Multiplication: Fluency with Facts

Strand: Computation and Estimation

Topic: Building multiplication fact fluency and number sense using arrays and decomposition of factors

Primary SOL: 4.4 The student will
a) demonstrate fluency with multiplication facts through 12 x 12, and the corresponding division facts.

Related SOL: 4.2a, 4.7

Materials

- Mystery Number Puzzle 1 activity sheet (attached)
- Mystery Number Puzzle 2 activity sheet (attached)
- Tracking My Progress with Multiplication Facts (attached)
- Applying the Factor Decomposition Strategy activity sheet (attached)
- Rectangle Rodeo Game Recording Sheet (attached)
- Multiplication Fact Cards for 3 x 3 through 12 x 12 (attached)
- Tiles
- Spaghetti
- Whiteboard or paper (one for each student)
- Demonstration tool (e.g., document camera, digital display)

Vocabulary

area, array, columns, decompose, factor, fluency, length, product, rectangular, rows, width

Student/Teacher Actions: What should students be doing? What should teachers be doing?

Note: The purpose of this lesson is to move students from using concrete and pictorial models of arrays for multiplication to using the relationship between the decomposition of arrays and the decomposition factors as a tool for mental multiplication and developing fluency with the multiplication facts. This lesson has been developed based on the description of computational fluency shared in the "2016 Mathematics Standards of Learning for Virginia Public Schools." That is, fluency is supported by flexible, efficient, and accurate methods for finding answers efficiently.

1. The first activity enables students to use decomposing or splitting arrays to understand the decomposition factors and to efficiently determine a multiplication fact. This process is supported by the distributive property, but fourth-graders are not required to know the name of the property. Students will need a whiteboard or a sheet of paper. Let students know they are going to see part of a rectangular shape. They will have 30 seconds to determine exactly how many tiles are needed to build the entire rectangular shape with tiles and write the number on their whiteboards. Project or draw the figure on Mystery Number Puzzle 1 for 30 seconds.
    a. Then, ask students to hold up their whiteboards. Write the different numbers on the class board.

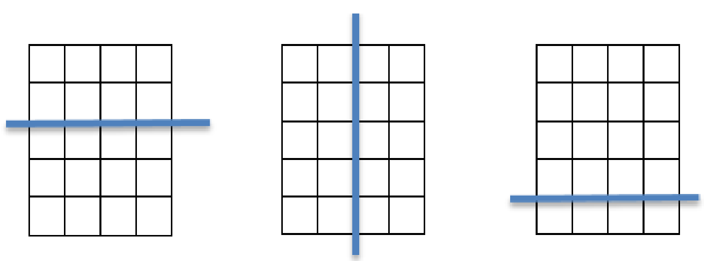
b. Without projecting the shape, call on students with different answers to explain in words and pictures how they determined the number of tiles that would be needed. Listen for terms such as rows, columns, array, 5 by 4, factor, product, counting by ones, counting by four, by five, or by other multiples, multiplication or other terms related to using arrays and multiplication and write these terms on the board.

c. Then, project the shape and, starting with the incorrect answers, call up volunteers to use the answers for the whiteboard to build the shape with tiles. For each incorrect answer, ask the class what someone might have done to get that number.

d. Once the correct answer has been demonstrated, provide tiles to all the students and have each build the shape and draw the representation in their notebooks. Then use the representation to call on students to describe how each term or word you wrote on the board relates to the array. Add other terms as needed. The students should write the terms and show in their picture what the term means.

e. Remind students that arrays are helpful in applying strategies to *figure out the product for factors when they are having trouble recalling the product from memory*. However, fluency requires that students understand and be able to explain their strategy and use the strategy efficiently to find the correct answer.

f. Review with students how to decompose arrays when they *cannot recall a product* and use the facts they can recall to quickly find the product for facts they may not recall. Provide students with a piece of spaghetti to use to show how they can decompose an array. Ask them to lay their spaghetti on their 5 by 4 array to show two smaller arrays within the larger array. Some of the decompositions you may see are the following.



g. Ask students to draw each of the above in their notebooks and write a description or a number sentence for how they would use the decomposition into two arrays to mentally find the product for 5 x 4 if they could not recall it quickly. Circulate around the room and identify three students to explain how they would use the decomposition. Then have a student write a number sentence to let them see a way of thinking about how decomposing the array determines how one of the factors is to be decomposed. Below is one way to write a correct number sentence and the mental math for each decomposition.

