Board of Education Agenda Item

Item: M.__ **Date:** February 17, 2011

Topic: Report on Virginia's College and Career Readiness Initiative and *College and Career Ready Mathematics Performance Expectations*

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Origin:

_X_ Topic presented for information only (no board action required)

____ Board review required by
- ____ State or federal law or regulation
- ____ Board of Education regulation
- ____ Other: _____

_X_ Action requested at this meeting ____ Action requested at future meeting: ___ (date)

Previous Review/Action:

_X_ No previous board review/action

___ Previous review/action
Date:
Action:

Background Information:

In January 2007, the Board of Education authorized the Virginia Department of Education (VDOE) to conduct studies to determine factors that contribute to success in postsecondary education. This critical component of Virginia's College and Career Readiness Initiative included an external analysis and validation of the *Standards of Learning* (*SOL*) in English and mathematics. As a result, Achieve, the College Board, and ACT conducted studies comparing their respective standards for postsecondary readiness to the Virginia *SOL* in English/Reading and Mathematics. In 2009 the Board adopted revised *SOL* in mathematics. The revised standards reflect the substantial input and recommended changes provided by college faculty and other experts from the College Board, ACT, Achieve (the American Diploma Project), and the business community.

In June 2010, the National Governors Association (NGA) and the Council of Chief State School Officers (CCSSO) released the *Common Core State Standards for Mathematics*. As Achieve, The College Board, and ACT were partners with NGA and CCSSO, their earlier work with states in the ADP Network provided a foundation upon which the *Common Core State Standards for Mathematics* were developed.

In the fall of 2010, Department staff and external reviewers conducted analyses to ensure Virginia's *Mathematics SOL* and *Curriculum Framework* met or exceeded the *Common Core State Standards for Mathematics*. The review identified some additional concepts that were recommended for incorporation into the *Mathematics SOL Curriculum Framework.* At its January 13, 2011, meeting, the Board of Education adopted a supplement to the revised *Mathematics SOL Curriculum Framework* and accepted the final Report of the Analysis of Virginia's 2009 *Mathematics Standards of Learning* compared to the *Common Core State Standards for Mathematics*.

Summary of Major Elements

The identification of college and career ready mathematics performance expectations has been a critical component of Virginia's ongoing College and Career Readiness Initiative to prepare all students for success in postsecondary education and careers. The expectations are intended to define the level of achievement students must reach to be academically prepared for entry-level, credit-bearing, college courses in mathematics and/or further career and technical training. To develop the performance expectations, VDOE worked in partnership with Virginia's higher education agencies, the Virginia Community College System (VCCS), and the State Council of Higher Education for Virginia (SCHEV).

To facilitate the collaborative work between VDOE, VCCS, and SCHEV partners, the Department identified preliminary college and career ready mathematics performance expectations using the *Mathematics Standards of Learning*, the *Mathematics SOL Curriculum Framework*, and other validated state and national college and career readiness standards, including the *Common Core State Standards for Mathematics*. It was found that certain *Mathematics Standards of Learning* from middle school grades and high school courses correlated highly with the national college and career ready standards.

With assistance from VCCS and SCHEV in recruitment, faculty from Virginia's two- and four-year institutions of higher education provided feedback on the preliminary college and career ready mathematics performance expectations. More than 100 higher education respondents participated in the survey.

A mathematics consensus/review team composed of faculty from two- and four-year higher education institutions and secondary content area experts analyzed the survey data and made recommendations to VDOE on which performance expectations reached the level of "important" or "critical" for college and career readiness. Virginia's *College and Career Ready Mathematics Performance Expectations*, included as Attachment A, represent the consensus/review team's recommendation to the Department. A correlation crosswalk between Virginia's *College and Career Ready Mathematics Performance Expectations* and the *Common Core State Standards for Mathematics* is provided as Attachment B.

The completion of the *Mathematics Performance Expectations* finalizes the work that is one of five components of Virginia's College and Career Readiness Initiative. In support of the initiative, the Department continues to conduct research to further understand associations between secondary outcomes and postsecondary success. The Department, in collaboration with its partners, has also made progress on several other components. Below is a list of each component and a status update of the ongoing work of Virginia's College and Career Readiness Initiative.

1) Defining college and career ready performance expectations aligned to national and international college and career ready standards.
    - VDOE, in collaboration with VCCS and SCHEV have collaboratively established Virginia's College and Career Ready Performance Expectations. The *English Performance Expectations* were completed in November 2010, and the *Mathematics Performance Expectations*, presented in this document, finalize this component of the work.

