Function Transformations

Strand: Algebra and Functions

Topic: Reflecting, Dilating and Translating Functions

Primary SOL: AFDA.2 The student will use knowledge of transformations to write an equation, given the graph of a linear, quadratic, exponential, and logarithmic function.

Related SOL: AFDA.1

Materials

- Function Transformations activity sheet (attached)
- Transformation Cut-outs activity sheet (attached)
- Transformations Practice activity sheet (attached)
- Graphing utility
- Graph paper

Vocabulary

dilation, exponential, quadratic, linear, logarithmic, reflection, translation

Student/Teacher Actions: What should students be doing? What should teachers be doing?
Time: 90 minutes

1. Distribute copies of the Transforming Functions activity sheet to students. Students should work through the problems using a graphing utility and sketching the graphs on the given coordinate planes. Check in with groups or stop the class periodically to review their work and clarify any misconceptions.
2. Distribute copies of the Transformations Cut-outs activity sheet to students. This activity can be done by each student individually or in groups. Students should match each description with the equation and the graph.
3. Distribute copies of the Transformations Practice activity sheet to students. This activity gives students additional practice in describing transformations, writing equations from transformations, and graphing transformations.

Assessment

- **Questions**
  - Why do the graphs of $f(x)=x^2$ and $g(x)=(-x)^2$ look the same?
  - Write equations of two different quadratic functions with the same vertex.
  - The point (1, 5) is on the function $k(x)$. What is the corresponding point on the function $k(x-2)+1$?
  - The exponential function $y=2^x$ has a horizontal asymptote at $y=0$. What transformation would make the horizontal asymptote at $y=-4$?
- **Journal/writing prompts**

  - Does the order in which we do our transformations matter? If so, why?
  - Which transformations affect the domain of the function? Why?

- **Other Assessments**
  - Use individual whiteboards to practice graphing equations given equations or writing equations given graphs.

Extensions and Connections

- Using a quadratic function, write an equation so that the function would have a y-intercept of 3 and an x-intercept of 1.

Strategies for Differentiation

- Use tracing paper to help students translate or reflect a given graph of a function.
- Scaffold the activity by reviewing transformations of geometric shapes before beginning transformations of parent functions.
- Use vocabulary cards for related vocabulary listed above.

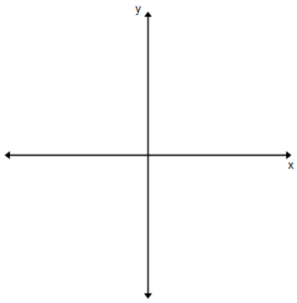
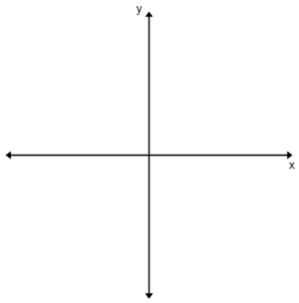
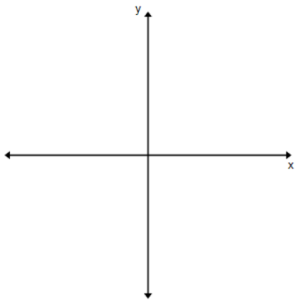
Note: The following pages are intended for classroom use for students as a visual aid to learning.

Function Transformations

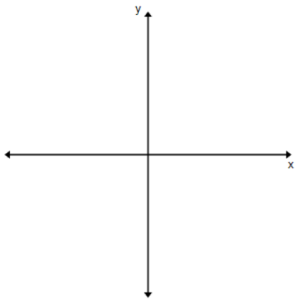
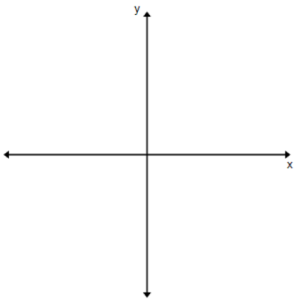
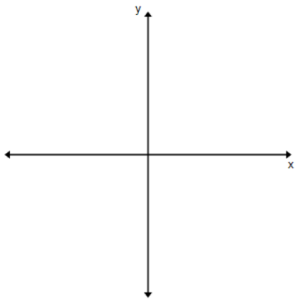
A ***reflection*** is a movement where a graph "flips" over an axis (or another designated line of reflection). It is called a reflection because it will be a mirror image of the original.

