

Surface Area and Volume

Strand:	Three-Dimensional Figures
Topic:	Deriving formulas for surface area and volume
Primary SOL:	G.13 The student will use surface area and volume of three-dimensional objects to solve practical problems.
Related SOL:	G.8, G.14

Materials

- Finding Formulas activity sheet (attached)
- Making Nets activity sheet (attached)
- Solving Problems activity sheet (attached)
- Graphing utility
- Cans
- Scissors
- Boxes made of lightweight cardboard
- Oranges
- Wax paper
- Knife (to cut oranges)
- Unit cubes
- Milk cartons or other boxes of the same shape but different sizes
- Geometric shape set (hollow)
- Sand, rice, or water
- Rulers
- Graduated cylinders

Vocabulary

area of a face, base, circumference, composite figures, cone, cylinder, derive, face of a 3-D figure, formula, height, hemisphere, lateral area, length, net, perimeter, prism, pyramid, rectangular prism, regular pyramid, scale, sphere, surface area, three-dimensional, two-dimensional, volume, width

Student/Teacher Actions: What should students be doing? What should teachers be doing?

1. Discuss with students: What are the formulas for determining the surface area of solid figures? How can these formulas be used to solve practical problems?
2. Have students work with partners to complete the Finding Formulas activity sheet, which has students derive the formulas for the surface area and volume of a rectangular prism, a cylinder, and a sphere.
3. Have students complete the Making Nets activity sheet to extend their knowledge of surface area of solid shapes.
4. Students will use the newly learned information to complete the Solving Problems activity sheet. This activity focuses on the application of formulas in practical contexts.
5. Discuss the relationship between surface area and two-dimensional perimeter and area.

6. Distribute milk cartons with the tops cut off, or open boxes of other kinds, along with unit cubes. Ask students to fill the boxes with cubes as completely as they can and estimate the volume of the box based on counting the total number of cubes used to fill the box.
7. Ask students to develop a quicker method than filling and counting the total number of cubes for figuring out how many whole cubes will fill the box. (They should decide on $length \times width \times height$.)
8. Have students generalize this finding to other prisms/cylinders by relating the area of the base to the height. Use a can as a model of a right circular cylinder. Students should calculate the area of the base and multiply this by the height to find the volume. Because $1 \text{ cm}^3 = 1 \text{ milliliter}$, a graduated cylinder can be used to compare the estimated volume with the actual volume.
9. Have students explore the relationship between the area of a pyramid and the area of a cube or prism with the same base and height, using solid representations of 3-D figures and sand, water, or rice.

Assessment

- **Questions**

- List the following three-dimensional figures in order, from the figure with the least volume to the figure with the greatest volume.
 - a pyramid with a square base with sides measuring 25 centimeters and height 30 centimeters
 - a sphere with radius of 10 centimeters
 - a rectangular prism measuring 10 centimeters x 20 centimeters x 30 centimeters
 - a cylinder with radius 10 centimeters and height 20 centimeters
- A water tank is 3 meters tall and has a diameter of 4 meters. The water level is 2 meters high. How many more cubic meters of water can be added to the tank? Justify your answer.
- A red cylinder has diameter that is twice that of a blue cylinder. The height of the red cylinder is half that of the blue cylinder. Compare the volumes of the two cylinders. Which holds more?

- **Journal/writing prompts**

- A soup can label is printed on a rectangular piece of paper. Explain how the radius and height of the soup can are related to the length and width of the rectangular piece of paper.
- Describe a practical example that uses surface area or volume.
- Write a practical problem and solution that uses surface area or volume.

- **Other Assessments**

- Have students bring in an example of packaging that does not form a solid that the class has studied (or allow students to select from a collection you have gathered). Have them determine (or estimate) the solid's surface area and volume and explain their process and reasoning.

Extensions and Connections

- Discuss the relationship of surface area and volume of three-dimensional solids and their relationship to area and perimeter of two-dimensional figures.
- Have students create a cereal box with maximum volume and minimum surface.
- Create a lesson that includes mock prices of popcorn containers in various three-dimensional shapes. Ask students to analyze the best value with justification for their reasoning.
- Have students search the Internet for more nets of platonic solids and investigate their surface areas and volumes.

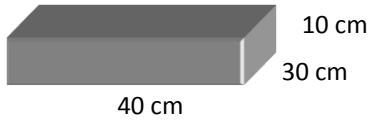
Strategies for Differentiation

- Use manipulatives to build figures from nets.
- Use enlarged graph paper for nets.
- Use presentation software to post multimedia classroom lectures, student conclusions, and notes online.
- Have students work in groups to make up their own review materials.
- Use presentation software to create a multimedia presentation to highlight key elements of the lesson.
- Have students use the presentation as a review at the end of the lesson.
- Using computer software, create visual representations of the formulas and figures, and randomly distribute them to students. Have students cut and paste to match the formulas to the correct figures.

Note: The following pages are intended for classroom use for students as a visual aid to learning.

Finding Formulas

1. After you purchase a gift for a friend, you decide to cover the sides and bottom of the gift box with wrapping paper. A diagram of the box with its dimensions appears below.



- a) How much wrapping paper will you need to cover the sides and bottom of the box? Explain how you arrived at your answer.
- b) Your gift box is called an open box because it has no top surface. If this were a closed box with a top surface, how much additional paper would be required to cover the top surface?
- c) How much total paper would be required to cover the box (a and b)?
- d) How can you generalize the process you used to find the surface area of the closed box?
- e) Let l = length, w = width, and h = height of the box. Compare the formula you and your partner developed to that of another group. Did you have the same result? You should be able to justify your formula to your classmates.

