Divide Fractions and Mixed Numbers

Strand: Computation and Estimation

Topic: Multiply and divide fractions (proper and improper) and mixed numbers.

Primary SOL: 6.5 The student will
b) multiply and divide fractions and mixed numbers.*

Materials

- Fraction strips
- Fraction rods
- Pattern blocks
- Paper

Vocabulary

array, area, denominator, factor, fraction, mixed number, multiplicative inverse (6.5), *numerator, product, quotient, simplest form, whole number* (earlier grades)

Student/Teacher Actions: What should students be doing? What should teachers be doing?

After working with students on multiplication, students should be able to easily transition to division of fractions. As with multiplication, help students connect the concept of fraction division to prior knowledge. Students should understand that there are two division meanings: measurement and partitioning.

1. When helping students recall the idea of division as measurement, you will want to include the ideas of repeated subtraction of equal groups. Ask students to solve the following problem with a partner: “If you have 64 ounces of soda, how many cups, each holding 8 ounces, can you fill?” When discussing this with students, talk about how this is a repeated-subtraction situation. Talking about the measurement model will be a good way for students to begin to develop the algorithm, so it is important to not skip this part of the conceptual understanding process.
2. Ask students how they can use the repeated-subtraction model to solve the following problem, “A serving of pizza is $\frac{1}{4}$ of the pizza. How many servings can I make from 2 pizzas?”
3. Next, have students solve the following: “A serving of pizza is $\frac{1}{4}$ of the pizza. How many servings can I make from $\frac{3}{4}$ of the pizza?” Ask, *“How you can have a whole-number quotient when you are working with fractions.”* During this time, when working with fractional pieces, it might be useful to allow students to use fraction rods or strips.

4. Next, have students solve the following: "A serving of pizza is $\frac{1}{4}$ of the pizza. How many servings can I make from $\frac{3}{8}$ of the pizza?" Students might want to answer $1\frac{1}{8}$, but it is important to remind students that a serving size is $\frac{1}{4}$, so $\frac{1}{8}$ is $\frac{1}{2}$ of $\frac{1}{4}$.
5. Ask students to work with a partner to solve the following: "You are going to have a Thanksgiving dinner, so you order 6 miniature pumpkin pies. If you are going to serve $\frac{3}{4}$ of a pie to each guest, how many guests can be served?" Have students share their strategies and answers with the class.
6. Students should also understand the partitioning method of division, which is the idea that you are sharing part of a whole. You can start by having students answer questions like, "How many minutes in an hour?" or "If you have 20 candy bars that you are sharing with 10 of your friends, how many does each receive?"
7. Begin introducing this with fractions divided by whole numbers. Students will be able to answer the questions, "How much is the whole?" or "How much for one?" Some good questions to ask students would be, "Jay has $1\frac{3}{4}$ of an hour left to complete his last four homework assignments. If he wants to give each assignment the same amount of time, how many hours can he give each assignment?" or "Ronaldo has $5\frac{1}{4}$ feet of a piece of string to make seven bracelets. How much string should he use for each bracelet if he wants to use the same amount of string for each?" In the first problem, students must partition the whole into smaller pieces than what is originally given. The first problem will require students to use sixteenths, and the second problem will require students to use the originally given partitioning: fourths. Using fraction bars or strips will be a good method for students to solve these types of problems.
8. In order to develop the algorithm, which should follow the conceptual understanding, for fraction division, one way is to find a common denominator and then divide the numerators. For example, if the problem is $\frac{1}{2} \div \frac{1}{3} = \frac{3}{6} \div \frac{2}{6} = \frac{3}{2}$. Why does this work? It is the same idea as whole-number operations, but you now can ask yourself, "How many sets of $\frac{2}{6}$ can you get in $\frac{3}{6}$?" You can get one whole $\frac{2}{6}$ and $\frac{1}{2}$ of a $\frac{2}{6}$, which equals $\frac{3}{2}$, or $1\frac{1}{2}$.
9. Another way to develop the more-traditional algorithm is to begin by having students look for patterns. Give the following problems and then ask the connected questions (keep a table of the responses):
   a. $4 \div \frac{1}{2}$ = (How many servings of $\frac{1}{2}$ in 4 containers?)
   b. $12 \div \frac{2}{3}$ = (How many servings of $\frac{2}{3}$ in 12 containers?)
   c. $9 \div \frac{3}{4}$ = (How many servings of $\frac{3}{4}$ in 9 containers?)

d. $2\frac{1}{4} \div \frac{1}{8}$ = (How many servings of $\frac{1}{8}$ in $2\frac{1}{4}$ containers?)

10. Once students have made the table, ask them to look for a pattern. Students should notice that they are multiplying by the denominator for the first problem (4 x 2 = 8). In the second problem, they will half as many—18. The third problem will have one-third as many—12, which means that you will divide by three. In the last problem, students should put all of the connections together: Make the first fraction an improper fraction, multiply by the denominator, and then divide by four.
11. Once they have figured out the pattern, they should now understand that the denominator will need to be multiplied by the numerator of the first fraction in order to figure out the total parts, and the numerator of the second fraction and the denominator of the first fraction will need to be used to divide by in order to figure out the final value. This should lead the students to understand that they can invert the second fraction (use the multiplicative inverse, or reciprocal) and multiply to find the solution of fraction-division problem.

Assessment

- **Questions**
  - What is the meaning of division?
  - How is division of fractions similar to whole-number division?
  - How can you use different representations to model division of fractions and mixed numbers? (arrays, paper folding, repeated addition/subtraction, fraction strips, fraction rods, pattern blocks, and area models)?
- **Journal/writing prompts (include a minimum of two)**
  - Describe how you would use a number line or an array to model $\frac{3}{4} \div \frac{1}{2}$.
  - How would you decide when a problem is asking you to divide fractions?
  - Why does it work to make common denominators and then divide the numerators for your answer?
  - Why can you invert the second fraction and multiply when solving a fraction-division problem?
- **qa**
  - While students are working in pairs, keep a checklist of specific skills that students need to master in order to understand fraction multiplication and division conceptually.
  - Have students solve problems on a whiteboard to do a quick check on understanding.
  - Ask students to respond with thumbs-up, sideways, or thumbs-down before moving to a new section of the topic to ensure comprehension (thumbs-up = 100 percent got it; thumbs sideways = kind of understand; and thumbs-down = don’t understand at all).

Extensions and Connections (for all students)

- Have the students create the algorithms along the way and then have them go through a series of tests to ensure its accuracy.
- Distribute pattern blocks to students and have them solve different fraction multiplication and division problems using this type of representation.

Strategies for Differentiation

- Manipulatives of different types (e.g., fraction rods, strips, etc.) can be used to help struggling students.
- Regroup students based on their strengths (e.g., one student might be good at a number-line model while another does well with repeated subtraction).
- Provide worked examples of similar problems for some students as needed.