Sketch the graph of each function below on the given graph.

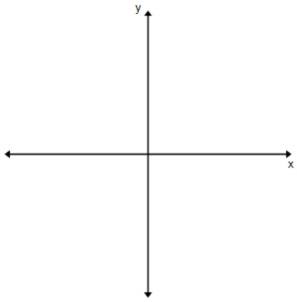
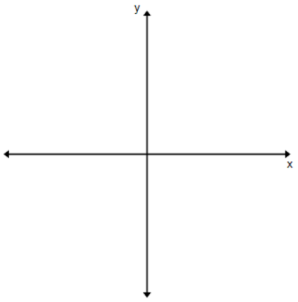
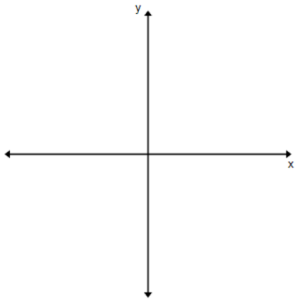
1. a) $y = x^2$ b) $y = -x^2$ c) $y = (-x)^2$

2. a) $y = e^x$ b) $y = -e^x$ c) $y = e^{-x}$

3. a) $y = \ln x$ b) $y = -\ln x$ c) $y = \ln(-x)$

What do you notice about the graphs of $-f(x)$ in each problem b above?

What do you notice about the graphs of $f(-x)$ in each problem c above?

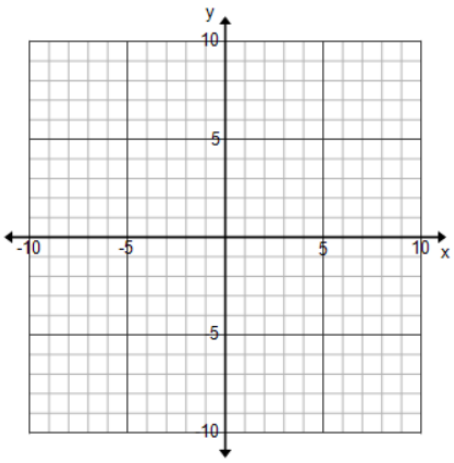
A ***dilation*** is a transformation that enlarges or shrinks a graph.

Graph each below on the same graph. Use different colored pencils to graph each, so you can compare your graphs.