2. If your gift were a can of tennis balls, the surface area would be the surface of the cylinder (the lateral area) plus the areas of the top and bottom (the bases). Use a can (soup can, soda can, tennis ball can) for this activity.

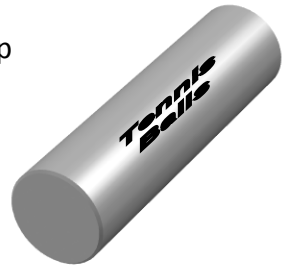
a) Wrap a piece of paper around the can, trim it to fit exactly, unwrap the paper from around the can and spread it out flat.

What shape is the piece of paper?

How can you find its area?

What relationship does the length of the paper have to the can?

What relationship does the width of the paper have to the can?



b) What shape are the bases of the can?

Are the two bases congruent?

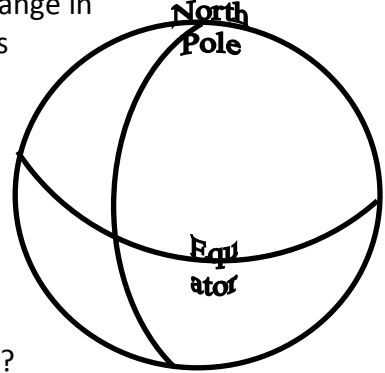
What is the area of each base?

c) The surface area of the can = the lateral area + the area of the two bases. What is the surface area of your can? Use your calculator to find decimal approximations to the nearest tenth.

3. The surface area of a sphere is more difficult to figure out. On a globe, a *great circle* is a circle drawn so that when the sphere is cut along the *great circle*, the cut passes through the center of the sphere. The equator is a *great circle* on Earth or on a globe.

a) Draw a great circle on an orange, and carefully cut the orange in half along the line of the great circle. Trace five cut halves on a piece of waxed paper.

b) Carefully peel both halves of the orange, and fill in as many circles as you can with the bits of orange peel. How many circles was your group able to fill?



How does this compare with the findings of other groups?

What is the class estimate for the number of great circles that can be filled in completely with the bits of orange peel?

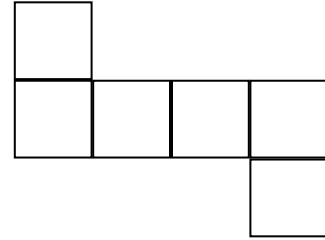
- c) Using your great circle tracing, find the radius of your orange and the area of the great circle.
- d) Given the area of one great circle and your estimate of the number of circles that can be filled in with the bits of orange peel, what is the surface area of the orange?
- e) What is the general formula for the surface area of a sphere in terms of its radius? Explain your thinking in determining the general formula.
- f) A hemisphere is defined as one-half of a sphere. What is the formula for finding the surface area of a hemisphere?

Making Nets

Name _____

Date _____

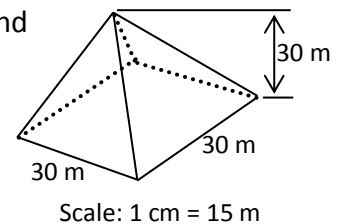
1. A *net* is a flattened paper model of a solid shape. For example, the net shown to the right, when folded, makes a cube. Can you draw a different net which, when folded, will also make a cube? If so, draw it, cut it out, and fold it to test your drawing.



2. A net is helpful because it represents the surface area of a shape. Take a box and cut it into a net. Note whether your box is open or closed. Sketch your box and its net. Use the formula derived in the Finding Formulas activity, problem 1, to find the surface area of your box. Explain to a classmate how your net relates to your formula.

3. Now sketch a net of the can you used in the Finding Formulas activity, problem 2. How does this net relate to the surface area formula you found?

4. Sketch a net of the pyramid shown to the right. Use your net to find the surface area of the pyramid.

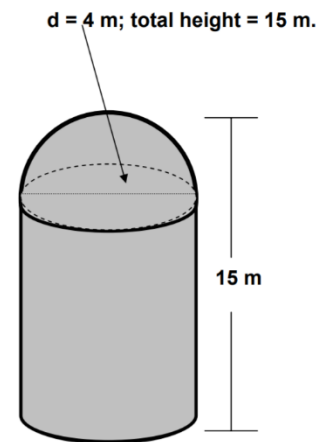


Solving Problems

Name _____ Date _____

1. Two cylindrical lampshades 40 centimeters in diameter and 40 centimeters high are to be covered with new fabric. The fabric chosen is 1 meter wide. If you purchase a 1.5-meter length of this fabric, will you have enough to cover both lampshades? Justify your answer.
2. An umbrella designer has created a new model for an umbrella that, when opened, has the shape of a hemisphere with a diameter of 1 meter. If a dozen sample models are to be made using a special waterproof material, approximately how much waterproof fabric will be needed (allowing 0.5 meter for seams and waste for each model)? Explain your plan, your strategies, and how you solved the problem.

3. A silo is a building used to store grain. It is cylindrical in shape with a dome on top. Given the height of 15 meter and a diameter of 4 meters, how much grain will this silo hold? If grain is sold for \$25.00 per cubic meter, what is the value of the grain in this silo?



Suppose the farmer needs to paint the silo. Find the surface area he will need to paint. If each can of paint covers 150 square feet, how many cans will the farmer need to paint the entire silo? The bottom of the silo is not painted.