

Curve of Best Fit

Strand: Statistics

Topic: Collecting and analyzing data, using curve of best fit

Primary SOL: All.9 The student will collect and analyze data, determine the equation of the curve of best fit in order to make predictions, and solve practical problems, using mathematical models of quadratic and exponential functions.

Related SOL: All.6

Materials

- Curve of Best Fit Introductory Exercise activity sheet (attached)
- Curve of Best Fit Exploration activity sheet (attached)
- Data and Regressions, Part 1 activity sheet (attached)
- Data and Regressions, Part 2 activity sheet (attached)
- Candy Experiment activity sheet (attached)
- Music and Mathematics activity sheet (attached)
- The Ewok-Jawa Problem activity sheet (attached)
- Graphing utility
- Sets of six card-stock circular regions with radii of 1 centimeter, 2 centimeters, 3 centimeters, 4 centimeters, 5 centimeters, and 6 centimeters
- One-centimeter grid paper
- Paper cups
- Large bag of M&Ms
- Paper towels

Vocabulary

equations, extraneous solution, index, line of best fit, mathematical model, radical algebraic radicand, scatterplot, parent functions

Student/Teacher Actions: What should students be doing? What should teachers be doing?

Time: 90 minutes

1. Distribute the Curve of Best Fit Introductory Exercise activity sheet. Have students complete it. (Note: The goal is to have students reflect on the general shape of each sample graph and use that knowledge to guide them in choosing a model that will likely best fit specific data.)
2. Distribute the Curve of Best Fit Exploration activity sheet. Have students work in pairs to draw curves of best fit for the sets of data. For each set of data, have them describe the model that best fits the data as either a quadratic or an exponential function.
3. Distribute the sets of six circular regions, the grid paper, and the Data and Regressions, Part 1, activity sheet. Have students work individually or in pairs to complete the handout. (Note: The more circular regions students have to measure, the better the scatterplots will look.)

4. Distribute the Data and Regressions, Part 2, activity sheet. Have students work individually or in pairs to complete it. (*Note: This data is interesting. During certain years, the regression “appears” to be linear, while at other times, it takes on different shapes. This could be used as an introduction to linearization in calculus.*)
5. Divide the class into groups of three or fewer. Give each group a large cup filled with M&Ms, a paper towel, and a copy of the Candy Experiment handout. Have each group complete the handout. (*Note: In the data-collection process, the number of candies remaining usually will not go to zero due to the manufacturing process. However, if the number does go to zero, this data point needs to be recorded as 0.01.*)
6. Distribute the Music and Mathematics activity sheet. Have students complete the activity. (*Note: Make sure that students understand the explanation given at the top of the handout. They will have studied this in Physical Science class (grade 8), so this should be a review of what they know.*)

Assessment

- **Questions**
 - What needs to be considered when determining a model for best fit?
 - What might you see in data that is best modeled by a logarithmic model? By an exponential model?
- **Journal/writing prompts**
 - When determining whether a set of data is best represented by a quadratic or exponential function, what type of characteristics are you looking for?
 - Explain how you would know, given a set of data points, that there is no line of best fit (either quadratic or exponential).
- **Other Assessments**
 - Pair students, and have one of them draw a set of data that correlates with a quadratic or exponential function. The other student then must state the line of best fit. Switch roles and repeat.
 - Create an exit slip which gives a set of data points, and have students determine the equation of the line of best fit.

Extensions and Connections

- Have students solve the Ewok-Jawa Problem activity sheet.
- Have students research what kind of analysis a statistician does when finding a curve of best fit.
- Discuss the method of least squares as a way for finding linear regression, including how it might apply to other nonlinear models.
- Have students put data in a graphing utility and try a few regression options. Direct them to check the correlation coefficient to determine the model that best fits the data.

Strategies for Differentiation

- Have students create and use flash cards with scatterplots on one side and the names of the appropriate curves of best fit on the other.

Mathematics Instructional Plan – Algebra II

- Use web applets to manipulate curves of best fit on an interactive whiteboard.
- Provide students with a curve on graph paper, and ask them to identify data points that would make it the curve of best fit for that data.

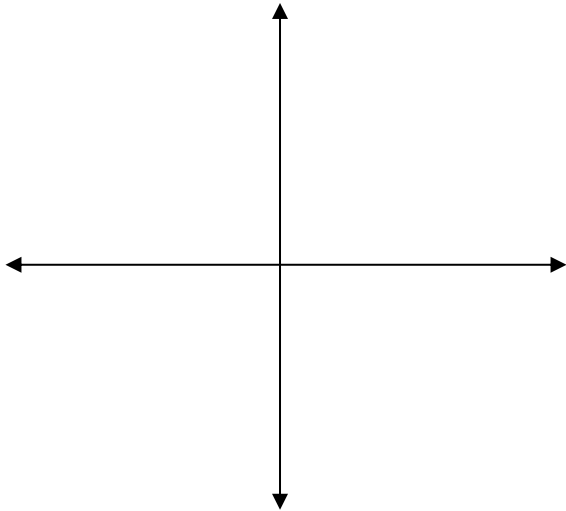
Note: The following pages are intended for classroom use for students as a visual aid to learning.

