Composition of Functions

Strand: Functions

Topic: Exploring Composition of Functions

Primary SOL: AII.7 The student will investigate and analyze linear, quadratic, absolute value, square root, cube root, rational, polynomial, exponential, and logarithmic function families algebraically and graphically. Key concepts include

k) composition of functions, algebraically and graphically.

Related SOL: AII.7a, g

Materials

- Composition of Functions activity sheet (attached)
- More Composition of Functions activity sheet (attached)
- Graphing utility

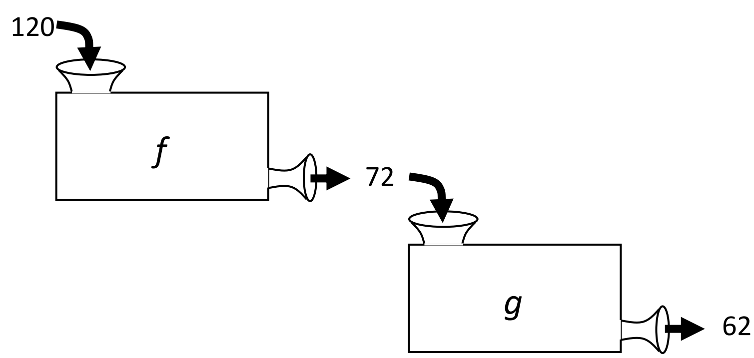
Vocabulary

composite functions, composition of functions, dependent variable, domain, independent variable, input, mapping, output, range

Student/Teacher Actions: What should students be doing? What should teachers be doing?

Time: 90 minutes

1. Present the following scenario to the whole class: Your favorite store at the mall has a huge sale offering a 40 percent discount on all merchandise. As a valued customer, you can also use the $10 coupon you received in the mail. How much will you pay for a pair of shoes, before tax, that are originally priced at $120 if
   - the 40 percent discount is applied first before the $10 coupon?
   - the $10 coupon is applied first before the 40 percent discount?
2. Demonstrate how an input-output diagram can be used to model composition of functions using the real-life problem presented. Stress the fact that inputs are domain values, whereas outputs represent range values. Suppose x represents the original price of the shoes,

   $f(x) = 0.6\,x \quad and \quad g(x) = x - 10$

   Then, $g\big(f(120)\big) = 62$.

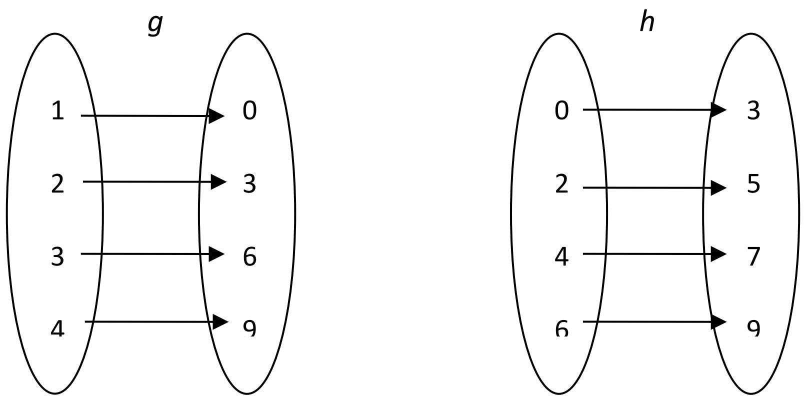
   Pose the following questions:
   - Using the diagram, how do you solve the composition of function $\big(f(120)\big)$?

- What does $f(g(120))$ mean? How do you evaluate the expression?
- Why are the values of the two expressions different?

3. After guiding students through several examples, distribute the Composition of Functions activity sheet. Have students complete problems 1-12.
4. Provide several examples and explain how to find the composition of functions in terms of *x*. Then, have students complete problems 13-16.
5. Present two graphs, then use them to explain the composition of functions graphically.
6. Distribute copies the More Composition of Functions activity sheet. Have students complete it individually or in small groups. The activity has students focusing on graphical methods to find the Inverse of functions.

Assessment

- **Questions**



  - In the mapping above, what are all points on the graph of $h[g(x)]$?
  - In the mapping above, what are the domain and range of $g[h(x)]$?
  - Given $g(x) = x^2$ and $h(x) = x - 3$, draw an input-output diagram to illustrate $g \circ h(-4)$. What is the output?
  - Given $f(x) = \sqrt{x}$, $g(x) = (x+1)^2$, and $h(x) = -2x$, evaluate the following:

    a. $f[g(x)]$ b. $h \circ f(8)$ c. $g[h(m)]$

    d. $f \circ h(-2)$ e. $g \circ f \circ h(-8)$

- **Journal/writing prompts**
  - Explain how to determine the output of a composition of functions, given the input.
  - When mapping to illustrate a composition of functions, describe a map in which there would be no output for an input value. Also, describe a map that would have the same number of outputs as inputs when the composition is performed.