4. a) $y = x^2$

   b) $y = 2x^2$

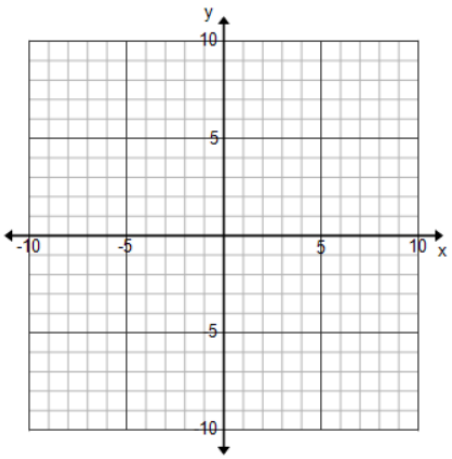
   c) $y = 0.5x^2$



5. a) $y = e^x$

   b) $y = 2e^x$

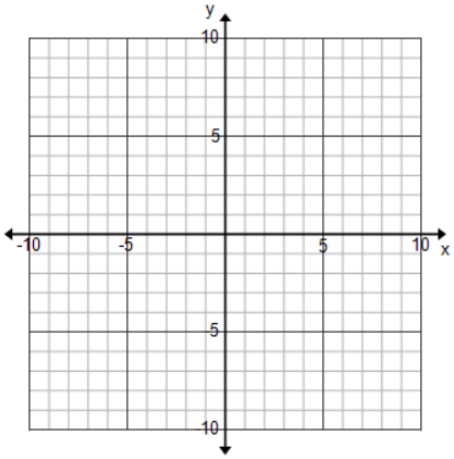
   c) $y = 0.5e^x$



6. a) $y = \ln x$

   b) $y = 2\ln x$

   c) $y = 0.5\ln x$



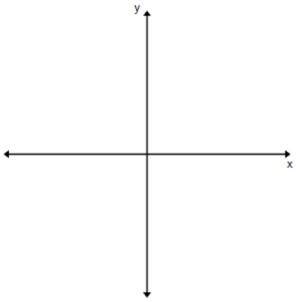
What do you notice about the graphs of $af(x)$ when $a > 1$, in each b above?

What do you notice about the graphs of $af(x)$ when $a < 1$, in each c above?

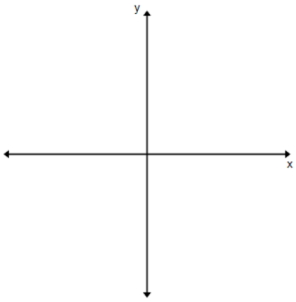
A ***translation*** is a transformation that involves sliding a graph vertically or horizontally.

Sketch the graph of each function below on the given graph.

