Distance, Midpoint, and Slope Formulas

Strand: Reasoning, Lines, and Transformations

Topic: Investigating and using distance, midpoint, and slope formulas

Primary SOL: G.3 The student will solve problems involving symmetry and transformation. This will include

a) investigating and using formulas for finding distance, midpoint, and slope.

Related SOL: G.3b, G.8

Materials

- Deriving the Distance Formula activity sheet (attached)
- Deriving the Midpoint Formula activity sheet (attached)
- Lake Geometria activity sheet (attached)

Vocabulary

average (mean), distance, hypotenuse, leg (of a right triangle), length, midpoint, ordered pair, Pythagorean Theorem, right triangle, slope, x*-coordinate,* y*-coordinate*

Student/Teacher Actions: What should students be doing? What should teachers be doing?

1. Have students work independently, with a partner, or in small groups to complete the Deriving the Distance Formula activity sheet. Each student should record his/her own findings. Have students discuss findings within their group, and then discuss findings as a whole class.
2. Have students work independently, with a partner, or in small groups to complete the Deriving the Midpoint Formula activity sheet. Each student should record his/her own findings. Have students discuss findings within their group, and then discuss findings as a whole class.
3. Have students work independently, with a partner, or in small groups to complete the Lake Geometria activity sheet. Each student should record his/her own findings. Have students discuss findings within their group, and then discuss findings as a whole class.

Assessment

- **Questions**
    - Identify two different points that are 10 units away from point P (8, −1). Explain your reasoning.
    - M (1, −2) is the midpoint of $\overline{AB}$, and A has coordinates (3, 4). Find the coordinates of B. Write the midpoint as an ordered pair. Explain your reasoning.
- **Journal/Writing Prompts**
    - Summarize the activity in your journal.
    - Write a practical problem and solution that uses the distance formula (or midpoint formula.)

- Explain why distance is always positive. Justify your answer algebraically.

- **Other Assessments**
  - Have groups of four students construct a short quiz covering the information presented in the class in this lesson and administer it to another group in the class.
  - Have students find the distance between two other islands on Lake Geometria.
  - Have students identify a point (or a point on land) that is the same distance from two other islands.

Extensions and Connections (for all students)

- Have students estimate the area of Lake Geometria or Euclid.
- Ask students to find a point that is the same distance from three islands (circumcenter).

Strategies for Differentiation

- Review the following vocabulary with students: *coordinates*, *square*, *square root*, *absolute value*, *segment*
- Give students the formulas, and have them go through the activity and see whether they get the same formulas.
- More space and larger graphics may be necessary.
- Use a grid on the floor.
- Partners check with a switch.
- Use dynamic geometry software with the Deriving the Distance Formula and Deriving the Midpoint Formula activity sheets.

Note: The following pages are intended for classroom use for students as a visual aid to learning.

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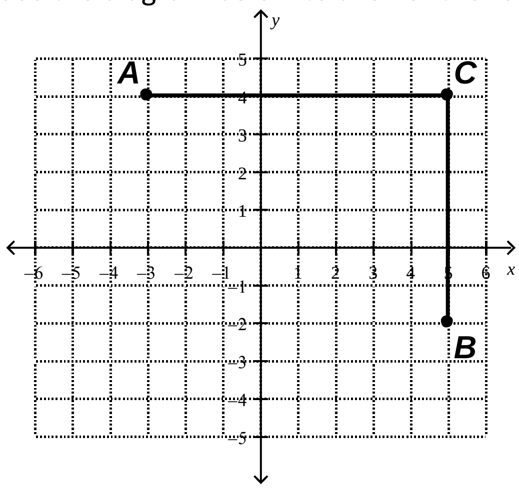
Deriving the Distance Formula

Name ______________________________________ **Date** _______________________

1. Use the diagram below to answer the following questions.



   a. What is *AC*?

   b. What are the coordinates of *A* and *C*?

   c. Use the coordinates of *A* and *C* to compute the length of $\overline{AC}$. Show your work.

   d. What are the coordinates of *C* and *B*?