Virginia Department of Education © 2018

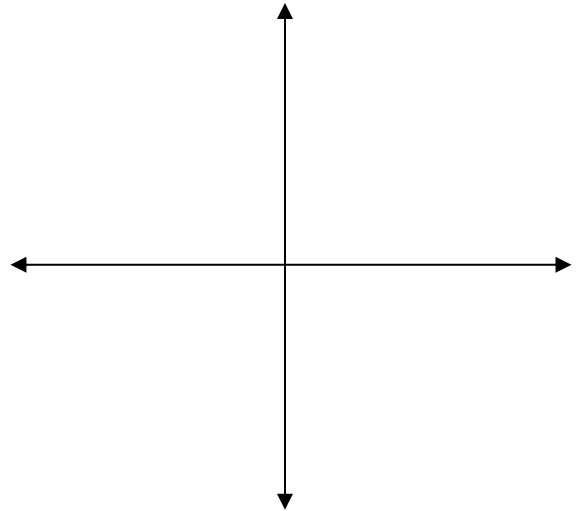
Curve of Best Fit Introductory Exercise

Draw two different sample graphs of both quadratic and exponential functions.

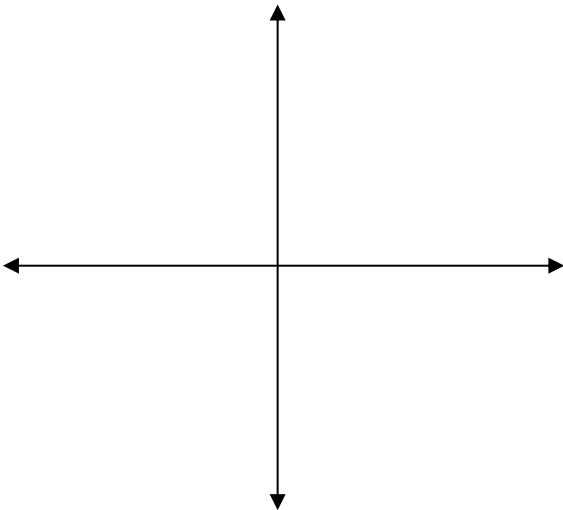
Quadratic Function



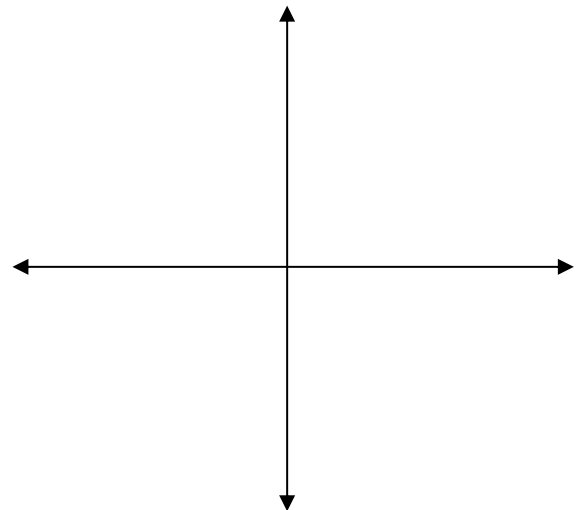
Quadratic Function



Exponential Function



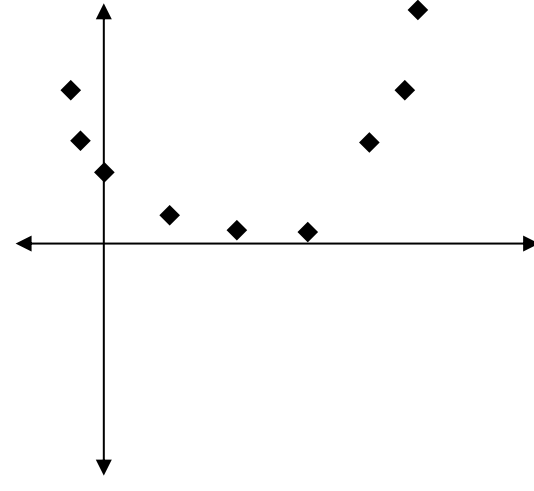
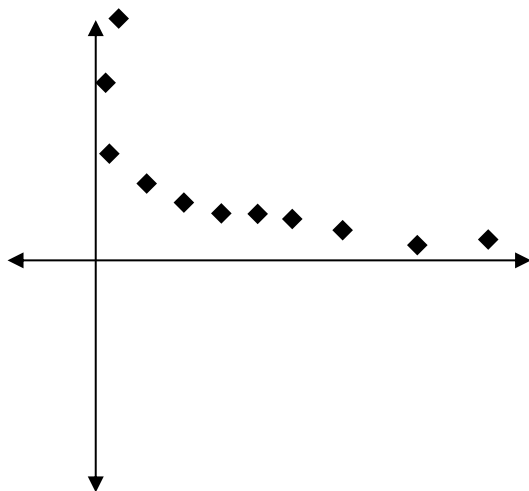
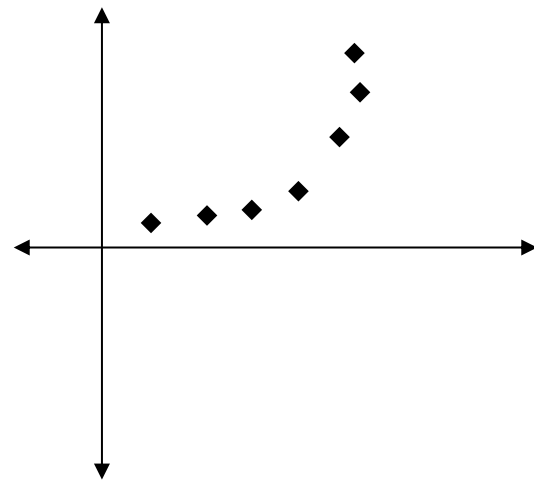
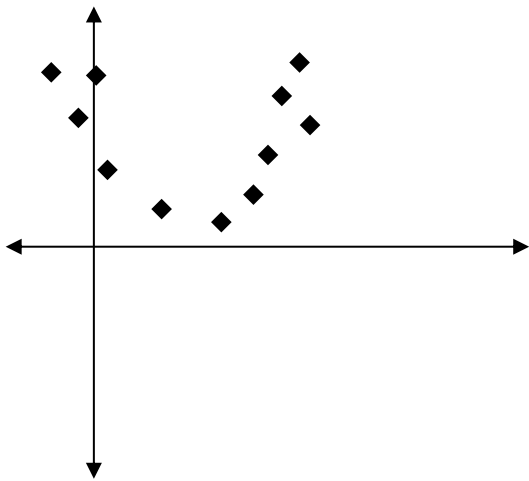
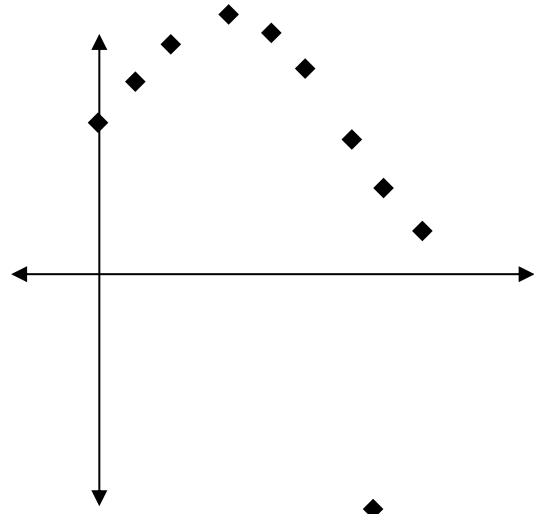
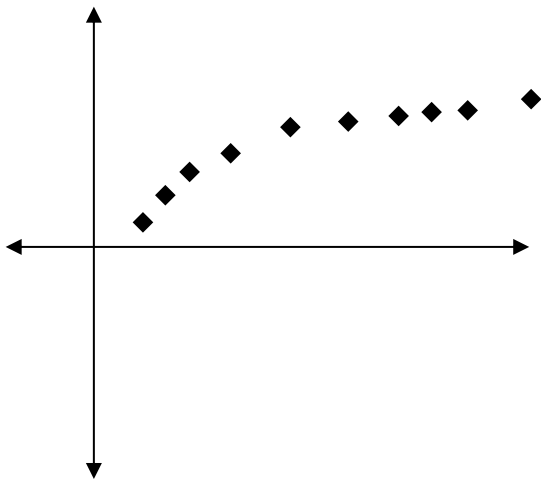
Exponential Function



Describe each graph in your own words.

Curve of Best Fit Exploration

Draw and describe the graph of a model that best fits each set of data.



Data and Regressions, Part 1

1. Place the provided circular regions on the centimeter grid paper and trace them.
2. Count the number of square centimeters in the area of each circular region, and record the data in the table.

Radius of circular region	Area: number of square centimeters
1 cm	
2 cm	
3 cm	
4 cm	
5 cm	
6 cm	

3. Input the data from the table above into a graphing utility or a virtual graphing tool (e.g. Desmos, Excel).

4. Generate the scatterplot.

5. Examine the scatterplot carefully, and decide whether the plot is best modeled by a quadratic or exponential function.