2) Developing elective "capstone courses" to support students who need additional instruction to meet college and career ready performance expectations before leaving high school
    - VDOE has drafted course descriptions, program objectives, sample teaching strategies, and delivery options to define the grade-12 capstone courses. Course codes have been identified.
    - In combination with technical assistance and professional development, the course development is intended to enable school divisions to implement the capstone courses in the fall of 2011. At least two school divisions are in the process of finalizing commitments with VDOE to pilot the courses with support from Virginia's institutions of higher education.

3) Providing technical assistance and professional development to Virginia's educators to support implementation of the revised English and mathematics standards and the college and career ready performance expectations.
    - The Department has commitments from four state universities to pilot the establishment of professional development centers to support schools in their efforts to improve students' preparation for college and careers. These centers will provide coursework and ongoing teacher support for the content of the college and career ready performance expectations. The work will be accomplished through federal teacher training funds. As part of their work, the centers will develop sample capstone course materials so that teachers can teach secondary courses more effectively and be ready to teach the capstone courses when their divisions implement the programs.
    - The State Council of Higher Education for Virginia has issued a request for proposals that establishes as a priority support for teacher professional development on the performance expectations. These grants require collaboration between the school divisions being served , and four-year universities, to include colleges of education and arts and sciences within the higher education institutions. The program is funded with federal funds from Title IIa, *Elementary and Secondary Education Act.*
4) Aligning the state assessments to measure student mastery of the more rigorous mathematics and English standards adopted in 2009 and 2010.
    - As new tests in mathematics and English are developed to align to Virginia's revised *Standards of Learning*, certain high school end-of-course tests are being designed to include quantitative indicators of whether students have met or exceeded the achievement levels needed to be successful in introductory mathematics and English courses in college.

5) Identifying accountability measures and incentives for schools to increase the percentage of students who graduate high school having demonstrated the academic and career skills needed to be successful in postsecondary education programs.

- Virginia's Virginia Index of Performance (VIP) is an incentive program that rewards schools and school divisions for exceeding minimum accountability requirements. From its inception, the program has included indicators of college and career readiness. The board took action on proposed revisions to the program at the February 2011 meeting to provide additional incentives for school divisions and schools to strengthen incentives to increase students' college and career readiness, as well as promote student achievement in science, technology, engineering, and mathematics (STEM) areas.
- The Department has started a crucial dialogue with its partners in the higher education community and policy makers to determine whether it is appropriate to provide additional incentives to schools that make gains in increasing students' preparation for college. As well, there might be incentives available directly to students who meet or exceed Virginia's CCR Performance Expectations, with a particular focus on student groups who have been underrepresented in postsecondary education and training programs. For example, the Southern Regional Education Board (SREB) has recommended that Virginia's public postsecondary institutions adopt a policy that would permit direct enrollment in entry-level, credit-bearing college courses for students who meet or exceed the readiness performance standards on the eleventh-grade English reading and writing assessments and the Algebra II end-of-course assessment. The policy would exempt these students from additional placement or readiness testing, thereby reducing the costs and time associated with such testing. Further, this policy would afford more students the opportunity to earn credits towards college graduation.

Superintendent's Recommendation:
The Superintendent of Public Instruction recommends that the Board of Education accept the Report on Virginia's College and Career Readiness Initiative and *College and Career Ready Mathematics Performance Expectations.*

Impact on Resources:
The work to date has been completed with existing Department resources and consulting staff support from the SREB through a grant from the Bill and Melinda Gates Foundation. Funding for the SREB grant will end February 28, 2011. Additional responsibilities related to this activity may affect other Department services.

Timetable for Further Review/Action:
The Department will update the Board of Education as additional components and materials for Virginia's *College and Career Readiness Initiative* are developed.

Virginia's College and Career Ready
Mathematics Performance Expectations

The Mathematics Performance Expectations (MPE) define the content and level of achievement students must reach to be academically prepared for success in entry-level, credit-bearing mathematics courses in college. They were developed through a process that involved faculty from Virginia's two- and four-year colleges and universities, members of the business community, and high school mathematics educators. The MPE are organized into four interacting and overlapping strands that include content in the areas of algebra and functions, statistics, geometry, mathematical analysis, and trigonometry. This particular strand structure is one of several ways the performance expectations can be organized. The structure is not intended to be a curriculum organizer, as each expectation interacts with many others in a range of problem-solving, modeling, and decision-making situations.

Problem Solving, Decision Making, and Integration

Students will apply algebraic, geometric, and statistical concepts and the relationships among them to solve problems, model relations, and make decisions using data and situations within and outside of mathematics. In accomplishing this goal, students will develop and enhance a repertoire of skills and strategies for solving a variety of problem types.