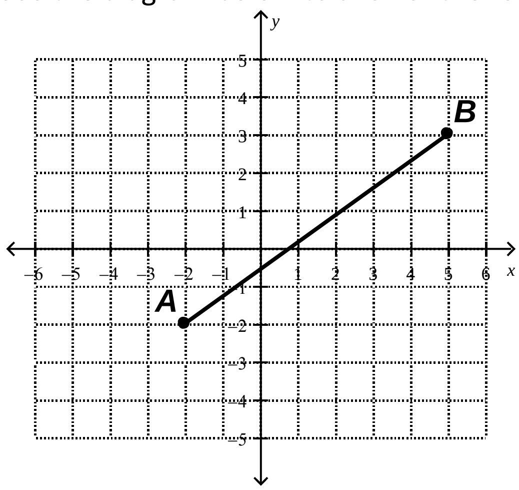
   e. Use the coordinates of *C* and *B* to compute the length of $\overline{CB}$. Show your work.

   f. Draw the segment $\overline{AB}$. What kind of triangle is ΔABC? For that kind of triangle, what are $\overline{AC}$ and $\overline{CB}$ called? What is $\overline{AB}$ called?

g. How can you find the length of the hypotenuse of a right triangle if you know the lengths of the two legs?

h. Use your answer to g to find the length of $\overline{AB}$.

2. Use the diagram below to answer the following questions.

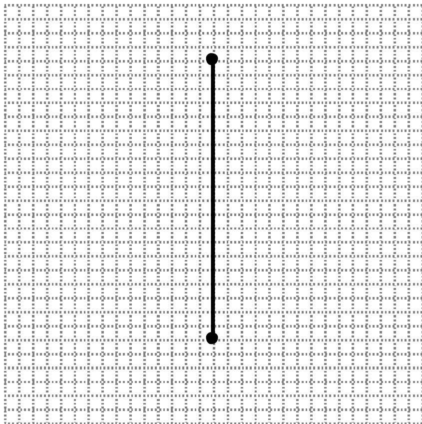


a. On the diagram above, create a right triangle with a horizontal leg, vertical leg, and hypotenuse $\overline{AB}$. Label the vertex of the right triangle *C*. Is this the only right triangle you could have drawn? Explain.

b. Find the length of $\overline{AC}$ and $\overline{CB}$.

c. Use the lengths of $\overline{AC}$ *and* $\overline{CB}$ to find the length of $\overline{AB}$.

3. The endpoints of a vertical segment $\overline{AB}$ are A (x_1, y_1) and B (x_2, y_2). Use this diagram for the following questions. *(Do not count. Graph is not to scale.)*



   a. Label the lower point *A* (x_1, y_1) and the upper point B (x_2, y_2). Because $\overline{AB}$ is a vertical segment, what can you say about x_1 and x_2 ?

   b. Express the length of $\overline{AB}$ in terms of y_1 and y_2 .

   c. Express the length of $\overline{AB}$ in a different way in terms of y_1 and y_2.

   d. Why is it necessary to use absolute value for the formulas above?

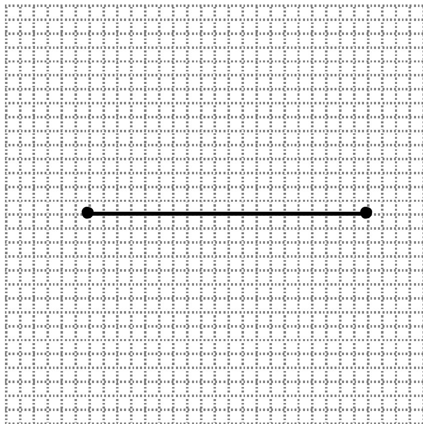
   e. Does it matter which of the two formulas above you use?

   f. Write a formula for the length of $\overline{AB}$ using either formula above.

      $\overline{AB}$ =

   g. The endpoints of a vertical segment are *G* (−10, 12) and *H* (−10, −22). Use one of your formulas to compute the length of $\overline{GH}$.

