Literal Equations and Formulas

Strand: Equations and Inequalities

Topic: Solving literal equations for a specified variable

Primary SOL: A.4 The student will solve
c) literal equations for a specified variable.

Related SOL: A.1a**,** A.4a, A.4e

Materials

- Colored construction paper
- Scissors and Glue
- Large white paper or individual dry-erase boards
- Markers
- Pattern Blocks (optional)
- Literal Equations activity sheet (attached)

Vocabulary

literal equations, properties of equality (A.4)

Student/Teacher Actions: What should students be doing? What should teachers be doing?

Note: This lesson is based on the premise that shapes that represent variables can help students connect the abstract to the concrete. The Concrete-Representational-Abstract (CRA) Model will help students develop conceptual understanding of literal equations.

1. Illustrate the equation $x + y - z = 4$ by suggesting that a shape could represent each variable (e.g., a red circle could represent *x,* a green triangle could represent *y,* and a purple square could represent *z*). Model the equation, using these three shapes cut out of colored construction paper and glued to a large sheet of white paper. Use a marker to write +, −, and =, as follows: ● + ▲ − ■ = 4
2. Solve the equation for ▲. Discuss the properties of equality as you solve the equation.
   ● + ▲ − ■ = 4

   ● + ▲ = 4 + ■

   ▲ = 4 + ■ − ●
3. Demonstrate coefficients in an equation such as 3x – 4y = 1, by having ● represent x-values and ▲ represent y values and modeling the equation as follows: 3● – 4 ▲ = 4
4. Solve the equation for ▲. Discuss the properties of equality as you solve the equation.
   3● – 4▲ = 4

   – 4▲ = 4 – 3●

   ▲ = -1 + $\frac{3}{4}$●

5. Give students the following example of work for solving a literal equation for a given variable. The student solved for b incorrectly in the literal equation. Between which two consecutive steps were the properties of equality applied incorrectly?

   Step 0 (GIVEN EQUATION): $-3(a + 4) - b = 6$

   Step 1: $-3a - 12 - b = 6$

   Step 2: $-3a - 12 + 12 - b = 6 + 12$

   Step 3: $-3a - b = 18$

   Step 4: $-3a + 3a - b = 18 + 3a$

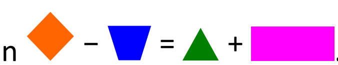
   Step 5: $-b = 18 + 3a$

   Step 6: $(-1)(-b) = (-1)(18 + 3a)$

   Step 7: $b = -18 + 3a$

6. Distribute the Literal Equations activity sheet, construction paper, scissors, and large white paper or a small dry-erase boards and markers. (Optional: Students could use pattern blocks in place of the construction paper.) Have students solve these equations, using the method just demonstrated.

Assessment

- **Questions**
  - Draw a pictorial representation of the equation $4x - 2y = 12$. In your drawing, show how to solve for x.
  - Create an equation to match the representation . Solve the equation for a selected variable, using algebra.
  - Solve the equation $\frac{1}{2}(6x + 4y) = \frac{3}{4}(-12x + 4y)$ for y step by step.
- **Journal/writing prompts**
  - The formula for the area of a triangle is $A = \frac{1}{2}bh$. If we know the area and height of a triangle, explain how to solve for the base, b.
  - Describe a formula that is often manipulated to solve for an unknown.
  - Explain how you can verify that you have solved for a variable in a literal equation *correctly.*
  - Explain why it is important to be able to solve for a specific variable in an equation.
- **Other Assessments**
  - Have students make a poster showing how to use shapes to solve literal equations.
  - Put students in pairs. Distribute a formula sheet. Each student should choose a formula and solve it for a specific variable. Students may use the same formula and solve for different variables. After each attempt, have students trade and

analyze their partner's work for errors. If any are discovered, partners should talk about how to correct the inaccuracy.

Extensions and Connections (for all students)

- Have students research formulas used in real-world situations and show how to use each formula to solve for a specific variable.
- Finding unknown variables and values is essential in mathematics and science.
- Formulas become more involved and extensive as more unknowns arise. In science, force, pressure, mass, and temperature equations use more than one variable. Depending on what information is known, the equations must be manipulated using properties of equality.