1) Solve practical problems involving rational numbers (including numbers in scientific notation), percents, ratios, and proportions.
2) Collect and analyze data, determine the equation of the curve of best fit, make predictions, and solve real-world problems using mathematical models. Mathematical models will include polynomial, exponential, and logarithmic functions.
3) Use pictorial representations, including computer software, constructions, and coordinate methods, to solve problems involving symmetry and transformation. This will include
    a) investigating and using formulas for finding distance, midpoint, and slope;
    b) applying slope to verify and determine whether lines are parallel or perpendicular;
    c) investigating symmetry and determining whether a figure is symmetric with respect to a line or a point; and
    d) determining whether a figure has been translated, reflected, rotated, or dilated, using coordinate methods.
4) Verify characteristics of quadrilaterals and use properties of quadrilaterals to solve real-world problems.
5) Solve real-world problems involving right triangles by using the Pythagorean Theorem and its converse, properties of special right triangles, and right triangle trigonometry.
6) Use formulas for surface area and volume of three-dimensional objects to solve real-world problems.

7) Use similar geometric objects in two- or three-dimensions to
    a) compare ratios between side lengths, perimeters, areas, and volumes;
    b) determine how changes in one or more dimensions of an object affect area and/or volume of the object;
    c) determine how changes in area and/or volume of an object affect one or more dimensions of the object; and
    d) solve real-world problems about similar geometric objects.
8) Compare distributions of two or more univariate data sets, analyzing center and spread (within group and between group variations), clusters and gaps, shapes, outliers, or other unusual features.
9) Design and conduct an experiment/survey. Key concepts include
    a) sample size;
    b) sampling technique;
    c) controlling sources of bias and experimental error;
    d) data collection; and
    e) data analysis and reporting.
10) Investigate and apply the properties of arithmetic and geometric sequences and series to solve real-world problems, including writing the first n terms, finding the nth term, and evaluating summation formulas. Notation will include Σ and a_n.
11) Use angles, arcs, chords, tangents, and secants to
    a) investigate, verify, and apply properties of circles;
    b) solve real-world problems involving properties of circles; and
    c) find arc lengths and areas of sectors in circles.

Understanding and Applying Functions

Students will be able to recognize, use, and interpret various functions and their representations, including verbal descriptions, tables, equations, and graphs to make predictions and analyze relationships in solving complex, real-world mathematical problems.

12) Transfer between and analyze multiple representations of functions, including algebraic formulas, graphs, tables, and words. Select and use appropriate representations for analysis, interpretation, and prediction.
13) Investigate and describe the relationships among solutions of an equation, zeros of a function, x-intercepts of a graph, and factors of a polynomial expression.
14) Recognize the general shape of function (absolute value, square root, cube root, rational, polynomial, exponential, and logarithmic) families and convert between graphic and symbolic forms of functions. Use a transformational approach to graphing. Use graphing calculators as a tool to investigate the shapes and behaviors of these functions.
15) Use knowledge of transformations to write an equation, given the graph of a function (linear, quadratic, exponential, and logarithmic).

16) Investigate and analyze functions (linear, quadratic, exponential, and logarithmic families) algebraically and graphically. Key concepts include
    a) continuity;
    b) local and absolute maxima and minima;
    c) domain and range, including limited and discontinuous domains and ranges;
    d) zeros;
    e) x- and y-intercepts;
    f) intervals in which a function is increasing or decreasing;
    g) asymptotes;
    h) end behavior;
    i) inverse of a function;
    j) composition of multiple functions;
    k) finding the values of a function for elements in its domain; and
    l) making connections between and among multiple representations of functions including concrete, verbal, numeric, graphic, and algebraic.

17) Determine optimal values in problem situations by identifying constraints and using linear programming techniques.

Procedure and Calculation

Students will be able to perform and justify steps in mathematical procedures and calculations and graph and solve a range of equations types. Students will reason from a variety of representations such as graphs, tables, and charts and will use displays of univariate data to identify and interpret patterns. Students will be able to calculate probabilities and analyze distributions of data to make decisions.

18) Given rational, radical, or polynomial expressions,
    a) add, subtract, multiply, divide, and simplify rational algebraic expressions;
    b) add, subtract, multiply, divide, and simplify radical expressions containing rational numbers and variables, and expressions containing rational exponents;
    c) write radical expressions as expressions containing rational exponents and vice versa; and
    d) factor polynomials completely.

19) Graph linear equations and linear inequalities in two variables, including
    a) determining the slope of a line when given an equation of the line, the graph of the line, or two points on the line; describing slope as rate of change and determine if it is positive, negative, zero, or undefined; and
    b) writing the equation of a line when given the graph of the line, two points on the line, or the slope and a point on the line.