4. The endpoints of a horizontal segment $\overline{CD}$ are *C* (x_1, y_1) and *D* (x_2, y_2)**.** Use this diagram for the following questions*. (Do not count. Graph is not to scale.)*



   a. Label the point on the left *C* (x_1, y_1) and the point on the right *D* (x_2, y_2)**.** Since $\overline{CD}$ is a horizontal segment, what can you say about y_1 and y_2?

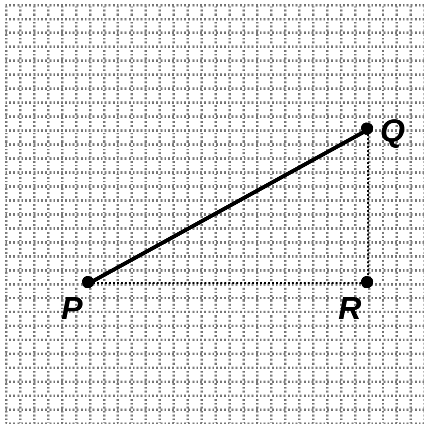
   b. Express the length of $\overline{CD}$ in two different ways in terms of x_1 and x_2.

   c. Write a formula for the length of $\overline{CD}$ using either formula above.

      $\overline{CD}$=

   d. The endpoints of a horizontal segment are *E* (−10, 12) and *F* (24, 12). Use your formula to compute the length of $\overline{EF}$.

5. The endpoints of a segment $\overline{PQ}$ are *P* (x_1, y_1) and *D* (x_2, y_2). Use this diagram for the



following questions. (Do not count. Graph is not to scale.)

a. Use the relationship between the legs and hypotenuse of a right triangle found in the Pythagorean theorem to complete the equation below:

$(PQ)^2 =$

b. Label the point *P* (x_1, y_1) as and point *Q* as (x_2, y_2). Find the coordinates of the point *R*.

c. Write formulas for the lengths of $\overline{PR}$ and $\overline{QR}$ using either formula above.

$\overline{PR} =$ $\qquad\qquad$ $\overline{QR} =$

d. Determine whether this equation is true. If it is true, explain why. If it is false, give a counterexample. (Hint: When you square a real number, is it ever negative?)

$|a|^2 = a^2$

e. Use the last three problems to get a formula for $(\overline{PQ})^2$ in terms of x_1, x_2 , y_1 , and y_2.

$(\overline{PQ})^2 =$

f. Take the square root of both sides of your last formula to write $\overline{PQ}$ in terms of x_1, x_2, y_1 , and y_2 .

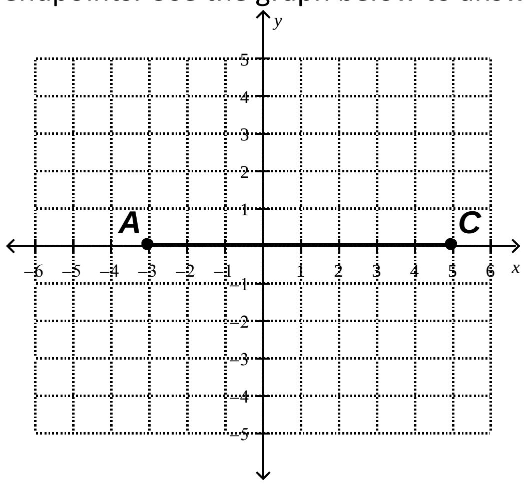
$\overline{PQ}$ =

g. The endpoints of a segment are *I* (8, 12) and *J* (2, 4). Use the formula you found to compute the distance between *I* and *J*.