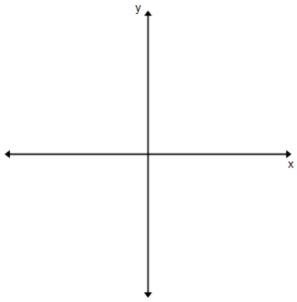
7. a) $y = x^2$



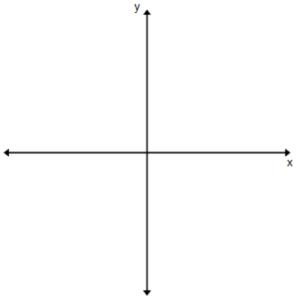
b) $y = x^2 + 3$



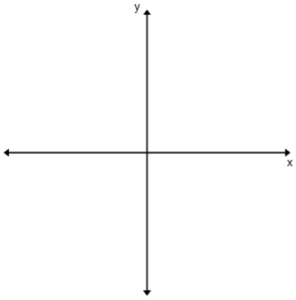
c) $y = x^2 - 3$



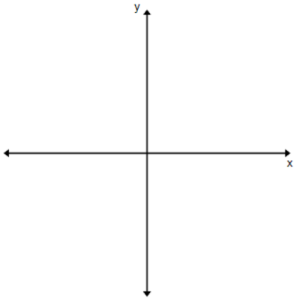
d) $y = (x-3)^2$



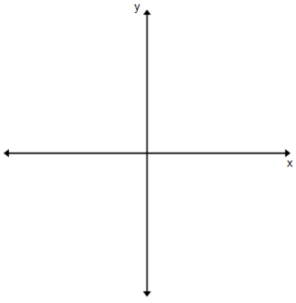
e) $y = (x+3)^2$



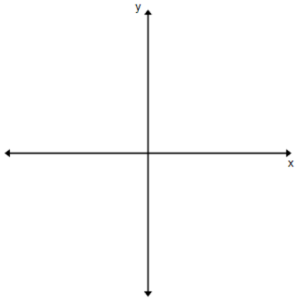
8. a) $y = e^x$



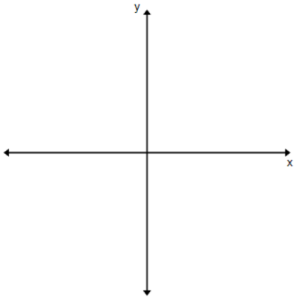
b) $y = e^x + 3$



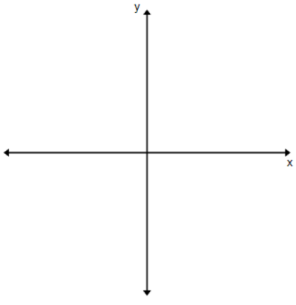
c) $y = e^x - 3$



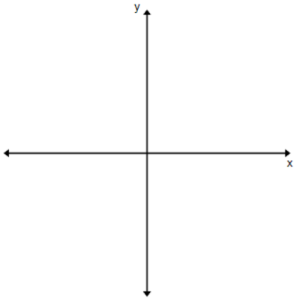
d) $y = e^{x-3}$



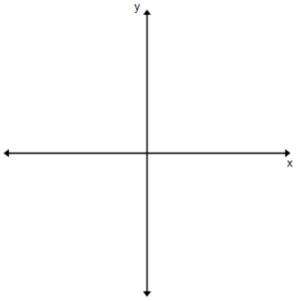
e) $y = e^{x+3}$



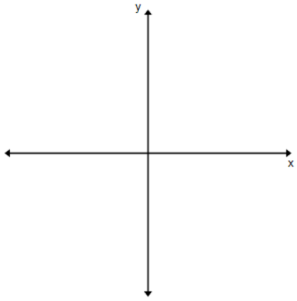
9. a) $y = \ln x$



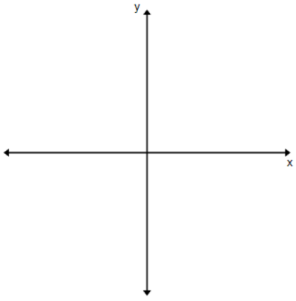
b) $y = \ln x + 3$



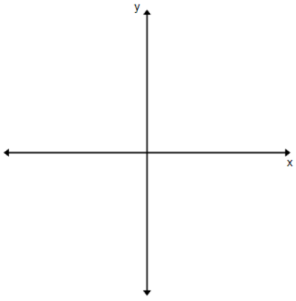
c) $y = \ln x - 3$



d) $y = \ln(x - 3)$



e) $y = \ln(x + 3)$

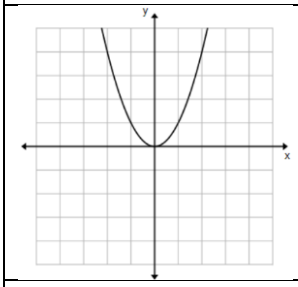
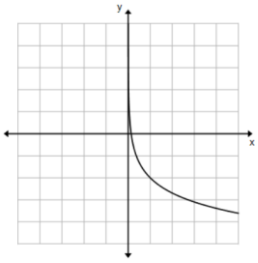
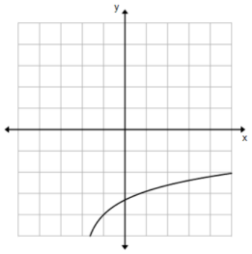
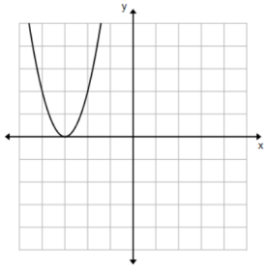
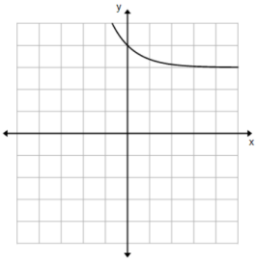
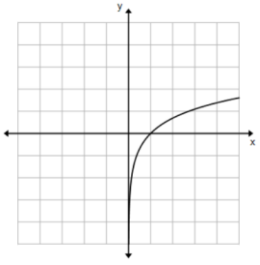
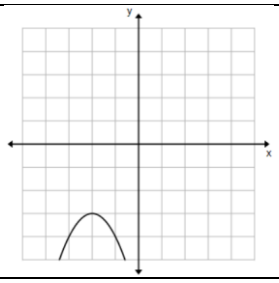
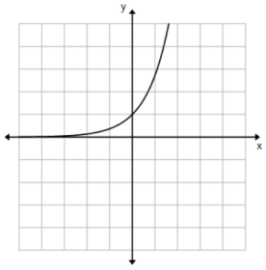
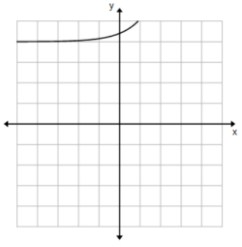
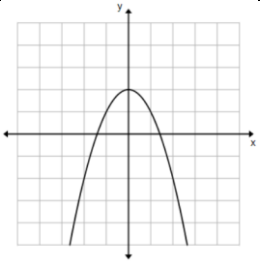
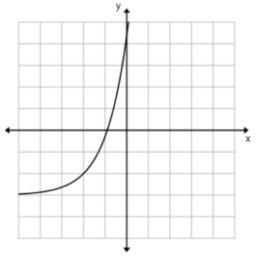
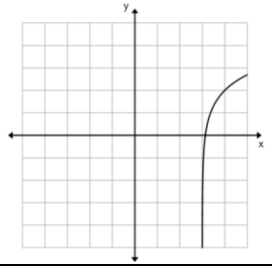