- 4 x 5 = (2 x 4) + (3 x 4)
  4 x 5 = 8 + 12
  4 x 5 = 20

- 4 x 5 = (5 x 2) + (5 x 2) or 4 x 5 = 2 x (2 x 5) is supported by the associative property of multiplication
  4 x 5 = 10 + 10   4 x 5 = 2 x 10
  4 x 5 = 20   4 x 5 = 20
- 4 x 5 = (4 x 4) + (1 x 4)
  4 x 5 = 16 + 4
  4 x 5 = 20

h. Let students know they will have 30 seconds to determine exactly how many tiles are needed to build the second rectangular shape and write the number on their whiteboards. Project or draw the figure on Mystery Number Puzzle 2.
  - Have student students hold up their whiteboards, and record the different answers on the board. Then show the figure again and ask students to build it with their tiles and explain to their shoulder partner how they determined the answer. The partners should decide which answer on the board is correct and develop a way to support why the answer is correct using the array.
  - Circulate around the room and determine which students you want to share their explanations.
  - Invite students to share their thinking.
  - In closing this part of the lesson, highlight that when we can usually recall the facts for five quickly and then we can use the idea of decomposing the array for 7 x 6 and realize that 7 can be decomposed or split into 5 and 2. Next, both the 5 and the 2 have to be multiplied by 6 and the products added together, that is (5 x 6) + (2 x 6). Another way to think about this (because some people recall the product of squares quickly) is decomposing 7 into 6 and 1 and then finding (6 x 6) + (1 x 6). But some people like to work with doubles, so another way is to recall that 3 x 7 is 21 so we can decompose 6 into 3 and 3 then finding the sum of (7 x 3) + (7 x 3).

2. The next activity will ask students to try to recall the fact from memory and then check the fact with an array. Students will work in pairs to complete the Applying the Factor Decomposition Strategy activity sheet. Each student will need a copy of the handout.
   a. Present students with a multiplication fact. Students first record what they think the product is and then record in words how they decomposed a factor. They also explain the process used to find the answer to the multiplication fact, to bridge from actually building the array to visualizing the array, to decomposing a factor. After completing each fact, the partners should compare and discuss their thinking and reach a consensus on the product.
   b. Circulate around the room to support students and to note which students to call on during the whole-class debrief. If students are struggling with decomposition, redirect them to think about what an array might look like for the multiplication fact and visualize what decomposition of a factor would allow them to work with facts they may recall.

   c. During the class, debrief to address the misconceptions or challenges students had while working on the problems. This activity prepares them for the Rectangle Rodeo game.
3. Students will work with a partner to play the Rectangle Rodeo Game. The object of the game is to color in an array or multiple arrays to represent the fact and the product. The space to show the arrays are limited, so students are forced at some point to decompose a factor and then they can color in two arrays based on their decomposition. Copy the Multiplication Fact Cards on card stock and cut them apart. Each team needs one set of cards, one Rectangle Rodeo Game recording sheet, two colors of crayons, and two pencils. The directions for playing the game are included on the recording sheet. The teacher may want to model how to play the game. Circulate around the room to redirect students as needed and to note who can recall facts quickly, who is able to decompose facts and use the strategy as needed, and who is able to draw the array for a fact quickly.
4. Provide a copy of Tracking My Progress with Multiplication Facts to each students. This tool can be used provide encouragement for students and a way to reflect on which facts they can give quickly answer and which facts they need to continue to work with in various ways, such as games and developing efficient strategies.
   a. Let students know the Tracking My Progress sheet is just for them and they will not have to share it with other students or even with you.
   b. First, show students the diagonal line that goes through the products of a factor times itself (the square number products) and then have them look at the products on both sides of the line. Students should realize that the products on one side are the same as those on the other side because facts such as 3 x 4 and 4 x 3 have the same product. This is supported by the commutative property of multiplication. Students do not need to recall the name of the property, but they do need to realize that order in multiplication does not change the quantity. Most importantly, it cuts in half the number of facts. You may want to direct students at this point to use their crayon to cross out the duplicates.
   c. Direct students to cross out any facts for 1 that they know. Then have them cross out any facts for 2 that they know.
   d. Direct students to review the facts they worked on during the lesson. If there were any they recalled from memory, they can cross them out too.
   e. Let students know that they can use the handout to keep track of what facts they have left. As they learn to recall any of the facts quickly or from memory, they can cross them out.

