

Special Right Triangles and Right Triangle Trigonometry

Strand:	Triangles
Topic:	Investigating special right triangles and right triangle trigonometry
Primary SOL:	G.8 The students will solve problems, including practical problems, involving right triangles. This will include applying b) properties of special right triangles; and c) trigonometric ratios.
Related SOL:	G.3a, G.5, G.7

Materials

- Special Right Triangles activity sheet (attached)
- Right Triangle Trigonometry activity sheet (attached)
- Dynamic geometry software package (Assignment can be modified to use paper, compass, and straightedge.)
- Demonstration tool (e.g., document camera, projector, calculator/graphing emulator, or other)
- Graphing utility
- Pencils
- Trigonometric table

Vocabulary

AA, adjacent, cosine, hypotenuse, isosceles right triangle, leg (of a right triangle), opposite, Pythagorean Theorem, radicals, rationalize denominator, reduce, reference angle, right triangle, scale factor, similar triangles, sine, special right triangle, tangent, trigonometric ratio, 45-45-90 right triangle

Student/Teacher Actions: What should students be doing? What should teachers be doing?

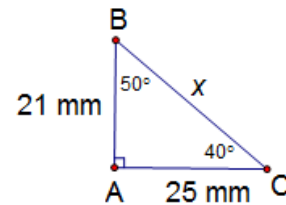
1. Have students work in pairs to the Special Right Triangles activity sheet. Each student should record his/her own findings. Have students discuss their findings with their partners. Discuss the findings as a whole group.
2. Have students work in pairs to complete the Right Triangle Trigonometry activity sheet. Each student should record his/her own findings. Have students discuss their findings with their partners. Discuss the findings as a whole group.
3. Use a demonstration tool to show students how to use their graphing utility, rather than a table, to evaluate trigonometric ratios.
4. Facilitate the class in determining a list of steps for solving right triangle trigonometry problems. Give two examples: one where the resulting equation has a variable in the numerator, and the other where the variable is in the denominator. Demonstrate the difference in solving the equations.

5. Give students an example in which the lengths of two sides of a right triangle are given. Have them use “guess and check” or use a trigonometry table to find the angle. Then show them how to use inverse trigonometric functions.
6. Introduce angle of elevation and angle of depression, using an example to show how they are related. Show students how to label the diagram so they can use the same method as before to get an equation and solve.

Assessment

• Questions

- How can you use the properties of a 30-60-90 right triangle to find the height of an equilateral triangle with sides 10 centimeters long?
- How can you use the properties of a 45-45-90 right triangle to find the length of a diagonal of a square with sides 10 centimeter long?
- An isosceles right triangle has hypotenuse 10 centimeters long. How can you use the properties of special right triangles to find the lengths of the legs? How else could you find the lengths of the legs?
- How do you decide whether to use sine, cosine, or tangent to solve a given problem?
- How do you know (without using a graphing utility) that there is an angle with tangent value equal to 1,000,000? What are possible lengths for the legs of such a right triangle?
- Write four different trigonometric equations to find the value of x in the diagram shown at right. How else could you find x ?



• Journal/writing prompts

- Complete a journal entry summarizing one of the activities.
- Given a right triangle with the length of one side and the measure of one angle given, explain how to find the length of another side using right-triangle trigonometry.
- Show students several similar right triangles and mark congruent acute angles as reference angles. Ask, “Why are the tangents of each of these reference angles equal?”
- Explain how to find the measure of an acute angle of a right triangle, given the lengths of two sides.
- Write a practical problem and solution that uses right-triangle trigonometry (or angle of elevation or depression).

• Other Assessments

- Have students complete Nos. 11–21 of the Right Triangle Trigonometry activity sheet for a different triangle.

- Show students a right triangle with two side lengths given (or three for an easier problem), and ask students to find the sine, cosine, and tangent of one of the acute angles. Encourage students to express their answers as ratios.
- How can you tell which angle is the 30-degree angle in a 30-60-90 triangle, remembering that diagrams are not always drawn to scale?

Extensions and Connections

- Invite a carpenter, builder, or city planner to come to the class and demonstrate the various job applications that use the current content or invite a CTE instructor or student.
- Have students use a local map to plot key destinations in the area (e.g., fire station, park, school). The destinations can be starting and end points for a classroom activity (Johnny Firefighter gets in his firetruck and drives 20 miles due north. He then drives five miles due west to put out a fire at Janet’s house. How far is he away from the fire station?).
- Have students restate the rules for special right triangles using ratios. ($1: 1: 1: \sqrt{2}$, $1: \sqrt{3}: 2$)
- Have students use a straightedge and compass to construct a 45-45-90 triangle and a 30-60-90 triangle.
- Have students explore parallax, a theodolite, or a clinometer for indirect measurement.
- Make tangrams, and look at similarity and types of triangles.
- Have a land or highway surveyor explain the use of a transit and the related trigonometry.
- Use trigonometric ratios to find the area of irregularly shaped buildings.

