Sieve of Eratosthenes: An Ancient Algorithm to Discover Prime Numbers

Strand: Number and Number Sense

Topic: Identifying and describing prime and composite numbers

Primary SOL: 5.3 The student will
a) identify and describe the characteristics of prime and composite numbers.

Related SOL: 5.3b

Materials

- Centimeter Grid Paper (attached)
- Rectangles Chart (attached)
- Hundred Chart (attached)
- Colored pencils

Vocabulary

area, composite number, dimensions, factor, multiple, prime number, rectangle, square

Student/Teacher Actions: What should students be doing? What should teachers be doing?

1. Review the difference between factors and multiples: Begin by writing the number 12 on the board and, in partners, ask one person to list factors of 12 and the other to list multiples of 12. Have partners compare lists and discuss the differences.

   Factors of 12: 1, 2, 3, 4, 6, 12

   Multiples of 12: 12, 24, 36, 48, 60, 72, 84, 96, 108, 120…

   Have volunteers share their ideas about the differences they discussed with the class. Ask questions like, *"Who will share the factors/multiples of 12 they wrote?" "How do you know those are factors/multiples?" "Does anyone else have any other factors/multiples of 12?"* Ensure that all students can differentiate factors (i.e., numbers that are multiplied to get a product) and multiples (i.e., counting by a number, or a sequence when a given number is multiplied by 1, 2, 3, 4, etc.).

Part 1, Exploring Factors of Numbers with Rectangles

2. Using centimeter grid paper, have partners draw as many rectangles as they can with an area of 12. Have volunteers share the dimensions of the rectangles they found (1 by 12, 2 by 6, and 3 by 4 rectangles). Some may mention 12 by 1 or 6 by 2 rectangles; ask students why this is so, making connections to what they learned in Grade 3 about the commutative property of multiplication (12 x 1 = 1 x 12, and 2 x 6 = 6 x 2).

   List all of the dimensions of the rectangles in order (1, 2, 3, 4, 6, 12). Students should recognize that these are the factors of 12 (numbers that are multiplied to yield the product of 12).

On the Rectangles Chart, demonstrate filling in the "Number of Rectangles" and "Factors" columns, telling students that they will fill in the "Prime or Composite" column later.

3. Allow small groups of students to explore the remaining numbers in the same fashion in order to complete the "Factors" and "Number of Rectangles" columns on the Rectangles Chart:
    - Draw as many rectangles as you can using the given area (the number in the first column), and fill in the number of rectangles you found in the second column. Count the rectangles made using the commutative property of multiplication as one rectangle (i.e., 1 x 12 and 12 x 1 are counted as one rectangle).
    - List the factors (dimensions of your rectangles) in order, from least to greatest, in the third column. Groups can be assigned to work with a few of these numbers, and the information from each group can be combine once the task is completed.

    It will be helpful to address rectangular area numbers like 4, 9, and 16. Besides 1 by 4, 1 by 9, and 1 by 16 rectangles, squares with the dimensions of 2 by 2, 3 by 3, and 4 by 4 can be created. These numbers form squares; thus, they are referred to as *square numbers*. Ask: *"How many different rectangles can you make to show the area of each number?" "Can you make a different rectangle to show the area of that number?"*
4. Allow groups to share their results on the Rectangles Chart and ask: *"What do you notice about the different numbers?" "What patterns do you see?"* They should notice that only one rectangle could be created with some of the numbers. Have students locate these numbers on the Rectangles Chart and discuss the characteristics of the factors of these numbers. Note that the factors of these numbers (the dimensions of these rectangles) are only the number and 1.
    - Ask students whether they know the name of these types of numbers (prime). *A prime number* is defined as *a natural number (counting number), other than one, that has exactly two different factors, one and the number itself.* Explain that the other numbers are called *composite numbers*, which are *natural numbers that have factors other than one and itself.*
    - Ask students whether they can find any numbers that are not prime or composite. Guide them to notice that the number 1 has only 1 factor (1 x 1), and thus is a unique number. Ask students to discuss whether there are any other numbers that are neither prime nor composite. (No, the number 1 is unique.)
    - Based on what they understand now about prime and composite numbers, have students complete the "Prime or Composite" column of the Rectangles Chart. Have volunteers explain why each number is prime or composite.

Part 2: Sieve of Eratosthenes

5. For the second part of this lesson, tell students about Eratosthenes, an ancient Greek mathematician who studied prime and composite numbers. He used a method now called the Sieve of Eratosthenes to help determine and show the prime and composite numbers. Distribute the Hundred Chart to the students, and have them proceed through the following directions to identify the prime and composite numbers from 1 to 100.

- Since the number 1 is not prime, color it purple on the Hundred Chart.
- Circle the first prime number, 2, with your pencil. Use yellow to color every multiple of 2 on your chart. Do not color the number 2.
- Circle the next prime, 3. Use red to color every multiple of 3 on your chart. (Some multiples of 3, such as 6 and 12, may already be colored yellow. Ignore these and look for the uncolored multiples. When you finish, you should have 16 red squares.)
- Circle the next prime, 5. Use blue to color any uncolored multiples of 5. (You should have six blue squares.)
- Circle the prime number, 7. Use green to color any uncolored multiples of 7. (You should have only three green squares.)
- Count the uncolored squares on your chart. Can you find 25 of them? If you can, then you have sifted out all the prime numbers under 100.

Conclusion

Ask students to explain the difference between prime and composite numbers. Who can name a composite number? How do you know it is composite? Who can name a prime number? How do you know it is prime?

Assessment

- **Questions**
  - What is the difference between a prime number and a composite number? Give examples of both, and draw pictures to help your explanation.
  - How can you determine whether a number is prime?
- **Journal/writing prompts**
  - Record in your mathematics journal the characteristics of prime and composite numbers, and describe using the Sieve of Eratosthenes to find the primes.
  - Explain what makes prime numbers special. Choose your favorite prime number and explain why it is your favorite.
- **Other Assessments**
  - Give students a blank Hundreds Chart, and have them identify all of the prime numbers. They may choose to replicate the Sieve of Eratosthenes process.
  - Give students a list of random numbers, such as 42, 27, 31, 56, and 83. Ask them to tell you whether each is prime or composite and to explain why.

Extensions and Connections

- Extend exploration of prime and composite numbers to numbers greater than 100. Give students a chart of numbers from 101 to 200, and see if they can identify the primes. Ask students to justify how they know those numbers are prime.
- Challenge students to explain how to determine whether a number is divisible by 2, 3, 4, 5, 6, 8, and 9. Ask them to develop divisibility rules for each of these divisors.

Strategies for Differentiation

- In the lesson, use colored tiles before using the grid paper.
- Model creating rectangles and squares for numbers, and model completing the charts in this lesson.
- Create cards using centimeter grid paper of rectangles and squares. Students sort according to composite or prime numbers and explain their sort.
- Game: Teacher says, “Group yourself into a factor of eight.” Students could group in numbers of one, two, four, or eight. Teacher could also say, “Group yourself into a prime number.” Repeat with various composite or prime numbers. Music is optional.

Note: The following pages are intended for classroom use for students as a visual aid to learning.

Rectangles Chart

Name ______________________________ **Date** ____________________

Rectangle Area	Factors	Number of Rectangles	Prime or Composite
1			
2			
3			
4			
5			
6			
7			
8			
9			
10			
11			
12			
13			
14			
15			
16			
17			
18			
19			
20			
21			
22			
23			
24			
25			

Hundreds Chart

Name ______________________________________ **Date** _______________________

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Centimeter Grid Paper