Assessment

- **Questions**
  - How can knowing that 4 x 6 is 24 help you find 8 x 6?
  - How can you use tiles to build an array for 9 x 7?
  - If you cannot recall the answer for 9 x 7, how can you use the array for 9 x 7 to figure out the answer for 9 x 7?

  - I see you have the statement 12 x 8 = (10 x 8) + (2 x 8). How are the two expressions related? How do you know they are equivalent?
  - I see you have this array (point to an array on their paper). Can you explain to me where the factors are represented and where the product is represented?
- **Journal/writing prompts**
  - You will be mentoring a third-grader this afternoon, and her teacher has asked that you help her understand how to use arrays in figuring out what multiplication is and how to find the answer for a multiplication fact. Describe what you will do and say while working with the student to help her and why you think the actions will be helpful.
  - You think that the multiplication facts for 2, 4, and 8 should be taught together. Persuade teachers why you think it is helpful for students to learn them together. Use words and pictures in your description.
  - Dillion said he thought the 11 and 12 facts were hard, but Mary told him she did not think so because you can just think about the 10 facts to make them easy. Describe and show what Mary is thinking about.
- **Other Assessments**
  - Mr. Jones has 48 bottles of water to arrange in a rectangular shape on a table. How could Mr. Jones arrange the water bottles?
  - Because you know that 3 x 8 is 24, how can you use that to help you find 9 x 8 quickly?
  - Look at this problem and decided whether you think it is correct or incorrect. If it is incorrect, describe the thinking that is incorrect and what changes would make the number sentence correct.

    8 x 7 = (4 x 5) + (4 x 2)

Extensions and Connections

- Provide additional opportunities to explore multiplication and division in context by using literature, various manipulatives, and different graphic representations of multiplication and division.
- Have students engage in a project that requires creating a product to share how arrays and multiplication facts are evident in a real-world context. They can record their findings with photographs, sketches, graphics from magazines, etc.
- If the school or local library has a copy of Greg Tang's trade book, *The Best of Times*, you may want to read it to the class or encourage students to read the book. The story and pictures support students to develop a deeper, more connected understanding of multiplication and the multiplication facts. This book develops fluency by using poems and pictures to communicate strategies that work for all numbers, not just small numbers. Students can then create a multiplication facts storybook for third-graders.
- Given a set of multiplication facts, students will write two story problems, one which requires multiplication to solve and the other which requires division to solve.

- Students explore situations, based on multiplication facts, where they are given the area and one dimension of a rectangle and need to find the other.

