Exponents and Radicals

Strand: Expressions and Operations

Topic: Performing operations with radical expressions containing rational exponents

Primary SOL: AII.1   The student will

                              b)   add, subtract, multiply, divide, and simplify radical expressions containing rational numbers and variables, and expressions containing rational exponents

Materials

- Activity sheets 1-3 (attached)
- Cardstock
- Scissors

Vocabulary

cube root, exponent, index, integer, nth root, power, radicand, radical, radical expression, rational, rationalizing denominators, variable, simplify, square root

Student/Teacher Actions: What should students be doing? What should teachers be doing?
Time: 90 minutes

1. Review with students that the inverse of raising a number to the nth power is finding the nth root of a number. A rational exponent is used as an alternate way of writing a radical. Introduce the concept of rational exponents and radicals, and provide examples that show how to write a radical expression in exponential form and vice versa.

2. Have students do a quick write by completing sentence frames similar to the examples below. Encourage students to be creative and accurate in writing their sentences. Ask the students to stand, find a partner, read the card to each other, exchange the card, and then find another partner. (Repeat the process up to four rounds.)

The cube of _3_ is 27 because the cube root of 27 is _3,_ which is the same as 27 to the power of 1/3.	The fourth root of _____ is _____ because _____. The fourth root of _____ is the same as	The square root of x³ is the same as _____.

3. Show examples of evaluating expressions with rational exponents (e.g., $16^{3/4}$, $-8^{2/3}$). Ask: *"Why is $(-8)^{2/3}$ defined in the set real numbers while $(-16)^{3/4}$ is not?"*

4. Show examples of simplifying expressions with rational exponents and radical expression, using rational exponents such as:

    a. $\left(x^{\frac{2}{3}}\right)\left(x^{\frac{-1}{4}}\right)$
                    b. $\dfrac{x^{\frac{1}{2}}+1}{x^{\frac{1}{2}}-1}$

    Ask: *"How would you simplify problem b using radicals?"*

    c. $\dfrac{\sqrt[3]{16}}{\sqrt{2}}$
                    d. $\sqrt[6]{8x^3}$

    Ask: *"Why are rational exponents necessary to simplify examples c and d?" "How can you simplify problem c and d without using rational exponents?"*

5. Distribute one "I Have. Who Has?" card to each student. If there are extras, continue handing out cards until all have been distributed. Ask the student with the starting card to read it aloud. After he or she reads, "Who has $4^{\frac{3}{2}}$?", write that expression on the board. Then, direct all students to write and simplify that expression. When they are finished, ask the student holding the card containing the simplified form of the expression to read his or her card: "I have 8. Who has $125^{\frac{2}{3}}$?" The game continues in this manner until the last card is read.

Assessment

- **Questions**
    - How do you know that an expression with rational exponents is in its simplest form?
    - How do you use rational exponents to show that $\sqrt[10]{a^5}$ is the same as $\sqrt{a}$?
    - When a radical is in its simplest form, what must be the relationship between the exponents under the radical and the index?
    - If a power includes a rational exponent, what will the numerator and denominator of that exponent represent if the power is written in radical form?

- **Journal/writing prompts**
    - In simplifying the expression $\dfrac{\sqrt[4]{27}}{\sqrt{3}}$, explain why it is easier to simplify the expression using rational exponents than using radicals.
    - Explain why $(-x)^{\frac{1}{3}} = -x^{\frac{1}{3}}$, but $(-x)^{\frac{1}{2}} \neq -x^{\frac{1}{2}}$.

- **Other Assessments**
    - Have students complete the Rational Exponents Matching Cards activity sheet.

Extensions and Connections (for all students)

- Have students complete the Rational Exponents activity sheet in practical situations.

Strategies for Differentiation

- Provide a handout of the problems contained in "I Have. Who Has?" game for students who need additional time to simplify.

- If students are confused about which part of the rational exponent is the index, create a postcard that shows a basic example (e.g., $\sqrt[3]{8^2} = 8^{\frac{2}{3}}$).
- Give students a sentence frame to support their articulation of exponential and rational expressions.
- Have students create and use flash cards for the properties of exponents.
- Create a laminated note sheet of exponent rules for students to use as a reference.
- Have students create a table of perfect squares and perfect cubes to use as an aide.
- For the "I Have. Who Has?" game, create a set of cards that involves only radicals or exponential expression simplification.
- Use colored pencils and highlighters to color code the same power in identical colors.
- Use vocabulary cards for related vocabulary listed above.
- Provide a list of multiple-choice answer options for the Rational Exponents handout.
- Provide an example bank (e.g., for five questions provide five answers).

Note: The following pages are intended for classroom use for students as a visual aid to learning.

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"I Have. Who Has?" Cards

Copy the cards on cardstock and cut out. Distribute to students.

I have the starting card. Who has $4^{\frac{3}{2}}$?	I have 8. Who has $125^{\frac{2}{3}}$?
I have 25. Who has $\sqrt[3]{32}$?	I have $2\sqrt[3]{4}$. Who has $\sqrt{72}$?
I have $6\sqrt{2}$. Who has $\sqrt{a^6 b^4}$?	I have $a^3 b^2$. Who has $\sqrt{27}$?
I have $3\sqrt{3}$. Who has $\dfrac{10}{\sqrt{2}}$?	I have $5\sqrt{2}$. Who has $\sqrt{a^5 b^3}$?

I have

$$a^2 b \sqrt{ab}$$.