What do you notice about the graphs of $f(x) + k$ in b) and c) above?

What do you notice about the graphs of $f(x - h)$ in d) above?

What about the graphs of $f(x + h)$ in e) above?

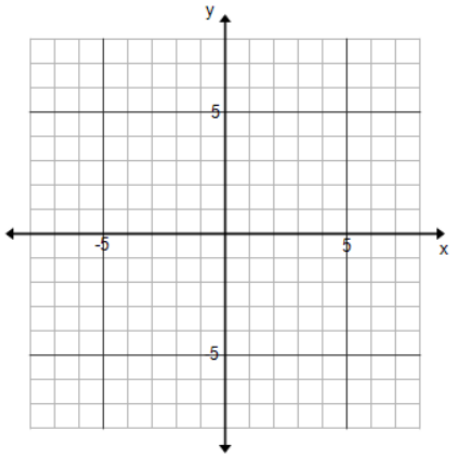
Transformation Cut-outs

Quadratic Reflect x-axis Translate up 2	Exponential Reflect y-axis Translate up 3	Logarithmic Translate up 2 Translate right 3	Quadratic Parent function
Exponential Parent function	Logarithmic Parent function	Quadratic Dilate by 2 Translate left 3	Exponential Translate down 3 Translate left 2
Logarithmic Reflect x-axis Translate down 2	Quadratic Reflect x-axis Translate down 3 Translate left 2	Exponential Dilate by 3 Translate right 2 Translate up 4	Logarithmic Translate left 2 Translate down 4
$y = -(x + 2)^2 - 3$	$y = e^{x+2} - 3$	$y = \ln(x - 3) + 2$	$y = x^2$
$y = e^x$	$y = -\ln(x) - 2$	$y = -x^2 + 2$	$y = e^{-x} + 3$
$y = \ln(x + 2) - 4$	$y = 2(x + 3)^2$	$y = 3e^{x-2} + 4$	$y = \ln x$
			
			
			

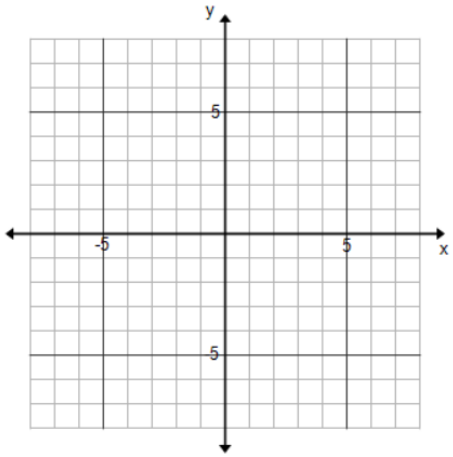
Transformations Practice

Describe the transformations of the parent function and sketch the graph.