- **Other Assessments**
  - Have students complete the following Khan Academy practice exercises:
    - "Evaluate Composite Functions: Graphs and Tables"

- "Evaluate Composite Functions"
- "Find Composite Functions"

Extensions and Connections

- Have students graph $f(x)=4x$ and $g(x)=\sqrt{x}$ simultaneously on their calculators. Then, have them graph $f[g(x)]$ on the same screen. Ask what they notice. Next, have them add the graph of $g[f(x)]$. Discuss why the general shapes of both compositions are the same.

Strategies for Differentiation

- Have students color-code input and output values.
- Create a handout of problems for students to work individually and then exchange with partners to check each other's work.
- Create "function machines" for students to use to produce outputs for given inputs. Then, have them exchange completed function machines with another group and determine the functions that were used to generate the outputs.
- Conduct a thorough review of evaluating functions for given input values.

Note: The following pages are intended for classroom use for students as a visual aid to learning.

Composition of Functions

Given $f(x) = -2x + 1$, $g(x) = x^2$, and $h(x) = -\frac{1}{2}x + \frac{1}{2}$, evaluate the following:

1. $f(-6)$ 2.
2. $g(-3)$
3. $h(4)$
4. $f[g(2)]$
5. $h[g(8)]$
6. $(g \circ f)(5)$
7. $(g \circ h)(7)$
8. $f[g(c)]$
9. $f[h(5)]$
10. $h[f(r)]$
11. $h[g[f(3)]]$
12. $(f \circ g \circ h)(3)$

Given $(x) = 2x^2 + 4$, $(x) = \sqrt{x - 4}$, and $h(x) = 4x - 2$, evaluate the following:

13. $f(g(x))$
14. $g(f(x))$
15. $f(h(x))$
16. $g(h(x))$

Refer to the maps below to determine the domain and range for each of $g \circ f(x)$ and $f \circ g(x)$. Name the points included in each composition of functions.

17. $(g \circ f)(x)$

domain_____________

range _____________

points _____________

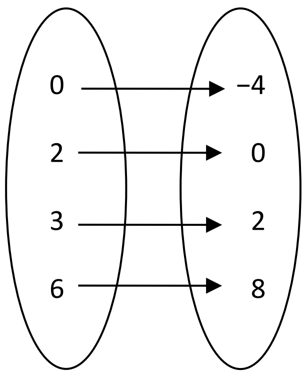
$(f \circ g)(x)$

domain _____________

range ______________

points ______________

f



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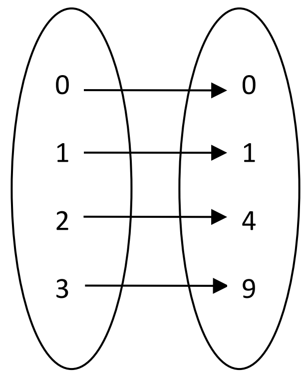