Strategies for Differentiation

- Have students come up with an acronym to help remember the trigonometric definitions. (Example: soh – cah – toa where sine = opposite/hypotenuse, cosine = adjacent/hypotenuse and tangent = opposite/adjacent)
- In groups of three to four, have students use a tape measure, pencil, and graphing paper to measure the sides of buildings around the school. Then have them graph the sides and, using the Pythagorean Theorem, find the distance between the diagonals.
- Present an indirect measurement application of Pythagorean Theorem on the board using magnets. Cut out little shapes of boats or whatever is used in the example, and glue them onto magnets. While students are working on similar problems, pull the weaker students individually to the board to work one-on-one with the indirect measurement exercise. After the student demonstrates mastery, they may return to their partner.
- Have three students form a right triangle using string. Use a fourth student to mark the reference angle by standing in the triangle at that vertex holding a sign saying, “reference angle.” Use signs labeled “hypotenuse,” “opposite side,” and “adjacent side” attached to clothespins to label those sides. Have the “reference angle” student move to the other acute angle, and have students adjust the other signs appropriately.

Mathematics Instructional Plan – Geometry

- Have students do a think-pair-share activity to introduce the lesson. “How might we use right triangles in determining distance?” Point at a map for illustration.
- Develop a graphic organizer or foldable that anchors the special right triangles and right-triangle trigonometry to the Pythagorean Theorem.
- Use an example/non-example strategy to teach concepts. “What is a special right triangle? What isn’t a special right triangle?”
- Use web resources to post multimedia classroom lectures and notes online.
- Color-code to differentiate various parts of the triangles.
- Use presentation software to isolate or highlight key points of the lesson.
- Have students work in groups to make up their own games, presentations, glossaries, or quizzes to reinforce the lesson.

Note: The following pages are intended for classroom use for students as a visual aid to learning.

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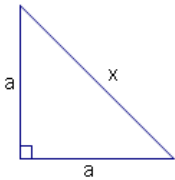
Special Right Triangles

45-45-90 right triangles (isosceles right triangles)

1. Draw and label an isosceles right triangle $\triangle ABC$, with the right angle at C . Draw a segment. Label the endpoints A and C . Construct a line through C perpendicular to \overline{AC} . Construct a circle with center C and radius \overline{AC} . Name one of the two points where this circle intersects the perpendicular line B . Construct triangle $\triangle ABC$. Hide everything but the triangle.
2. Measure each angle and each side. Compute the ratio $\frac{AB}{BC}$ (as a decimal). Record your findings in the table below.
3. Sketch your diagram here. Show all angle measures and segment lengths.
4. Adjust points A and B , or draw another triangle using the same directions if using paper, and record the new values in the table below. Repeat one more time.

$m\angle A$	$m\angle B$	$m\angle C$	\overline{BC} (leg)	\overline{AC} (leg)	\overline{AB} (hypotenuse)	$\frac{AB}{BC}$

5. What do you notice about the acute angles? Explain why this must be true for $\triangle ABC$.
6. What do you notice about the legs \overline{AC} and \overline{BC} ? Explain why this must be true for $\triangle ABC$.
7. Use the Pythagorean Theorem on the following triangle to express the length of the hypotenuse, x , in terms of the lengths of the legs, a . Be sure to reduce radicals.



8. How is the ratio $\frac{AB}{BC}$ related to the result in No. 7?
9. What is the formal relationship among the sides of a 45-45-90 right triangle?
10. Find the length of the two sides of a 45-45-90 right triangle where the hypotenuse is 7.

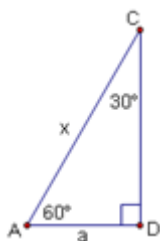
30-60-90 right triangles

1. Draw and label an equilateral triangle $\triangle ABC$. (Draw a segment and label the endpoints A and B . Construct a circle with center A and radius \overline{AB} . Construct a circle with center B and radius \overline{AB} . The two circles intersect at two points. Name one of those points of intersection C . Construct triangle $\triangle ADC$ and $\triangle BDC$. Hide everything but the triangle $\triangle ADC$.)
2. What is the measure of each of the angles in this equilateral triangle? (Measure them if you don't know.)
3. Select the segment \overline{AB} , and construct the midpoint of \overline{AB} . Label it D . Construct the segment \overline{CD} .
4. What is the relationship between the length of \overline{AB} and the length of \overline{AD} ? (Remember that D is the midpoint of \overline{AB} .)
5. Measure the angles $\angle ACD$ and $\angle BCD$. $m\angle ACD = \underline{\hspace{2cm}}$, $m\angle BCD = \underline{\hspace{2cm}}$
6. Hide everything but $\triangle ADC$. Measure each angle measure and segment length. Compute the ratios $\frac{AC}{CD}$ and $\frac{AD}{CD}$. (You may be able use your dynamic geometry software to do this.) Record your findings in the table below.
7. Sketch your diagram here. Show all angle measures and segment lengths.

