Probability Density

Strand: Statistics

Topic: Probability

Primary SOL: AFDA.7c The student will
                 c) apply properties of normal distributions to determine probabilities associated with areas under the standard normal curve.

Related SOL: AFDA.6

Materials

- Probability Density activity sheet (attached)
- Graphing utility
- Highlighters
- Colored pencils
- Rulers
- Graph paper

Vocabulary

conditional probability, dependent events, independent events, law of large numbers, mutually exclusive events, normal distribution, normal curve, probability, z-score

Student/Teacher Actions: What should students be doing? What should teachers be doing?

Time: 90 minutes

1. Students will investigate how a standard normal curve can be used to calculate probabilities. Students will connect the ideas of probability and percentile. Students will also link the concept of area under the curve with probability and learn to shade the appropriate region under a standard normal curve to represent a certain probability.

2. Graphing utilities are helpful in visualizing probability densities. Desmos and Geogebra are useful graphing utilities for this activity. Geogebra has a helpful tutorial.

3. Distribute copies of the Probability Density activity sheet. Students can work individually or in pairs to complete the exercises. The teacher should monitor students while working to provide feedback and support.

4. Students will share their responses to the exercises with a neighbor. Each student shares their learning related to the following questions:
   a. How did you calculate the probability of where a specific data point lies in the normal distribution?
   b. How is the process similar or different to calculating probabilities of events?
   c. What information would have been helpful to know before attempting to solve the exercises?

Assessment

- **Questions**
  - How do you represent probability as area under the curve of a standard normal distribution?
  - How can you use the table of standard normal probabilities and a graphing utility to determine normal distribution probabilities?

- **Journal/writing prompts**
  - What is the relationship between the area under the curve and probabilities?
  - How can we calculate the probability of an event using a normal distribution?

- **Other Assessments**
  - Have students create a problem scenario related to probability density (i.e., infant birth weights). Students will share the problem with another group. The second group will come up with a solution and present it to the class.
  - Have students practice the concepts using Khan Academy exercises.
    1) "Normal Distributions Practice: Empirical Rule"
    2) "Normal Distribution Calculations Practice: Normal Distribution: Area Above or Below a Point"
    3) "Normal Distribution Calculations Practice: Normal Distribution: Area Between Two Points"
    4) "Normal Distribution Calculations: Normal Calculations in Reverse"

Extensions and Connections

- Have students connect their work in calculating probability densities to specifying the probability of the random variable falling within a particular range of values instead of taking on any one value.
- Ask students to research the connections between science and probability density. There are relationships in chemistry, biology, and physics.

Strategies for Differentiation

- Students who need a larger graph can create their own on graph paper.
- Colored pencils can be used to shade in the area under the curve. Different colors can be used for greater than or less than.
- Use vocabulary cards for related vocabulary listed above.
- Highlighters or rulers can be used to track lines in probability tables.
- Graphing calculators support computation for students who struggle with setting up the probability density function in a graphing utility.

Note: The following pages are intended for classroom use for students as a visual aid to learning.

Probability Density

If an experiment generates outcomes that are normally distributed, the standard normal curve can be used to calculate probabilities of specific outcomes.

For example, the life span of a particular machine gasket is normally distributed, with a mean of 750 hours with a standard deviation of 25 hours. Use a graphing utility, such a Desmos or Geogebra, to re-create the graph below. The graph of the distribution is below:
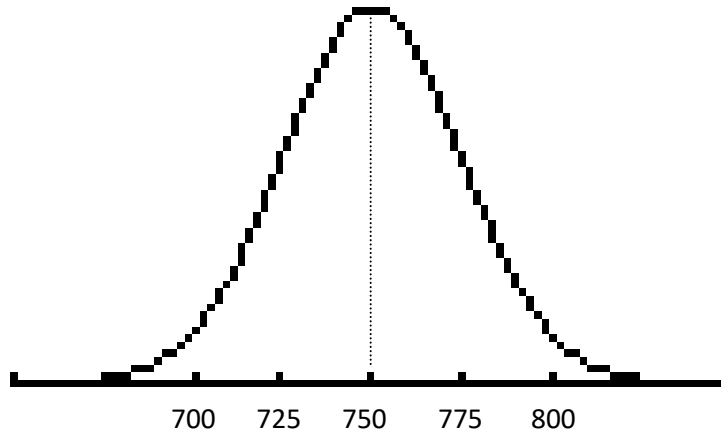
The accumulated area up to 775 represents the **probability** that the gasket will last 775 or fewer hours.