Deriving the Midpoint Formula

Name ____________________________________ **Date** ______________________

1. The *midpoint* of a segment is the point on the segment that is the same distance from both endpoints. Use the graph below to answer the following questions.



   a. What are the *x*-coordinates of *A* and *C*? ________ and _________

   b. What is the average (or mean) of the *x*-coordinates of A and C? ________

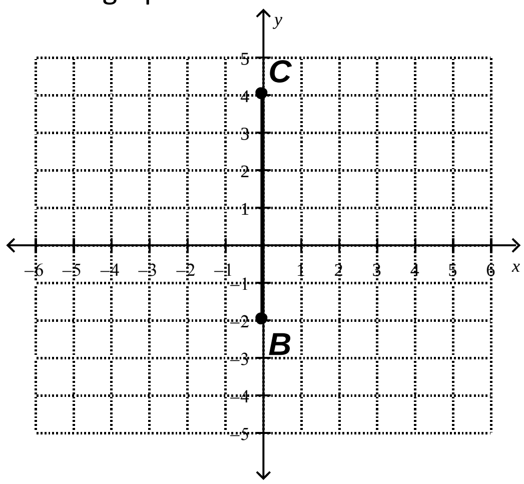
   c. What number is halfway between the *x*-coordinates of A and C? ________

   d. What are the coordinates of the midpoint of $\overline{AC}$? ________ Graph and label the midpoint.

   e. Explain the relationships among the answers to questions b, c, and d.

   f. The endpoints of a vertical segment are *G* (−10, 12) and *H* (–10, –22). Use your formula to compute the midpoint of $\overline{EF}$.

2. Use the graph below to answer the following questions.



a. What are the *y*-coordinates of *B* and *C*? ________ and _________

b. What is the average (or mean) of the *y*-coordinates of B and C? ________

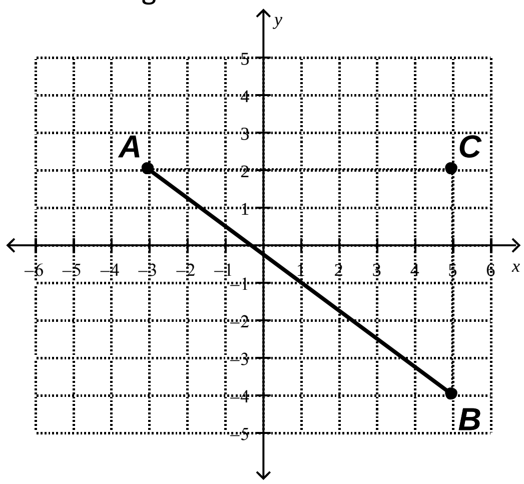
c. What number is halfway between the *y*-coordinates of B and C? ________

d. What are the coordinates of the midpoint of $\overline{BC}$? ________ Graph and label the midpoint.

e. Explain the relationships among the answers to questions b, c, and d.

f. B (0, –2) is the midpoint of segment $\overline{CD}$. If point C is located at (0, 4) as in the figure above, what would be the coordinate of point D?

3. Use the diagram below to answer the following questions.



a. Graph the midpoint of $\overline{AC}$. Label it *P*. What is the *x*-coordinate of this point? ______

b. What is the average (or mean) of the *x*-coordinates of A and C? ______ How is this related to your answer to a?

c. Graph the midpoint of $\overline{BC}$. Label it Q. What is the y-coordinate of this point? ______

d. What is the average (or mean) of the *y*-coordinates of B and C? ______How is this related to your answer to c?

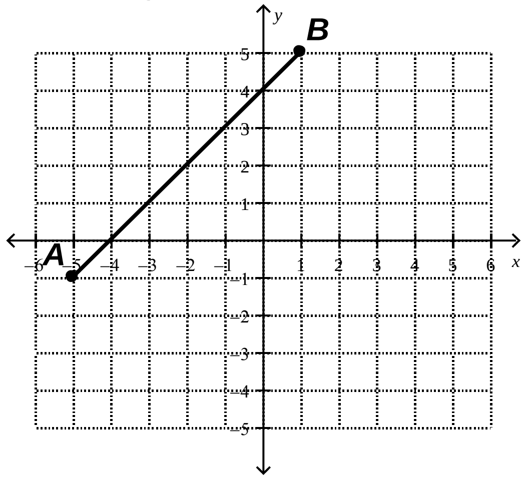
e. What is the average (or mean) of the *x*-coordinates of A and B? ______

f. What is the average (or mean) of the *y*-coordinates of A and B? ______

g. What is the midpoint of $\overline{BC}$?

h. How is the midpoint of $\overline{AB}$ related to the answers to 3e and 3f?