Who has

$$\sqrt{\dfrac{25}{2}} \ ?$$

I have

$$\dfrac{5\sqrt{2}}{2}$$.

Who has

$$3\sqrt{48} \ ?$$

I have

$$12\sqrt{3}$$.

Who has

$$(3 - \sqrt{2})^2 ?$$

I have

$$11 - 6\sqrt{2}$$.

Who has

$$(a^2 b^2)^3 ?$$

I have

$$a^6 b^6$$.

Who has

$$2(a^2 b)^3 ?$$

I have

$$2a^6 b^3$$.

Who has

$$2b(2a^3 b)^2 ?$$

I have

$$8a^6 b^3$$.

Who has

$$\sqrt[3]{54} \ ?$$

I have

$$3\sqrt[3]{2}$$.

Who has

$$(ab^2)^2 ?$$

I have

$$a^2 b^4.$$

Who has

$$\frac{7}{3+\sqrt{2}} ?$$

I have

$$3-\sqrt{2}.$$

Who has

$$81^{\frac{2}{3}} ?$$

I have

$$9\sqrt[3]{9}.$$

Who has

$$\frac{8}{\sqrt{3}} ?$$

I have

$$\frac{8\sqrt{3}}{3}.$$

Who has

$$(5-\sqrt{6})^2 ?$$

I have

$$31-10\sqrt{6}.$$

Who has

$$\frac{2-\sqrt{3}}{5-\sqrt{7}} ?$$

I have

$$\frac{10+2\sqrt{7}-5\sqrt{3}-\sqrt{21}}{18}.$$

Who has

$$2(a^4 b)^4 ?$$

I have

$$2a^{16} b^4.$$

Who has

$$\frac{8}{\sqrt{2}} ?$$

I have

$$4\sqrt{2}.$$

Who has

the last card?

Rational Exponents Matching Cards

Copy cards on cardstock and cut out.

$\sqrt[3]{x^2} \cdot \sqrt[4]{x^3}$	$9^{\frac{3}{2}}$	$\sqrt[4]{y^2}$
$x^{\frac{2}{3}} \cdot x^{\frac{3}{4}}$	$\left(\sqrt{9}\right)^3$	$y^{\frac{2}{4}}$
$x^{\frac{8}{12}} \cdot x^{\frac{9}{12}}$	3^3	$y^{\frac{1}{2}}$
$x^{\frac{17}{12}}$	27	$\sqrt{y}$
$\sqrt[12]{x^{17}}$	$4x^{\frac{1}{4}}$	$4\sqrt[4]{x}$

$\sqrt{5} \cdot \sqrt[3]{6}$	$\sqrt[6]{5^3 \cdot 6^2}$	$\left(16y^4\right)^{\frac{3}{4}}$
$5^{\frac{1}{2}} \cdot 6^{\frac{1}{3}}$	$\sqrt[6]{125 \cdot 36}$	$16^{\frac{3}{4}} y^3$
$5^{\frac{3}{6}} \cdot 6^{\frac{2}{6}}$	$\sqrt[6]{4500}$	$y^3 \left(\sqrt[4]{16}\right)^3$
$\sqrt[6]{5^3} \cdot \sqrt[6]{6^2}$	$\left(\sqrt[4]{m}\right)^5$	$y^3 (2)^3$
$m^{\frac{5}{4}}$	$\sqrt[4]{m^5}$	$8y^3$

$m^{-\frac{1}{3}}$	$4^{-\frac{1}{2}}$	$-(32)^{\frac{1}{5}}$
$\dfrac{1}{m^{\frac{1}{3}}}$	$\dfrac{1}{4^{\frac{1}{2}}}$	$-\sqrt[5]{32}$
$\dfrac{1}{\sqrt[3]{m}}$	$\dfrac{1}{\sqrt{4}}$	-2
$\dfrac{\sqrt[3]{m^2}}{m}$	$\dfrac{1}{2}$	

Rational Exponents

1. The volume, *V*, of a soap bubble is related to its surface area, *A*. In cubic centimeters, this relationship is shown by the formula $V = 0.094A^{3/2}$. What is the surface area of a bubble with a volume of 5 cm³?

2. The number of hours, *H*, that milk stays fresh is a function of the surrounding temperature, *T*. In degrees centigrade, this relationship is shown by the formula $H(T) = 180 \cdot 10^{(-.04T)}$. How many hours will newly pasteurized milk stay fresh when stored at 8° C?

3. A ship's speed, *S*, in knots varies directly as the seventh root of the power, *P*, in horsepower being generated by the engine. This is expressed by the formula $S = 6.492P^{1/7}$. If a ship is traveling at a speed of 25 knots, about how much power is the engine generating?

4. A formula that the police use for finding the speed, *S*, in miles per hour that a car was going from the length, *L*, in feet of its skid marks is $S = \sqrt{5L}$. The *Guinness Book of Records* reports that in 1960, a Jaguar in England had the longest skid mark ever recorded: 950 feet.
   a. What was the Jaguar's approximate speed?
   b. About how far will a car travel if it skids from 50 mph to a stop?

5. Mike wants to save $500 for a big trip in the summer after he graduates from high school. His goal is to find an investment vehicle that will allow his money to double to $1,000 in four years. What compound annual interest rate, *R*, will make this happen? This can be expressed by the formula $A = P(1 + R)^T$, where *P* is the original principal, *R* is the annual interest rate, *T* is the number of years, and *A* is the total value of the investment.