The shaded area below represents the **probability** that the gasket will last 775 or fewer hours. The corresponding numerical probability is determined using a z-score.
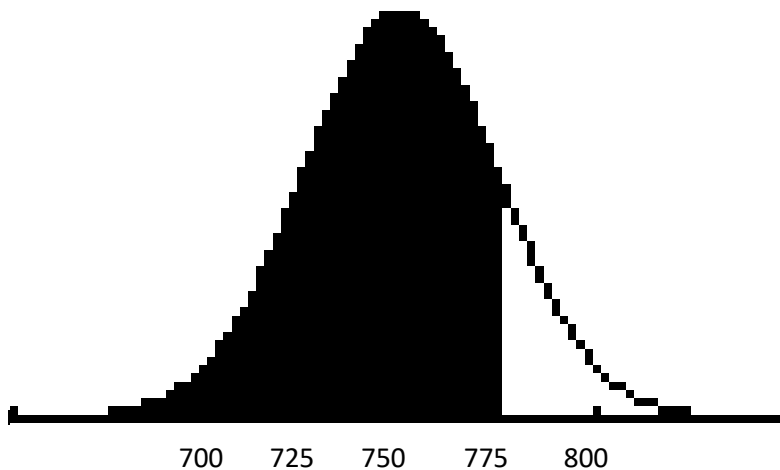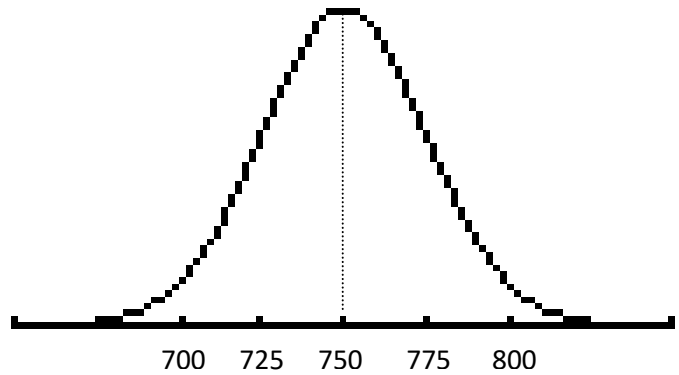

With $z = \dfrac{775 - 750}{25} = 1$ and using the Standard Normal Table, we can see

P(x < 775) = 0.841

Using the same machine gasket example, shade the region representing the given probability and then determine the numerical probability using the table or a calculator.
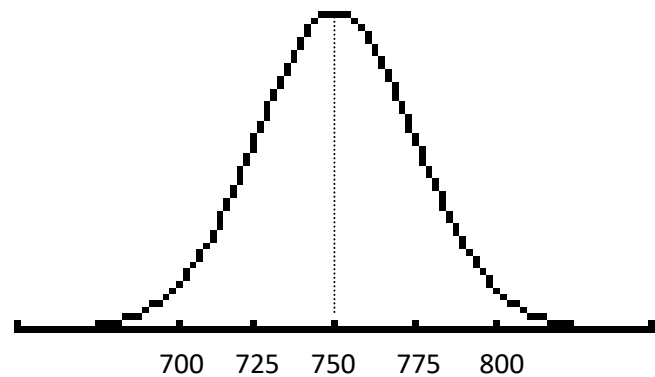
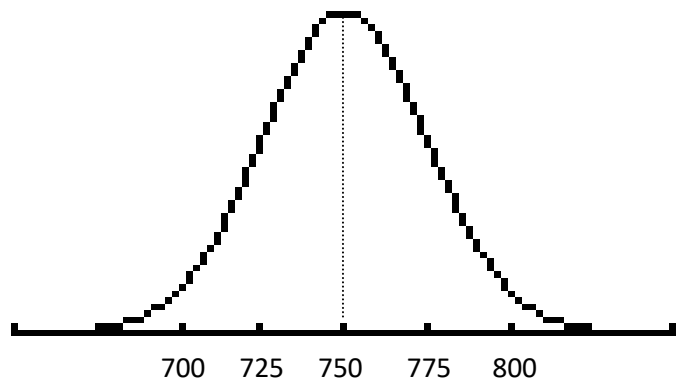1. The probability the gasket will last more than 730 hours.

   P(x > 730)

   700   725   750   775   800

2. The probability the gasket will last less than 785 hours.

   P(x < 785)

   700   725   750   775   800

3. The probability the gasket will last
    between 730 and 785 hours.

    P(730 < x < 785)

4. The probability that the gasket will last
    more than 780 hours or less than
    720 hours.

    P(780 < x or x < 725)