20) Given a point other than the origin on the terminal side of an angle, use the definitions of the six trigonometric functions to find the sine, cosine, tangent, cotangent, secant, and cosecant of the angle in standard position. Relate trigonometric functions defined on the unit circle to trigonometric functions defined in right triangles.

21) Given the coordinates of the center of a circle and a point on the circle, write the equation of the circle.

22) Analyze graphical displays of univariate data, including dotplots, stemplots, and histograms, to identify and describe patterns and departures from patterns, using central tendency, spread, clusters, gaps, and outliers. Use appropriate technology to create graphical displays.

23) Analyze the normal distribution. Key concepts include
    a) characteristics of normally distributed data;
    b) percentiles;
    c) normalizing data, using z-scores; and
    d) area under the standard normal curve and probability.

24) Describe orally and in writing the relationships between the subsets of the real number system.

25) Perform operations on complex numbers, express the results in simplest form using patterns of the powers of *i*, and identify field properties that are valid for the complex numbers.

26) Solve, algebraically and graphically,
    a) absolute value equations and inequalities;
    b) quadratic equations over the set of complex numbers;
    c) equations containing rational algebraic expressions; and
    d) equations containing radical expressions.

    Use graphing calculators for solving and for confirming the algebraic solutions.

27) Given one of the six trigonometric functions in standard form,
    a) state the domain and the range of the function;
    b) determine the amplitude, period, phase shift, vertical shift, and asymptotes;
    c) sketch the graph of the function by using transformations for at least a two-period interval; and
    d) investigate the effect of changing the parameters in a trigonometric function on the graph of the function.

28) Find, without the aid of a calculator, the values of the trigonometric functions of the special angles and their related angles as found in the unit circle. This includes converting angle measures from radians to degrees and vice versa.

29) Investigate and identify the characteristics of conic section equations in (h, k) and standard forms. Use transformations in the coordinate plane to graph conic sections.

30) Using two-way tables, analyze categorical data to describe patterns and departure from patterns and to find marginal frequency and relative frequencies, including conditional frequencies.

31) Calculate probabilities. Key concepts include
    a) conditional probability;
    b) dependent and independent events;
    c) addition and multiplication rules;
    d) counting techniques (permutations and combinations); and
    e) Law of Large Numbers.

Verification and Proof

Students will recognize verification and proof as fundamental aspects of mathematical reasoning. Students will integrate and apply inductive and deductive reasoning skills to make, test, and evaluate mathematical statements. This applies equally through simple mathematical calculations, in geometric applications, and more abstract statistical and algebraic processes. Students will use logical reasoning to analyze an argument and to determine whether conclusions are valid.

32) Use the relationships between angles formed by two lines cut by a transversal to
    a) determine whether two lines are parallel;
    b) verify the parallelism, using algebraic and coordinate methods as well as deductive proofs; and
    c) solve real-world problems involving angles formed when parallel lines are cut by a transversal.

33) Given information in the form of a figure or statement, prove two triangles are congruent, using algebraic and coordinate methods as well as deductive proofs.

34) Given information in the form of a figure or statement, prove two triangles are similar, using algebraic and coordinate methods as well as deductive proofs.

35) Construct and justify the constructions of
    a) a line segment congruent to a given line segment;
    b) the perpendicular bisector of a line segment;
    c) a perpendicular to a given line from a point not on the line;
    d) a perpendicular to a given line at a given point on the line;
    e) the bisector of a given angle,
    f) an angle congruent to a given angle; and
    g) a line parallel to a given line through a point not on the given line.

36) Verify basic trigonometric identities and make substitutions, using the basic identities.

Comparison of Virginia's College and Career Ready Mathematics Performance Expectations with the Common Core State Standards for Mathematics

February 17, 2010

Common Core State Standards for Mathematics Mathematics Standards for High School[1] Number and Quantity	Virginia's Mathematics Performance Expectation
The Real Number System	
Extend the properties of exponents to rational exponents	
1. Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. *For example, we define* $5^{1/3}$ *to be the cube root of 5 because we want* $(5^{1/3})^3 = 5^{(1/3)3}$ *to hold, so* $(5^{1/3})^3$ *must equal 5.*	**MPE.18 Given rational, radical, or polynomial expressions,** **a) add, subtract, multiply, divide, and simplify rational algebraic expressions;** **b) add, subtract, multiply, divide, and simplify radical expressions containing rational numbers and variables, and expressions containing rational exponents;** **c) write radical expressions as expressions containing rational exponents and vice versa; and** **d) factor polynomials completely.** **(SOL AII.1)**
2. Rewrite expressions involving radicals and rational exponents using the properties of exponents.	
Use properties of rational and irrational numbers	
3. Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.	**MPE.24 Describe orally and in writing the relationships between the subsets of the real number system.** **(SOL 8.2)**

[1] According to the *Common Core State Standards (CCSS) for Mathematics*, the CCSS high school standards specify the mathematics that all students should study in order to be college and career ready. The *CCSS for Mathematics* also includes additional mathematics that students should learn in order to take advanced courses such as calculus, advanced statistics, or discrete mathematics, as indicated by "(+)". This document includes all CCSS high school standards and the CCSS additional mathematics standards that align with Virginia's *College and Career Ready Mathematics Performance Expectations*.