Strategies for Differentiation

- Have students highlight the variable and/or shape for which they are solving.
- Have students create problems for an interactive whiteboard that involve solving literal equations (formulas) for a given variable.
- Use an interactive whiteboard to show each step of solving for a variable, "moving" values in an equation using properties of equality.
- Ask students to reflect on errors they may make and create their own "error analysis" problem. Then have them explain to a partner where their mistake was made and why they may have made that error.
- Allow students to work with a partner to complete the Literal Equations activity.
- Allow for the use of manipulatives, such as pattern blocks, if a student prefers.
- Provide a formula sheet for journal/writing prompts.

Note: The following pages are intended for classroom use for students as a visual aid to learning.

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Literal Equations

Name:____ Date: ____________

Directions: Solve for the indicated variable in each formula below. Assign a shape to represent each variable. Rearrange the shapes, using the properties of equality, to solve for the indicated shape. Write your algebraic solution in the space provided.

1. $i = prt$ (interest = principal · rate · time)

   a) Solve for p: ______________b) Solve for r: ______________c) Solve for t: ___________

2. $V = \pi r^2 h$ (volume of a cylinder = pi · radius^2 · height)

   a) Solve for h: ______________b) Solve for r: ______________

3. $A = \frac{1}{2}bh$ (area of a triangle = $\frac{1}{2}$ base · height)

   a) Solve for b: ______________b) Solve for h: ______________

4. $A = \frac{1}{2}h(b_1 + b_2)$ (area of a trapezoid = $\frac{1}{2}$ height · [base$_1$ + base$_2$])

   a) Solve for h: ______________b) Solve for b_1: _____________c) Solve for b_2: __________

5. $Ax + By = C$ (general form of a linear equation)

   a) Solve for y: ______________b) Solve for x: ______________

6. $y = mx + b$ (slope-intercept form for the equation of a line)

   a) Solve for x: ______________b) Solve for m: _____________c) Solve for b: ___________

Some values or coefficients for the formulas above have been determined. With some of these values given, solve for the indicated variable.

7. $3y = \frac{1}{2}x + b$. *Solve for b:*

8. *36 = -pt. Solve for t:*

9. $A = \frac{1}{2}h\,(2 + -4b)$. *Solve for h:*

11. $112 = \frac{1}{2}bh$. *Solve for b:*

10. -7x + 7y = 21. *Solve for y:*

12. $V = 9\pi h$. *Solve for h:*

Error Analysis. Students were given equations and asked to solve for specific variables. In each solution, a mistake has been made. For each, determine between which two consecutive steps were either the properties of real numbers or the properties of equality applied incorrectly.

13. Solve for y in the equation: Step 0: $\frac{1}{2}(2x - 4y) = 3x + 4$ Step 1: $x - 2y = 3x + 4$ Step 2: $x - x - 2y = 3x - x + 4$ Step 3: $-2y = 2x + 4$ Step 4: $\frac{-2y}{-2} = \frac{2x+4}{-2}$ Step 5: $y = x - 2$ Error made between step _____ and step _____	14. Solve for x in the equation: Step 0: $\frac{2}{3}x = 3x + 4y$ Step 1: $3\left(\frac{2}{3}x\right) = 3(3x + 4y)$ Step 2: $2x = 9x + 12y$ Step 3: $2x - 9x = 9x - 9x + 12y$ Step 4: $-7x = 12y$ Step 5: $\frac{-7x}{-7} = \frac{12y}{7}$ Step 6: $x = \frac{12}{7}y$ Error made between step _____ and step _____
15. Solve for b in the equation: Step 0: $4a - 11 + c = 3(b - 5)$ Step 1: $4a - 11 + c = 3b - 5$ Step 2: $4a - 11 + 5 + c = 3b - 5 + 5$ Step 3: $4a - 6 + c = 3b$ Step 4: $\frac{4a-6+c}{3} = \frac{3b}{3}$ Step 5: $\frac{4}{3}a - 2 + \frac{1}{3}c = b$ Error made between step _____ and step _____	16. Solve for y in the equation: Step 0: $y - 3 = -2(x + 7)$ Step 1: $y - 3 = -2x - 14$ Step 2: $y - 3 + 3 = -2x - 14 - 3$ Step 3: $y = -2x - 17$ Error made between step _____ and step _____