8. Adjust points A and B , and record the new values in the table below. Repeat one more time.

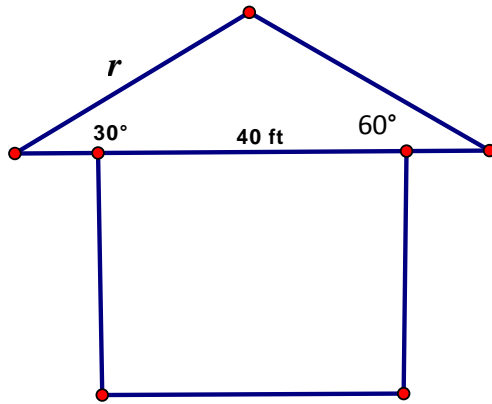
$m\angle A$	$m\angle C$	$m\angle D$	\overline{CD} (short leg)	\overline{AD} (long leg)	\overline{AC} (hypotenuse)	$\frac{AC}{AD}$	$\frac{CD}{AD}$

9. Use your answer to No. 4 or the ratio $\frac{AC}{CD}$ to express the length x in the diagram below in terms of a .



10. Use the Pythagorean Theorem on the triangle below to express y in terms of a . Be sure to reduce radicals.
11. How is the ratio $\frac{CD}{AD}$ related to the result in No. 10?

12. What is the formal relationship among the sides of a 30-60-90 right triangle?
13. A builder needs to find the length of a roof rafter r . How can you use the properties of special right triangles to find the length? Show your work.



Right Triangle Trigonometry

Name _____ Date _____

Vocabulary

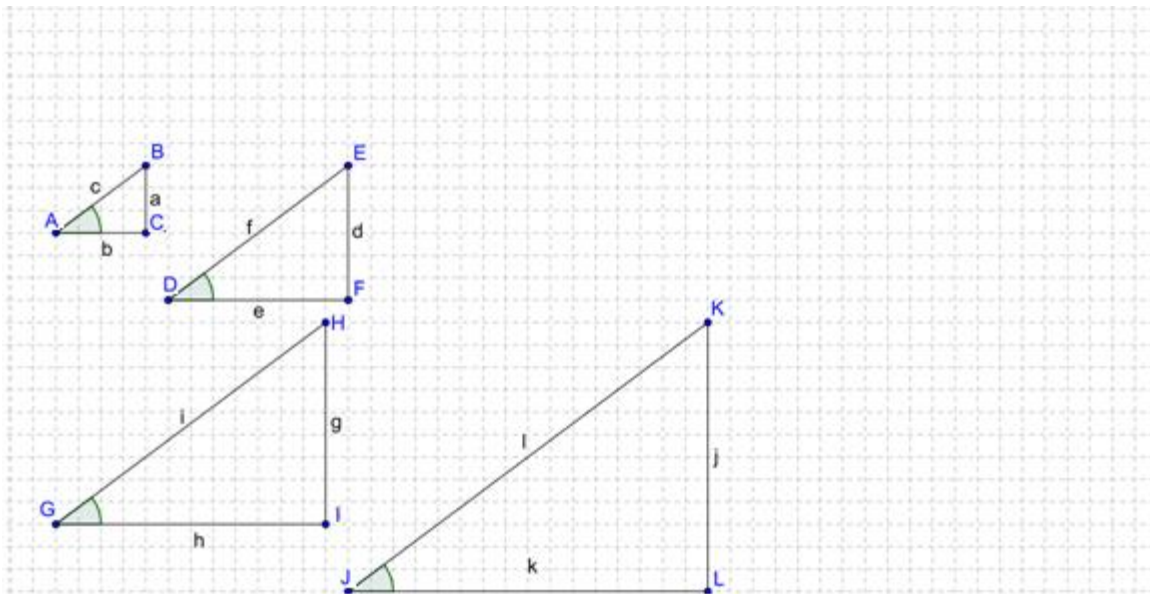
Reference Angle: the marked angle (In $\triangle ABC$, $\angle C$ is the marked angle.)

Hypotenuse (HYP): the side opposite the right angle in a right triangle

Opposite (OPP): the side opposite the reference angle

Adjacent (ADJ): the side adjacent to (next to) the reference angle

Use the diagram of four right triangles, $\triangle ABC$, $\triangle DEF$, $\triangle GHI$, and $\triangle JKL$ to complete the tables and answer the following questions. Remember to reduce fractions!