Common Core State Standards for Mathematics Mathematics Standards for High School[1] Number and Quantity	Virginia's Mathematics Performance Expectation
Quantities	
Reason quantitatively and use units to solve problems	
1. Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.	**MPE.1 Solve practical problems involving rational numbers (including numbers in scientific notation), percents, ratios, and proportions.** **(SOL 8.3, 8.1b)**
2. Define appropriate quantities for the purpose of descriptive modeling.	
3. Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.	
The Complex Number System	
Perform arithmetic operations with complex numbers	
1. Know there is a complex number i such that $i^2 = -1$, and every complex number has the form $a + bi$ with a and b real.	**MPE.25 Perform operations on complex numbers, express the results in simplest form using patterns of the powers of *i*, and identify field properties that are valid for the complex numbers.** **(SOL AII.3)**
2. Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.	
3. (+) Find the conjugate of a complex number; use conjugates to find moduli and quotients of complex numbers.	
Use complex numbers in polynomial identities and equations	
7. Solve quadratic equations with real coefficients that have complex solutions.	**MPE.26 Solve, algebraically and graphically,** **a) absolute value equations and inequalities;** **b) quadratic equations over the set of complex numbers;** **c) equations containing rational algebraic expressions; and** **d) equations containing radical expressions.** **Use graphing calculators for solving and for confirming the algebraic solutions.** **(SOL AII.4)**
8. (+) Extend polynomial identities to the complex numbers. *For example, rewrite* $x^2 + 4$ *as* $(x + 2i)(x - 2i)$.	

Common Core State Standards for Mathematics Mathematics Standards for High School[1] Number and Quantity	Virginia's Mathematics Performance Expectation
9. (+) Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials.	**MPE.13 Investigate and describe the relationships among solutions of an equation, zeros of a function, *x*-intercepts of a graph, and factors of a polynomial expression.** **(SOL AII.8)**

Common Core State Standards for Mathematics Mathematics Standards for High School Algebra	Virginia's Mathematics Performance Expectation
Seeing Structure in Expressions	
Interpret the structure of expressions	
1. Interpret expressions that represent a quantity in terms of its context.	
a. Interpret parts of an expression, such as terms, factors, and coefficients.	**MPE.26 Solve, algebraically and graphically,** **a) absolute value equations and inequalities;** **b) quadratic equations over the set of complex numbers;** **c) equations containing rational algebraic expressions; and** **d) equations containing radical expressions.** **Use graphing calculators for solving and for confirming the algebraic solutions.** **(SOL AII.4)**
b. Interpret complicated expressions by viewing one or more of their parts as a single entity. *For example, interpret* $P(1+r)^n$ *as the product of P and a factor not depending on P.*	
2. Use the structure of an expression to identify ways to rewrite it. *For example, see* $x^4 - y^4$ *as* $(x^2)^2 - (y^2)^2$, *thus recognizing it as a difference of squares that can be factored as* $(x^2 - y^2)(x^2 + y^2)$.	**MPE.18 Given rational, radical, or polynomial expressions,** **a) add, subtract, multiply, divide, and simplify rational algebraic expressions;** **b) add, subtract, multiply, divide, and simplify radical expressions containing rational numbers and variables, and expressions containing rational exponents;** **c) write radical expressions as expressions containing rational exponents and vice versa; and** **d) factor polynomials completely.** **(SOL AII.1)**
Write expressions in equivalent forms to solve problems	
3. Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.	
a. Factor a quadratic expression to reveal the zeros of the function it defines.	**MPE.16 Investigate and analyze functions (linear, quadratic, exponential, and logarithmic families) algebraically and**