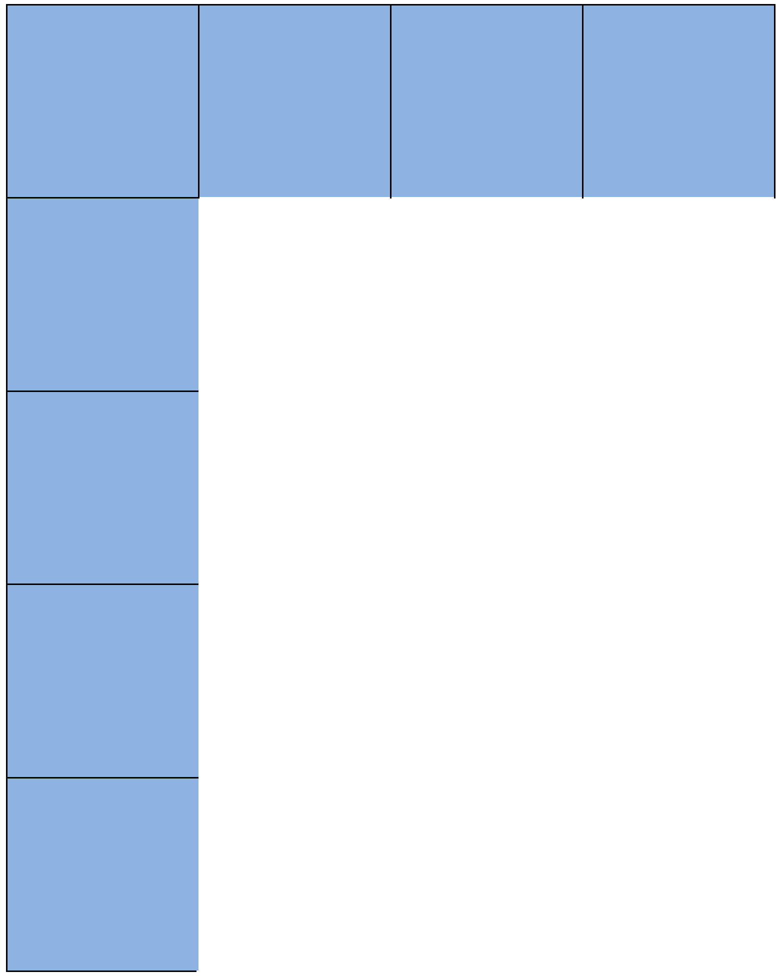
Strategies for Differentiation

- Provide options for developing pictorial and concrete representations using literature, various manipulatives, and different graphic representations of multiplication.
- Provide learning experiences with related multiplication facts and help students make explicit connections. For example, the 2, 4, and 8 facts; the 3, 6, and 12 facts; the 5 and 10 facts; and the 9 and 10 facts.
- Encourage students to use tiles and grid paper to explore the strategy of using arrays and decomposition of a factor to use known facts to solve an unknown fact.
- Help students investigate patterns and develop strategies such as one group more than (i.e., 7 x 6 is 7 x 5 and one more group of 7). This is closely connected to one group less than (i.e., 9 x 9 is one group of 9 less than 10 x 9).
- Students use an interactive multiplication table to explore the array that models a given multiplication fact. URL: http://www.visnos.com/demos/times-tables
- The Math Cats site shares a choice of four interactive learning tools that allows students to investigate arrays and the multiplication fact that is related to the array. URL: http://www.mathcats.com/explore/multiplicationtable.html

Note: The following pages are intended for classroom use for students as a visual aid to learning.

