Multiply Fractions and Mixed Numbers

Strand: Computation and Estimation

Topic: Multiply and divide fractions (proper and improper) and mixed numbers.

Primary SOL: 6.5 The student will
a) multiply and divide fractions and mixed numbers.

Materials

- Fraction strips
- Fraction rods
- Pattern blocks
- Paper
- Graph paper

Vocabulary

array, area, denominator, factor, fraction, mixed number, multiplicative inverse (6.5), *numerator, product, quotient, simplest form, whole number* (earlier grades)

Student/Teacher Actions: What should students be doing? What should teachers be doing?

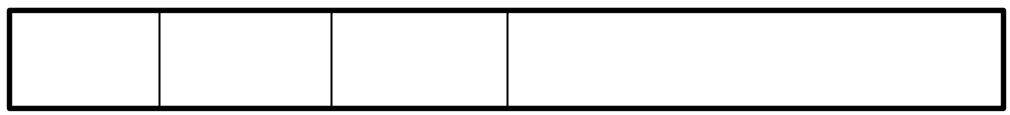
For students to master their understanding of fraction multiplication and division, modeling is an important step. It is also important to start with finding a fraction of a whole number. In order to build onto prior understanding when beginning a unit on multiplication and division of fractions, teachers should begin with reviewing the concept of whole-number multiplication as learned in grades 3 and 4 (i.e., 4 • 5 is four groups of five). For division, students will need to understand, in order to find the quotient for $\mathbf{18 \div 6}$, "How many groups of six are in 18?"

1. Using a number line, have students show how they could find 2 x 3. Students should make two groups of three on a number line and end up with six. Once students are able to show their conceptual understanding of multiplication, ask them to discuss with a partner how they might find the product of $2 \bullet \frac{1}{4}$ using the same method. Students should make two groups of $\frac{1}{4}$ and find that the answer equals $\frac{2}{4}$; however, the teacher will want to encourage them to give the response in simplest form.
2. Once students understand this concept on a number line, ask students to use any manipulative or drawing to figure out the answer to the next problem. "Jason filled four glasses of with $\frac{2}{3}$ cup of juice in each. How much juice did Jason use?" Have them work with a partner to discuss how to answer the question, and have students present their solutions to the class using the different representations. Students should notice that their answer can be written in simplest form as a whole number. It might be useful at this point to also have the students show their work with repeated addition to connect that idea as well (i.e., $4 \bullet \frac{2}{3} = \frac{2}{3} + \frac{2}{3} + \frac{2}{3} + \frac{2}{3}$).

3. After students have had practice with fractions of a whole or wholes of fractions, then they are ready to find a fraction of a fraction. Give students the following problem, "You have $\frac{1}{2}$ of a pizza leftover from the night before. If you give $\frac{1}{3}$ of it to your sister, how much of the whole pizza will she get?" Have them work with a partner to use the materials they have to try to solve the problem. Remind students to think about the meaning of multiplication while attempting to solve the problem.