Common Core State Standards for Mathematics Mathematics Standards for High School Algebra	Virginia's Mathematics Performance Expectation
	graphically. Key concepts include **a) continuity; (SOL AFDA.1)** **b) local and absolute maxima and minima; (SOL AFDA.1)** **c) domain and range, including limited and discontinuous domains and ranges;** **d) zeros;** **e) x- and y-intercepts;** **f) intervals in which a function is increasing or decreasing;** **g) asymptotes;** **h) end behavior;** **i) inverse of a function;** **j) composition of multiple functions;** **k) finding the values of a function for elements in its domain; (SOL A.7) and** **l) making connections between and among multiple representations of functions including concrete, verbal, numeric, graphic, and algebraic. (SOL A.7)** **(SOL AII.7)** **MPE.18 Given rational, radical, or polynomial expressions,** **a) add, subtract, multiply, divide, and simplify rational algebraic expressions;** **b) add, subtract, multiply, divide, and simplify radical expressions containing rational numbers and variables, and expressions containing rational exponents;** **c) write radical expressions as expressions containing rational exponents and vice versa; and** **d) factor polynomials completely.** **(SOL AII.1)**

Common Core State Standards for Mathematics Mathematics Standards for High School Algebra	Virginia's Mathematics Performance Expectation
	MPE.26 Solve, algebraically and graphically, **a) absolute value equations and inequalities;** **b) quadratic equations over the set of complex numbers;** **c) equations containing rational algebraic expressions; and** **d) equations containing radical expressions.** **Use graphing calculators for solving and for confirming the algebraic solutions.** **(SOL AII.4)**
b. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.	**MPE.16 Investigate and analyze functions (linear, quadratic, exponential, and logarithmic families) algebraically and graphically. Key concepts include** **a) continuity; (SOL AFDA.1)** **b) local and absolute maxima and minima; (SOL AFDA.1)** **c) domain and range, including limited and discontinuous domains and ranges;** **d) zeros;** **e) x- and y-intercepts;** **f) intervals in which a function is increasing or decreasing;** **g) asymptotes;** **h) end behavior;** **i) inverse of a function;** **j) composition of multiple functions;** **k) finding the values of a function for elements in its domain; (SOL A.7) and** **l) making connections between and among multiple representations of functions including concrete, verbal, numeric, graphic, and algebraic. (SOL A.7)** **(SOL AII.7)**

Common Core State Standards for Mathematics Mathematics Standards for High School Algebra	Virginia's Mathematics Performance Expectation
	MPE.18 Given rational, radical, or polynomial expressions, **a) add, subtract, multiply, divide, and simplify rational algebraic expressions;** **b) add, subtract, multiply, divide, and simplify radical expressions containing rational numbers and variables, and expressions containing rational exponents;** **c) write radical expressions as expressions containing rational exponents and vice versa; and** **d) factor polynomials completely.** **(SOL AII.1)**
c. Use the properties of exponents to transform expressions for exponential functions. *For example the expression* 1.15^t *can be rewritten as* $(1.15^{1/12})^{12t} \approx 1.012^{12t}$ *to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.*	**MPE.2 Collect and analyze data, determine the equation of the curve of best fit, make predictions, and solve real-world problems using mathematical models. Mathematical models will include polynomial, exponential, and logarithmic functions.** **(SOL AII.9)** **MPE.18 Given rational, radical, or polynomial expressions,** **a) add, subtract, multiply, divide, and simplify rational algebraic expressions;** **b) add, subtract, multiply, divide, and simplify radical expressions containing rational numbers and variables, and expressions containing rational exponents;** **c) write radical expressions as expressions containing rational exponents and vice versa; and** **d) factor polynomials completely.** **(SOL AII.1)**
4. Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems. *For example, calculate mortgage payments.*	**MPE.10 Investigate and apply the properties of arithmetic and geometric sequences and series to solve real-world problems, including writing the first *n* terms, finding the *n*th term, and**

Common Core State Standards for Mathematics Mathematics Standards for High School Algebra	Virginia's Mathematics Performance Expectation
	evaluating summation formulas. Notation will include Σ and a_n. (SOL AII.2)
Arithmetic with Polynomials and Rational Expressions	
Perform arithmetic operations on polynomials	
1. Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.	**MPE.18 Given rational, radical, or polynomial expressions,** **a) add, subtract, multiply, divide, and simplify rational algebraic expressions;** **b) add, subtract, multiply, divide, and simplify radical expressions containing rational numbers and variables, and expressions containing rational exponents;** **c) write radical expressions as expressions containing rational exponents and vice versa; and** **d) factor polynomials completely.** **(SOL AII.1)** **MPE.26 Solve, algebraically and graphically,** **a) absolute value equations and inequalities;** **b) quadratic equations over the set of complex numbers;** **c) equations containing rational algebraic expressions; and** **d) equations containing radical expressions.** **Use graphing calculators for solving and for confirming the algebraic solutions.** **(SOL AII.4)**
Understand the relationship between zeros and factors of polynomials	
2. Know and apply the Remainder Theorem: For a polynomial $p(x)$ and a number a, the remainder on division by $x - a$ is $p(a)$, so $p(a) = 0$ if and only if $(x - a)$ is a factor of $p(x)$.	**MPE.18 Given rational, radical, or polynomial expressions,** **a) add, subtract, multiply, divide, and simplify rational algebraic expressions;** **b) add, subtract, multiply, divide, and simplify radical expressions**