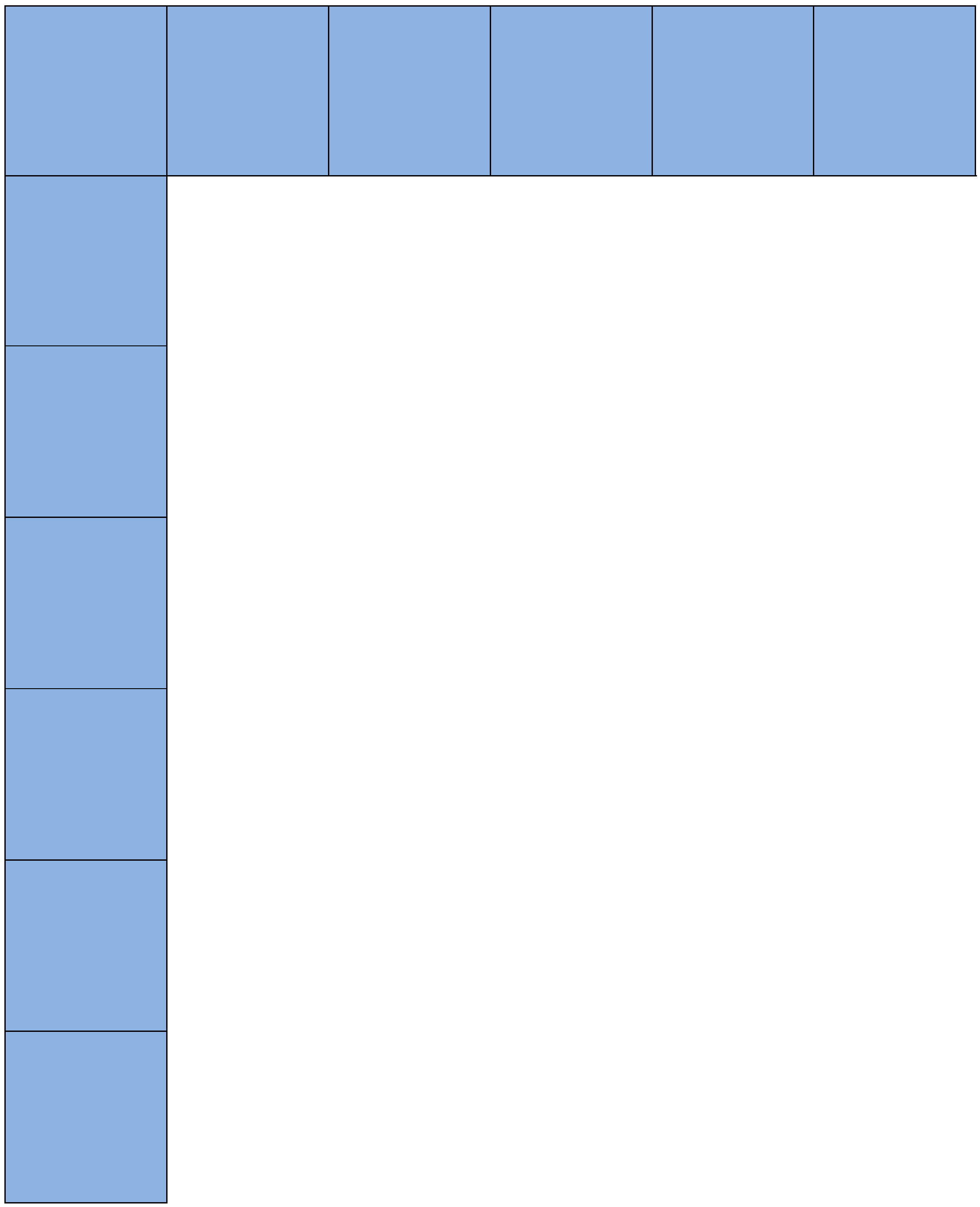
Mystery Number Puzzle 1

How Many Tiles Do I Need to Build the Rectangular Shape?



Mystery Number Puzzle 2

How Many Tiles Do I Need to Build the Shape?



Tracking My Progress with Multiplication Facts

X	1	2	3	4	5	6	7	8	9	10	11	12
1	**1**	2	3	4	5	6	7	8	9	10	11	12
2	2	**4**	6	8	10	12	14	16	18	20	22	24
3	3	6	**9**	12	15	18	21	24	27	30	33	36
4	4	8	12	**16**	20	24	28	32	36	40	44	48
5	5	10	15	20	**25**	30	35	40	45	50	55	60
6	6	12	18	24	30	**36**	42	48	54	60	66	72
7	7	14	21	28	35	42	**49**	56	63	70	77	84
8	8	16	24	32	40	48	56	**64**	72	80	88	96
9	9	18	27	36	45	54	63	72	**81**	90	99	108
10	10	20	30	40	50	60	70	80	90	**100**	110	120
11	11	22	33	44	55	66	77	88	99	110	**121**	132
12	12	24	36	48	60	72	84	96	108	120	132	**144**

Applying the Factor Decomposition Strategy

Multiplication Fact	**What I Think the Product Is**	**Decomposition of One of the Factors to Check My Answer (Remember, it does not matter which factor.)**
7 x 6	42	In words: Break 7 into 5 and 2, and then 5 times 6 is 30 and 2 times 6 is 12. Add 30 and 12 to get 42. or As a number sentence: 7 x 6 = (5 x 6) + (2 x 6) or 7 x 6 = 30 + 12 = 42. • **(There are other decompositions)**
8 x 7		
6 x 9		
12 x 8		
7 x 7		
6 x 8		

Rectangle Rodeo Game Recording Sheet

1. Partners will share a Rectangle Rodeo Game recording sheet and a set of multiplication fact cards. Each player writes their name on one side of the sheet and shuffles and places the fact cards facedown between the two players.
2. Each player will need a different color of crayon or marker to color in a rectangle that shows the product of the fact they draw. For example, if the fact is 2 x 5, then color a 2 by 5 array to capture 10 smaller rectangles and write 2 x 5 = 10 on the rectangle. Write the fact with the product on the array.
3. You are allowed to decompose, or split, one of the factors and make two arrays if there is not enough space. For example, 2 x 5 can be split in several ways such as (2 x 3) + (2 x 2) = 10, so you can color in a 2-by-3 array and a 2-by-2 array and write the number sentence showing the decomposition on both arrays.
4. A partner can challenge an answer immediately. If the answer is incorrect, the rectangle is crossed out and the player cannot recapture that space.
5. The goal of the game is to capture as much of your space as possible. The game ends when all fact cards have been used, or it is not possible to use any more, or when the teacher calls time.

Name_________________________									**Name**_________________________							

Multiplication Fact Cards

3 x 3	3 x 4
3 x 6	3 x 7
3 x 8	3 x 9
3 x 11	3 x 12

4 x 4	4 x 6
4 x 7	4 x 8
4 x 9	4 x 11
4 x 12	6 x 6

6 x 7	6 x 8
6 x 9	6 x 11
6 x 12	7 x 7
7 x 8	7 x 9

7 x 11	7 x 12
8 x 8	8 x 9
8 x 11	8 x 12
9 x 9	9 x 11

9 x 12	10 x 11
10 x 12	11 x 11
11 x 12	12 x 12
7 x 0	1 x 1