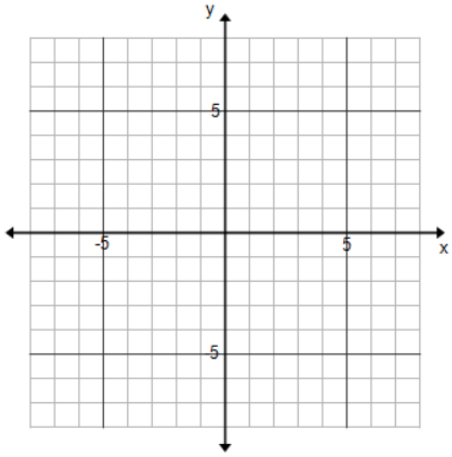
1. $y=(x-3)^2+4$



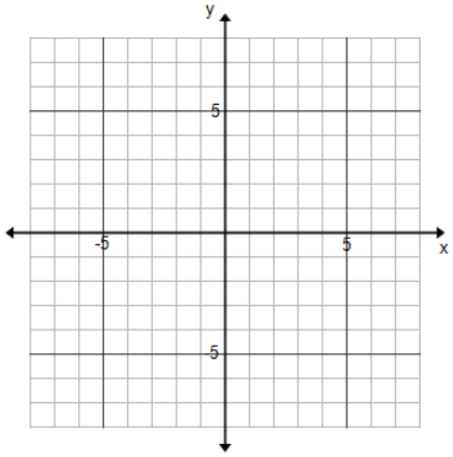
2. $y=2e^{x+4}$



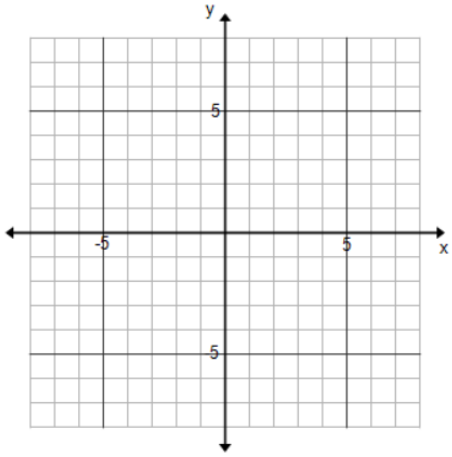
3. $y=-\ln(x-3)+1$



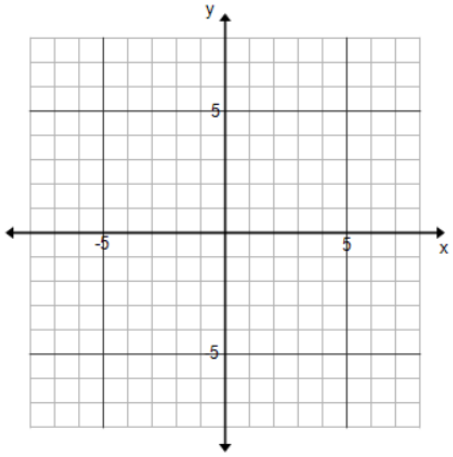
4. $y=\ln(-x)-4$



5. $y=0.5(x+4)^2-2$



6. $y=-x+3$



Write the equation of each function described below.

7. An exponential function reflected across the y-axis and translated up 3.

8. A quadratic function that is reflected across the x-axis, translated right 2 and down 4.

9. A logarithmic function dilated by a factor of 2 and translated left 1 and up 5.

10. A linear function translated right 2.