Common Core State Standards for Mathematics Mathematics Standards for High School Algebra	Virginia's Mathematics Performance Expectation
	containing rational numbers and variables, and expressions containing rational exponents; **c) write radical expressions as expressions containing rational exponents and vice versa; and** **d) factor polynomials completely.** **(SOL AII.1)**
3. Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.	**MPE.13 Investigate and describe the relationships among solutions of an equation, zeros of a function, *x*-intercepts of a graph, and factors of a polynomial expression.** **(SOL AII.8)** **MPE.16 Investigate and analyze functions (linear, quadratic, exponential, and logarithmic families) algebraically and graphically. Key concepts include** **a) continuity; (SOL AFDA.1)** **b) local and absolute maxima and minima; (SOL AFDA.1)** **c) domain and range, including limited and discontinuous domains and ranges;** **d) zeros;** **e) x- and y-intercepts;** **f) intervals in which a function is increasing or decreasing;** **g) asymptotes;** **h) end behavior;** **i) inverse of a function;** **j) composition of multiple functions;** **k) finding the values of a function for elements in its domain; (SOL A.7) and** **l) making connections between and among multiple representations of functions including concrete, verbal, numeric,**

Common Core State Standards for Mathematics Mathematics Standards for High School Algebra	Virginia's Mathematics Performance Expectation
	graphic, and algebraic. (SOL A.7) **(SOL AII.7)**
Use polynomial identities to solve problems	
4. Prove polynomial identities and use them to describe numerical relationships. *For example, the polynomial identity* $(x^2 + y^2)^2 = (x^2 – y^2)^2 + (2xy)^2$ *can be used to generate Pythagorean triples.*	**MPE.18 Given rational, radical, or polynomial expressions,** **a) add, subtract, multiply, divide, and simplify rational algebraic expressions;** **b) add, subtract, multiply, divide, and simplify radical expressions containing rational numbers and variables, and expressions containing rational exponents;** **c) write radical expressions as expressions containing rational exponents and vice versa; and** **d) factor polynomials completely.** **(SOL AII.1)**
Rewrite rational expressions	
6. Rewrite simple rational expressions in different forms; write $a(x)/b(x)$ in the form $q(x) + r(x)/b(x)$, where $a(x)$, $b(x)$, $q(x)$, and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$, using inspection, long division, or, for the more complicated examples, a computer algebra system.	**MPE.18 Given rational, radical, or polynomial expressions,** **a) add, subtract, multiply, divide, and simplify rational algebraic expressions;** **b) add, subtract, multiply, divide, and simplify radical expressions containing rational numbers and variables, and expressions containing rational exponents;** **c) write radical expressions as expressions containing rational exponents and vice versa; and** **d) factor polynomials completely.** **(SOL AII.1)**
7. (+) Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.	
Creating Equations	
Create equations that describe numbers or relationships	
1. Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear*	**MPE.16 Investigate and analyze functions (linear, quadratic, exponential, and logarithmic families) algebraically and**

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and quadratic functions, and simple rational and exponential functions.	**graphically. Key concepts include** **a) continuity; (SOL AFDA.1)** **b) local and absolute maxima and minima; (SOL AFDA.1)** **c) domain and range, including limited and discontinuous domains and ranges;** **d) zeros;** **e) x- and y-intercepts;** **f) intervals in which a function is increasing or decreasing;** **g) asymptotes;** **h) end behavior;** **i) inverse of a function;** **j) composition of multiple functions;** **k) finding the values of a function for elements in its domain; (SOL A.7) and** **l) making connections between and among multiple representations of functions including concrete, verbal, numeric, graphic, and algebraic. (SOL A.7)** **(SOL AII.7)** **MPE.26 Solve, algebraically and graphically,** **a) absolute value equations and inequalities;** **b) quadratic equations over the set of complex numbers;** **c) equations containing rational algebraic expressions; and** **d) equations containing radical expressions.** **Use graphing calculators for solving and for confirming the algebraic solutions.** **(SOL AII.4)**
2. Create equations in two or more variables to represent relationships between quantities; graph equations on	**MPE.19 Graph linear equations and linear inequalities in two variables, including**