6. Find the regression equation that you believe best fits your data, using the regression equation capabilities of the graphing utility and your knowledge of function families. Graph the curve through the data points.

7. Using the equation of the curve, predict the area of a circular region with a radius of 12 cm.

8. What would be the radius of a circular region that covers 450 square cm? (Hint: Work backwards to find the answer.)

1. Using the data in the table at right, enter the year, the total number of marriages, and the total number of divorces into a graphing utility (table).
2. Calculate the divorce rate for each year and add that data column to your table.
3. Which type of regression best fits the divorce rate data from 1960 to 1976?
4. If the pattern continued in this way, what would be the predicted divorce rate in 1994?

Data and Regressions, Part		
2 Marriages and divorces in the United States, 1960–1992		
Year	Marriages	Divorces
1960	1,523,000	393,000
1962	1,557,000	413,000
1964	1,725,000	450,000
1966	1,857,000	499,000
1968	2,069,258	584,000
1970	2,158,802	708,000
1972	2,282,154	845,000
1974	2,229,667	977,000
1976	2,154,807	1,083,000
1978	2,282,272	1,130,000
1980	2,406,708	1,182,000
1982	2,495,000	1,180,000
1984	2,487,000	1,155,000
1986	2,400,000	1,159,000
1988	2,389,000	1,183,000
1990	2,448,000	1,175,000
1992	2,362,000	1,215,000

5. Which type of regression best fits the divorce rate data from 1978 to 1992?
6. Based on the data, what would be the predicted divorce rate for 2020?

Extension

7. What do you notice about the data trends?
8. Do you find anything “odd” about this data? If so, explain.

Candy Experiment

The following experiment is designed to simulate a natural occurrence.

Part I. Data Collection

1. Count the M&Ms in the cup given to your group and record this number in the chart to the right of toss number zero.
2. Shake the cup, and carefully dump the candies on the paper towel. Remove and set aside those candies that landed with the *M* showing. Count the pieces remaining, and record this number in the chart to the right of toss 1. Pour these pieces back in the cup.
3. Again, shake the cup, dump the pieces, remove and set aside those with the *M* showing, count the remainder, record the number, and pour the remainder back in the cup.
4. Keep repeating the process until no *M*s appear.
5. Graph on graph paper the collected data from the chart.

Toss number	Number remaining
0	
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	
11	
12	
13	
14	

Part II. Data Analysis

1. The data collected can be modeled with an exponential curve of the form $y = ab^x$. Using the graph from Part I, develop an equation to fit the data collected.

2. Give the coordinates of the beginning point in your data.

3. What information does this give about the equation being developed?

4. Locate a second point on the graph. Using this specified point and the original data point, find the value of b in the general equation.

- Write the completed equation for the graph.
- How well does this equation predict the remaining data on the graph? Check the generated value for y and the graphed value for y for several chosen values of x .
- How “good a fit” is the equation that you developed?

Music and Mathematics

Stringed instruments, such as violins and guitars, produce the different pitches of a musical scale through different vibration frequencies. The frequency of a string depends on the vibrating length of the string and the tension on it. For a string under a constant tension, the frequency varies inversely with its vibrating length (i.e., the shorter [smaller] the string, the higher [larger] its frequency; the longer [larger] the string, the lower [smaller] its frequency).

- Complete the table to find the vibrating length of the string for each pitch of a C-major scale. Round your answers to the nearest whole number. Assume that each string is under the same tension.

Pitch	C	D	E	F	G	A	B	C
Vibrating frequency (cycles/sec.)	523	587	659	698	784	880	988	1046
Vibrating length of string (mm)	420							

- Write a function that models this variation.
- Describe how the values of the frequency change in relation to the vibrating length of the string.
- Make a scatterplot of your data. Describe the graph.

The Ewok-Jawa Problem

1. A band of 45 Ewoks crash-landed in the forest last night. This sounds like a small problem, but the population will grow at the rate of 22 percent per year. Write an equation to describe the population in any given year. Also, create a table that shows the Ewok population every five years from this year to the year 2050.
2. Coincidentally, a band of Jawas crash-landed near the Ewoks. The Jawas have a population growth modeled by the equation $A = 105(0.91)^t$, where A is the population at any time, t , given in years. Is this population increasing? Decreasing? Stagnant? Explain your reasoning.
3. In 2015, the two tribes begin fighting, and the casualties of war offset the Ewok birth rate. Write the compound equation to describe the Ewok population from 2015 to 2050.
4. The Jawas lose 15 percent of their members each year in battle. When will their tribe be extinct?
5. Using a graphing utility, graph and label the equations for each of the four scenarios above. Include a brief explanation of each graph's implications.