4. Use the diagram below to answer the following questions.



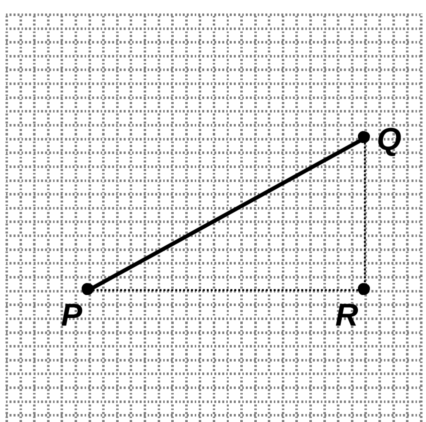
   a. What is the average (or mean) of the *x*-coordinates of *A* and *B*? ______

   b. What is the average (or mean) of the *y*-coordinates of *A* and *B*? ______

   c. What are the coordinates of the midpoint of $\overline{AB}$? ____________

   d. Explain the relationships among the answers to questions b, c, and d.

5. The endpoints of a segment $\overline{PQ}$ are *P* (x_1, y_1) and Q (x_2, y_2).



   a. Label the point *P* as (x_1, y_1) and point *Q* as (x_2, y_2).

b. Write a formula for the average (or mean) of the x-coordinates of P and Q.

c. Write a formula for the average (or mean) of the y-coordinates of P and Q.

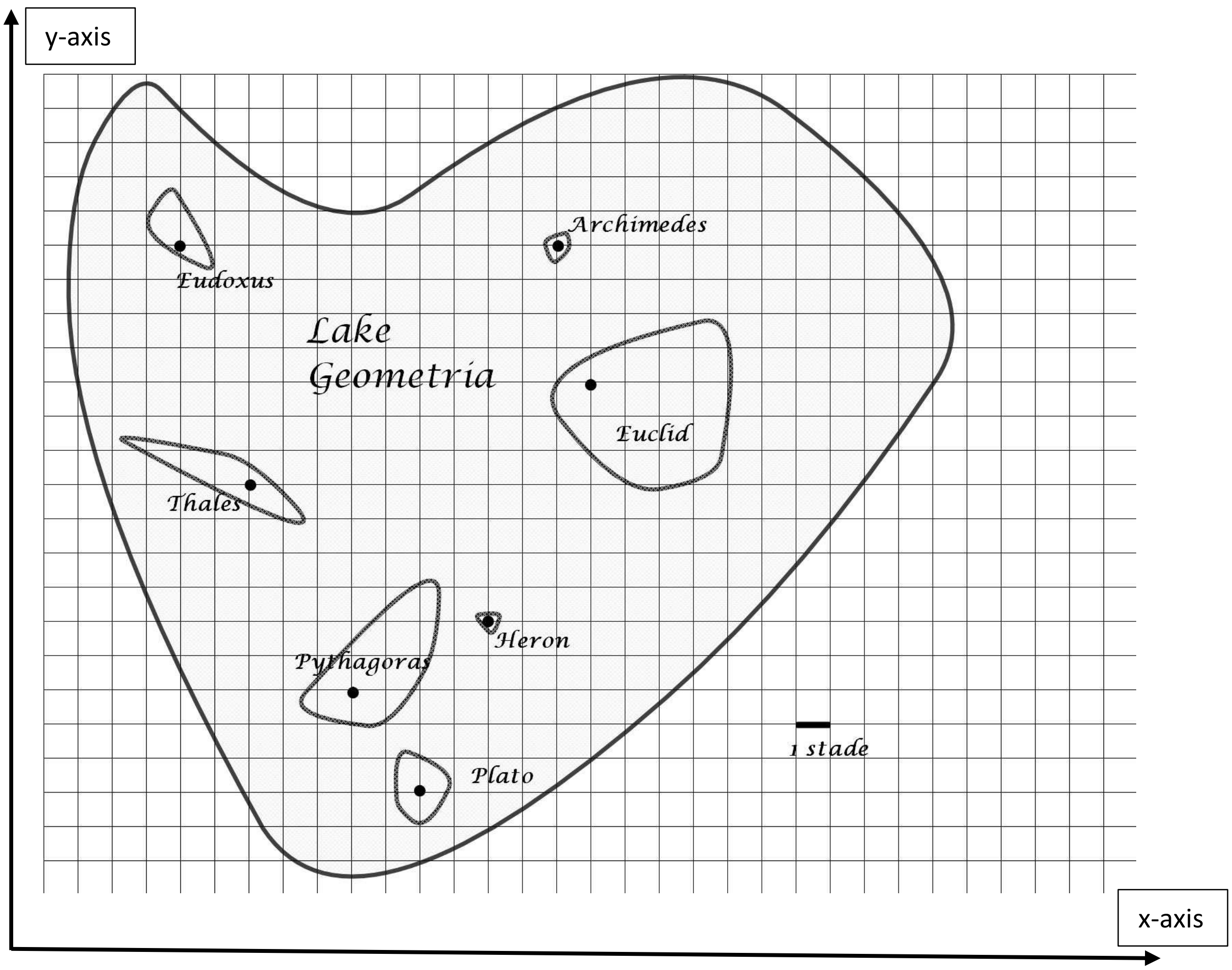
d. One way to think of the midpoint of $\overline{PQ}$ is as follows: average (or mean) of the x-coordinates, average (or mean) of the y-coordinates. Use this to derive a formula for the midpoint of $\overline{PQ}$.

