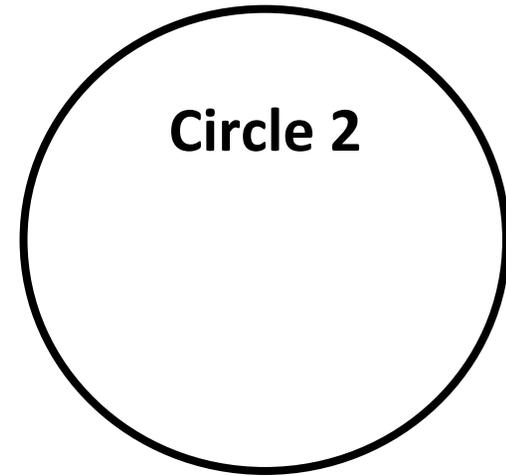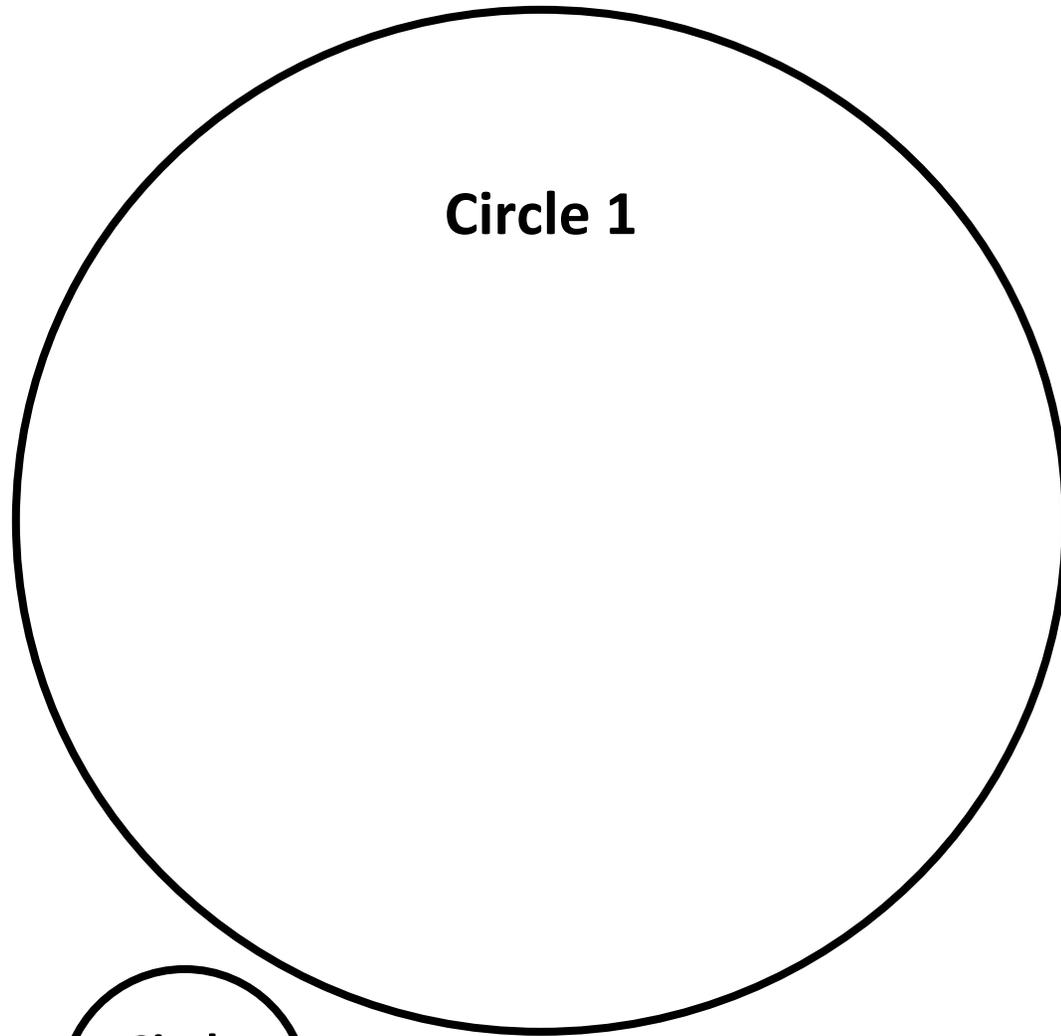
Circle 3

Circle 4

Circling Measures

Name _____ **Date** _____

Circle	Length of Yarn (Circum-ference)	Length of Line (Diameter)	Length of Line Divided by 2 (Radius)	Error! (pi)		
1						
2						
3						
4						

Bubbles Galore

Name _____ **Date** _____

Directions: Use the construction paper (or card stock) to catch a bubble. Use your ruler to measure (centimeters). When the bubble bursts, it will leave a dark circle from the liquid. Answer the questions below.

Circumference Focus (Part I):
1. What is the measure of your bubble's diameter? Describe how you found the diameter.

2. What is the measure of your bubble's radius? Describe how you found or calculated the radius.

3. Create a chord other than the diameter. What is the measure of the chord?

4. What is the measure of your bubble's circumference? Apply the appropriate formula and demonstrate your work. Explain how you arrived at your solution.

5. Given the proportional relationship between circumference and diameter, explain why $\frac{C}{d} = \pi$.

    Would the results of this proportional relationship be the same as $\frac{C}{2r} = \pi$? Explain why or why not.

Area Focus (Part II):
6. What is the measure of your bubble's area? Apply the appropriate formula and demonstrate your work. Explain how you arrived at your solution.

7. Which is greater, the circumference or area of your circle?

Summarize (Part III):
8. Is there a relationship between the circumference and the area of your circle? Explain and justify your reasoning.

Missing Measures and Practical Applications

Name _____ **Date** _____

Directions: Find the missing measure and apply appropriate units. Round all answers to the *nearest hundredth*. Demonstrate all work below and provide a brief description of your process for solving. Then, create a practical application problem using the values that you found and share it with a partner or with your small group.

Item Number	Missing Measures	Process Description	Practical Application Problem
1	$d = \frac{3}{4}$ in, $r =$ _____, $C =$ _____		
2			The top of Nicole's water bottle is a circle. The radius of the circle measures 4.52 cm. What is the diameter and the circumference of Nicole's water bottle top?
3	$d =$ _____, $r =$ _____, $C =$ 24 in		
4			A circular field measures 74 meters through the center, connecting two points on the circumference. What is the radius and the area of the field?
5	$d = 3.8$ cm, $r =$ _____, $A =$ _____		