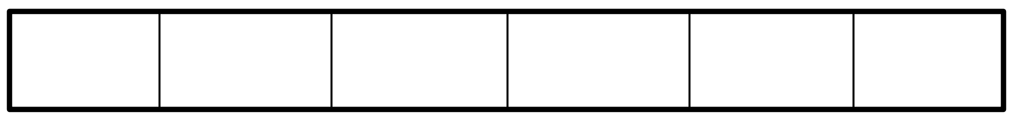
$\frac{1}{2}$ of the pizza

Incorrect



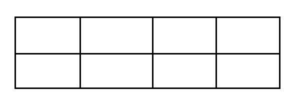
$\frac{1}{2}$ of the pizza

Correct

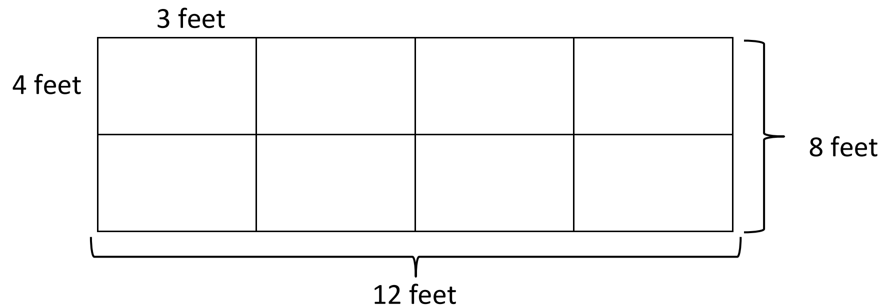


4. Students should be able to draw $\frac{1}{2}$ of the pizza, mark it on a number line, or represent it with the manipulatives given. Students might find that they better understand this idea with fraction rods or strips rather than number lines; however, it is important that they are also able to understand it using a number-line model. Once they have done that, they should attempt to figure out what $\frac{1}{3}$ of that half would be. They might need to be reminded that whenever they are splitting a part of a whole, the other section must be split as well. Once students have split their pizza into the correct parts, then they should be able to see that $\frac{1}{3}$ of $\frac{1}{2}$ should equal $\frac{1}{6}$, because the whole had to be divided into sixths and only one of those pieces is shaded in.

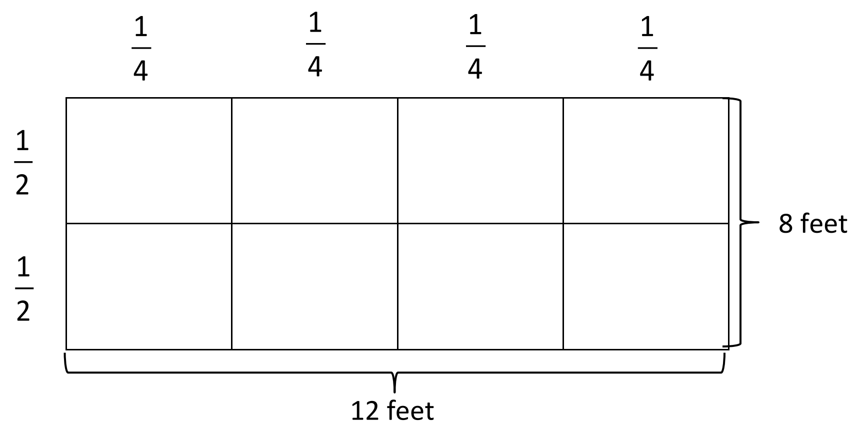
5. Another way to have students model fraction-by-fraction multiplication is an array. Have students create an area (or array) model of 2 x 4 which means "2 rows of 4."



6. Have students use grid paper to sketch a drawing of a quilt that will be 12 feet by 8 feet. Each group will draw a picture that will be 4 feet by 3 feet for the quilt. Have the students work in partners to discuss what fraction of the quilt will be created by each group.



7. Once students have completed question 6, ask students to discuss what fraction of the quilt was created. Explain to the students that they are going to prepare a section of the quilt that is $\frac{1}{4}$ of the length and $\frac{1}{2}$ of the width. Have the students sketch out the quilt and the portion that the group is responsible for preparing.



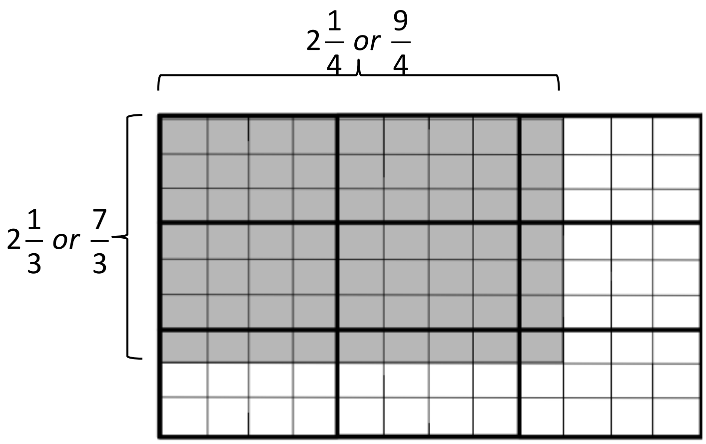
8. Help students to make the connections between the idea that area is length times width and they need one-fourth the length x half the width to find the area they are responsible for completing $\left(\frac{1}{4} \bullet \frac{1}{2} = \frac{1}{8}\right)$.