18. $(g \circ f)(x)$

domain_____________

range _____________

points _____________

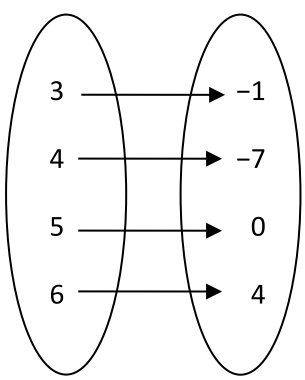
$(f \circ g)(x)$

domain _____________

range ______________

points ______________

f



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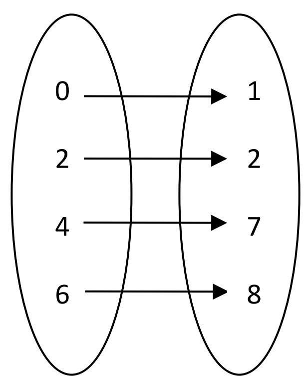
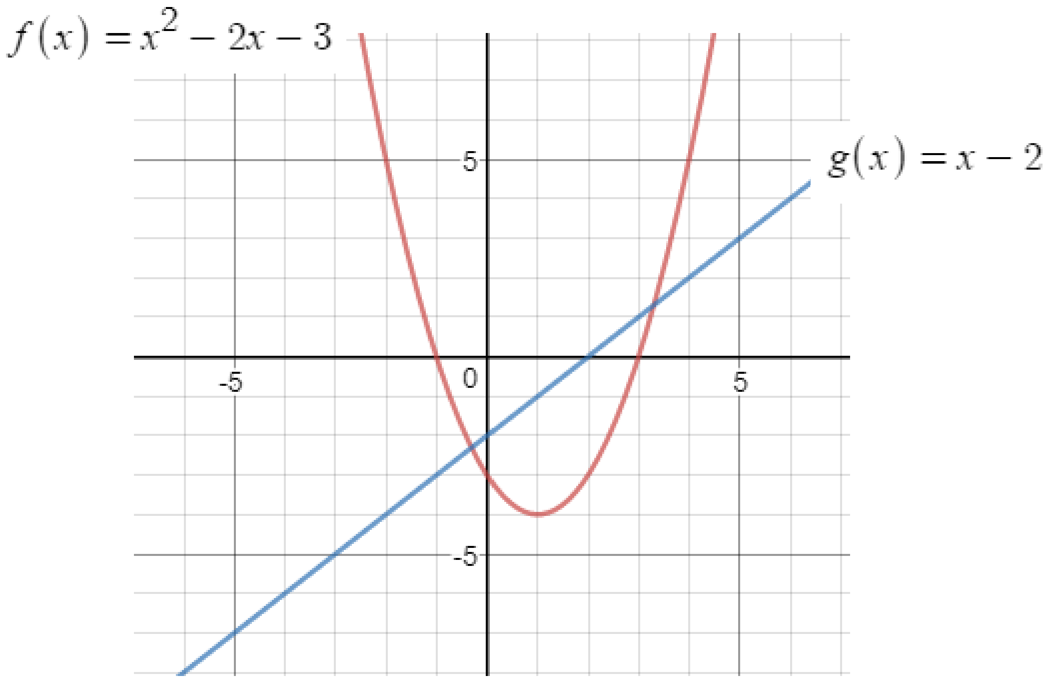


19. How do you find the domain of the composite function?

More Composition of Functions

Use the graph of $f(x)$ and $g(x)$ to find the composition of functions.



1. Find
   a. $f(g(x))$
   b. $g(f(x))$

2. Evaluate
   a. $f(g(2))$
   b. $[f \circ g](0)$
   c. $g(f(2))$
   d. $[g \circ f](-1)$
   e. $g(g(4))$
   f. $[f \circ f](3)$
   g. $f\left(g\left(f(1)\right)\right)$
   h. $g(f(\ g(0))$

3. For what value(s) of *x* is $f\left(g(x)\right) = -3$?

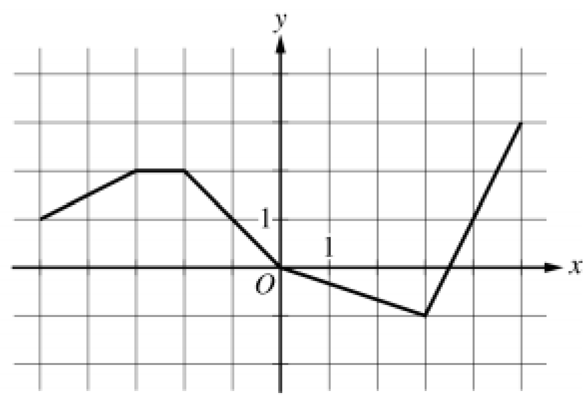
4. For what value(s) of *x* is $g\left(f(x)\right) = -3$?

5. For what value(s) of *x* is $[f \circ g](x) = -4$?

The table below shows values of $f(x)$ at selected values of x. The function $g(x)$ is shown in the graph below.

x	$f(x)$
-3	10
-2	5
-1	2
0	1
1	2
2	5
3	10

Graph of *g(x)*



Let *h* be the function defined by $h(x) = 2|x - 4|$.
Find:

1. $y = h(f(2))$

2. $y = h(g(3))$

3. $y = g(f(-2))$

4. $y = \ f(g(-3))$

5. $y = g(f(h(3)))$

6. Find $h(f(g(0)))$

7. Let $m(x)$ be defined by $m(x) = h(f(x))$. What is $m(-2)$?

8. Let $n(x)$ be defined by $n(x) = h\left(g(x)\right)$. What is $n(3)$?

9. For what value(s) of x is $g(f(x)) = 3$?

10. For what value(s) of x if $f(g(x) \ = 2$?