1. Count, and use the Pythagorean Theorem to complete the table.

	$\triangle ABC$	$\triangle DEF$	$\triangle GHI$	$\triangle JKL$
OPP	a =	d =	g =	j =
ADJ	b =	e =	h =	k =
HYP	c =	f =	i =	l =

2. What do you notice about the lengths of the sides of the triangles?
3. What can you say about the triangles? (Hint: They are not congruent, but...) Why do you know this?
4. What is the scale factor of $\triangle DEF$ to $\triangle ABC$? _____
 $\triangle GHI$ to $\triangle ABC$? _____

$\triangle JKL$ to $\triangle ABC$? _____

- What do you know about $\angle C$, $\angle F$, $\angle I$, and $\angle L$? $\angle A$, $\angle D$, $\angle G$, and $\angle J$? Why do you know this?
- Draw a triangle $\triangle MNO$, using the point M in the diagram, so that the scale factor of $\triangle MNO$ to triangle $\triangle ABC$ is 5.
- Complete the table below. (The definitions of these terms can be found in the vocabulary section at the beginning of this activity sheet.)

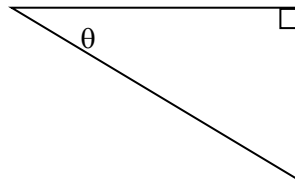
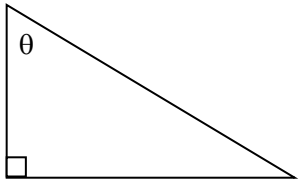
	Write the names of the Trig ratios:					
	OPP	ADJ	HYP	$\frac{\text{OPP}}{\text{HYP}}$	$\frac{\text{ADJ}}{\text{HYP}}$	$\frac{\text{OPP}}{\text{ADJ}}$
$\triangle ABC$						
$\triangle DEF$						
$\triangle GHI$						
$\triangle JKL$						
$\triangle MNO$						

- Make sure the ratios in the last three columns have been reduced. Do you notice any patterns?

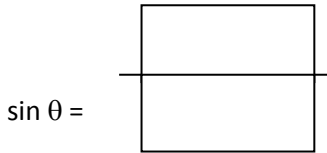
More Vocabulary

- Trigonometric Ratio: a ratio of the lengths of sides of a right triangle
- Sine (of the reference angle): the ratio $\frac{\text{OPP}}{\text{HYP}}$
- Cosine (of the reference angle): the ratio $\frac{\text{ADJ}}{\text{HYP}}$
- Tangent (of the reference angle): the ratio $\frac{\text{OPP}}{\text{ADJ}}$

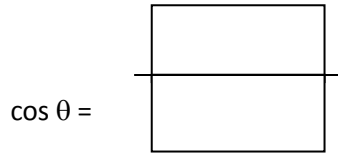
9. From the reference angle, θ , label the opposite side (OPP), adjacent side (ADJ), and hypotenuse (HYP) for each right triangle. (Label on the triangle)



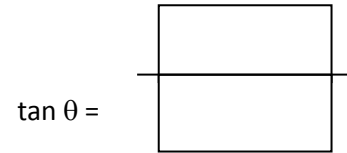
10. Define $\sin \theta$



- Define $\cos \theta$



- Define $\tan \theta$



11. Now, investigate sine, cosine, and tangent in right triangles, using a dynamic geometry software package. (If available) You will also need a trigonometric table.
12. Draw and label a right triangle $\triangle ABC$. (You can do this by drawing a segment \overline{AC} , and constructing a line perpendicular to the segment \overline{AC} , through the endpoint C . Create a point B on the perpendicular line and create $\triangle ABC$.) Mark the right angle as $\angle C$.
13. Measure the length of one side—any side—and the measure of one angle (not the right angle).
14. Draw your diagram here, and record your measurements on the diagram.
15. From your diagram, label the measured side as OPP, HYP, or ADJ to the angle you've measured.
16. Decide which length you would like to find next. Label that side with a variable (like x). Also label that side as OPP, HYP, or ADJ.
17. Using the definitions for sine, cosine, and tangent above, decide which trigonometric ratio uses the side you have and the side you're looking for. Set up and write down your trigonometric equation here.
18. Solve the equation for the variable. Using the trigonometric table, find the value of the trigonometric ratio, and use that value to find the length of the segment of interest.

19. Once you have your value, check your work by measuring the segment using the dynamic geometry software. What did you find? Were you correct or incorrect? Why?

20. Do this same process for the remaining side. Record your work here.

21. Discuss with your partner another method for finding the length of the third side when you have the lengths of two lengths of the triangle.

22. Draw a different triangle, and do this entire process again (steps 1–10).