9. Students at this point should also begin to develop the algorithm. Ask students to solve the following problems:

   $\frac{1}{2} \bullet \frac{5}{6}$ $\quad$ $\frac{2}{4} \bullet \frac{1}{5}$ $\quad$ $\frac{1}{3} \bullet \frac{3}{7}$

10. Ask students to connect the illustration to the computation. One question that will help guide the students is, "How can you figure out the unit of the fraction (or the denominator) for the product?" Always go back to the models from previous examples while having these conversations with the students.

11. The next step in fraction multiplication would be to include mixed numbers. Students should use estimation strategies before beginning. For the following problem, $2\frac{1}{3} \bullet 2\frac{1}{4}$, ask students to estimate the product. They should realize that it is about $2 \bullet 2$. Therefore, the answer should be around 4. Students also might relate this back to whole number by fraction multiplication.
12. Once students understand how to estimate, students can then start applying the area model to the problem.



There are seven rows and nine columns, or $7 \bullet 9$ parts in the product.

The *whole* has three rows and four columns, or $3 \bullet 4$ parts.

$$2\frac{1}{3} \bullet 2\frac{1}{4} = \frac{7}{3} \bullet \frac{9}{4} = \text{PRODUCT}$$

$$\frac{\textit{Number of parts}}{\textit{Kind of parts}} = \frac{7 \bullet 9}{3 \bullet 4} = \frac{63}{12} = \frac{21}{4}$$

Technology note: While exploring fraction multiplication and division, the [National Library of Virtual Manipulatives](#) offers many fraction explorations. Consider using Fractions–Rectangle Multiplication applet.

Assessment

- **Questions**
  - What is the meaning of multiplication?
  - How is multiplication of fractions similar to whole-number multiplication?
  - How can you use different representations to model multiplication of fractions? (arrays, paper folding, repeated addition/subtraction, fraction strips, fraction rods, pattern blocks, and area models)
- **Journal/writing prompts**
  - Communities A and B are building playgrounds in empty lots by nearby schools that are each 50 yards by 100 yards on grassy areas. Predict which community would have the bigger grassy area on the playground based on the following information (illustrate and explain):
    - Community A has decided to convert $\frac{3}{4}$ of the lot to a playground, and $\frac{2}{5}$ of it will be covered in blacktop.

  - Community B will build the playground on $\frac{2}{5}$ of the lot, and $\frac{3}{4}$ of it will be covered with blacktop.
- Describe how you would use a number line or an array to model $\frac{3}{4} \bullet \frac{1}{2}$.

- **Other Assessments**
  - Have students create posters while they are solving the different problems to explain their thinking.
  - While students are working in pairs, keep a checklist of specific skills that students need to master in order to understand fraction multiplication and division conceptually.
  - Have students solve problems on a whiteboard to do a quick check on understanding.
  - Ask students to respond with thumbs-up, sideways, or thumbs-down before moving to a new section of the topic to ensure comprehension (thumbs-up = 100 percent got it; thumbs sideways = kind of understand; and thumbs-down = don't understand at all).

Extensions and Connections (for all students)

- Have the students create the algorithms along the way and then have them go through a series to tests to ensure its accuracy.
- Distribute pattern blocks to students and have them solve different fraction multiplication and division problems using this type of representation.

Strategies for Differentiation

- Manipulatives of different types (e.g., fraction rods, strips) can be used to help struggling students.
- Regroup students based on their strengths (i.e., one student might be good at a number-line model while another does well with the array model).
- Preteach/review essential vocabulary for certain students, as needed.
- Provide students with worked examples of similar problems to use as reference, if needed.