e. The endpoints of a segment are E (8, 12) and F (2, 4). Use your formula to compute the midpoint of $\overline{EF}$.

Lake Geometria

Name ____________________________________ **Date** ____________________________

The islands of Lake Geometria are shown below. A cabin is marked on each island. The scale, using units called stades, is shown in the lower right. A stade measures about 600 feet, so Lake Geometria is not very big. Use the grid to help you answer the following questions. All island



measurements should be made from cabin to cabin.

1. What is the distance in stades from Eudoxus to Archimedes? (Round to the nearest stade.) Describe how you found your answer.

2. What is the distance in stades from Thales to Euclid? (Round to the nearest stade.) Describe how you found your answer.

3. Which is closer to Thales—Pythagoras or Heron? Describe how you found your answer.

4. Find a point **in the water** that is the same distance from Archimedes and Eudoxus. Label the point *M*. Describe how you found this point.

5. Now find a point **on land** that is the same distance from Archimedes and Eudoxus. Label this point *N*.

6. How many points can you find that are the same distance from Archimedes and Eudoxus? Explain.

7. Find a point **on land** that is the same distance from Thales and Pythagoras. Is your point on the mainland or on an island? Does it have to be? Explain.

8. Groups staying on any of the islands are provided with solar-charged walkie-talkies, but their ranges are only about 1 mile. There are about 9 stades in a mile. With which islands could someone staying at the cabin on Thales expect to be able to communicate? Show your work, or explain how you found your answer.

9. Estimate the shortest distance from Thales to the mainland. (You may use stades, feet, or miles.) Show your work or explain how you found your answer.

10. Estimate how many miles wide Lake Geometria is at its widest point. Explain how you found your answer.

11. If you sailed from Thales to Pythagora, and your friend sailed from Eudoxus to Heron, would you be sailing parallel to each other? How do you know? Show you work.

12. Is the route from Pythagora to Euclid, perpendicular to the route from Pythagora to Plato? How do you know? Show your work.

13. One ship leaves Archimedes and sails to Euclid, while another ship leaves Eudoxus and sails to Thales. Are they sailing parallel to each other? How do you know? Justify your answer.

14. A buoy located at the midpoint between the islands of Eudoxus and Archimedes. Place an ‘X’ on the grid where the buoy would be located.

15. A lighthouse is located at the midpoint between the islands of Pythagora and Euclid. Place a small circle on the grid where the lighthouse would be located.