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coordinate axes with labels and scales.	**a) determining the slope of a line when given an equation of the line, the graph of the line, or two points on the line; describing slope as rate of change and determine if it is positive, negative, zero, or undefined; and** **b) writing the equation of a line when given the graph of the line, two points on the line, or the slope and a point on the line.** **(SOL A.6)** **MPE.26 Solve, algebraically and graphically,** **a) absolute value equations and inequalities;** **b) quadratic equations over the set of complex numbers;** **c) equations containing rational algebraic expressions; and** **d) equations containing radical expressions.** **Use graphing calculators for solving and for confirming the algebraic solutions.** **(SOL AII.4)**
3. Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. *For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.*	**MPE.17 Determine optimal values in problem situations by identifying constraints and using linear programming techniques.** **(SOL AFDA.5)** **MPE.26 Solve, algebraically and graphically,** **a) absolute value equations and inequalities;** **b) quadratic equations over the set of complex numbers;** **c) equations containing rational algebraic expressions; and** **d) equations containing radical expressions.** **Use graphing calculators for solving and for confirming the algebraic solutions.** **(SOL AII.4)**
4. Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. *For example,*	**MPE.26 Solve, algebraically and graphically,** **a) absolute value equations and inequalities;**

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rearrange Ohm's law V = IR to highlight resistance R.	**b) quadratic equations over the set of complex numbers;** **c) equations containing rational algebraic expressions; and** **d) equations containing radical expressions.** **Use graphing calculators for solving and for confirming the algebraic solutions.** **(SOL AII.4)**
Reasoning with Equations and Inequalities	
Understand solving equations as a process of reasoning and explain the reasoning	
1. Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.	**MPE.26 Solve, algebraically and graphically,** **a) absolute value equations and inequalities;** **b) quadratic equations over the set of complex numbers;** **c) equations containing rational algebraic expressions; and** **d) equations containing radical expressions.** **Use graphing calculators for solving and for confirming the algebraic solutions.** **(SOL AII.4)**
2. Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.	
Solve equations and inequalities in one variable	
3. Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.	**MPE.17 Determine optimal values in problem situations by identifying constraints and using linear programming techniques.** **(SOL AFDA.5)** **MPE.26 Solve, algebraically and graphically,** **a) absolute value equations and inequalities;** **b) quadratic equations over the set of complex numbers;** **c) equations containing rational algebraic expressions; and** **d) equations containing radical expressions.** **Use graphing calculators for solving and for confirming the algebraic solutions.**

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	(SOL AII.4)
4. Solve quadratic equations in one variable.	
a. Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x - p)^2 =$ q that has the same solutions. Derive the quadratic formula from this form.	**MPE.14 Recognize the general shape of function (absolute value, square root, cube root, rational, polynomial, exponential, and logarithmic) families and convert between graphic and symbolic forms of functions. Use a transformational approach to graphing. Use graphing calculators as a tool to investigate the shapes and behaviors of these functions.** **(SOL AII.6)** **MPE.26 Solve, algebraically and graphically,** **a) absolute value equations and inequalities;** **b) quadratic equations over the set of complex numbers;** **c) equations containing rational algebraic expressions; and** **d) equations containing radical expressions.** **Use graphing calculators for solving and for confirming the algebraic solutions.** **(SOL AII.4)**
b. Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b.	**MPE.26 Solve, algebraically and graphically,** **a) absolute value equations and inequalities;** **b) quadratic equations over the set of complex numbers;** **c) equations containing rational algebraic expressions; and** **d) equations containing radical expressions.** **Use graphing calculators for solving and for confirming the algebraic solutions.** **(SOL AII.4)**
Solve systems of equations	
5. Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a	**MPE.26 Solve, algebraically and graphically,** **a) absolute value equations and inequalities;**

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multiple of the other produces a system with the same solutions.	**b) quadratic equations over the set of complex numbers;** **c) equations containing rational algebraic expressions; and** **d) equations containing radical expressions.** **Use graphing calculators for solving and for confirming the algebraic solutions.** **(SOL AII.4)**
6. Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.	
7. Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. *For example, find the points of intersection between the line* $y = -3x$ and the circle $x^2 + y^2 = 3$.	**MPE.17 Determine optimal values in problem situations by identifying constraints and using linear programming techniques.** **(SOL AFDA.5)** **MPE.26 Solve, algebraically and graphically,** **a) absolute value equations and inequalities;** **b) quadratic equations over the set of complex numbers;** **c) equations containing rational algebraic expressions; and** **d) equations containing radical expressions.** **Use graphing calculators for solving and for confirming the algebraic solutions.** **(SOL AII.4)**
Represent and solve equations and inequalities graphically	
10. Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).	**MPE.12 Transfer between and analyze multiple representations of functions, including algebraic formulas, graphs, tables, and words. Select and use appropriate representations for analysis, interpretation, and prediction.** **(AFDA.4)** **MPE.14 Recognize the general shape of function (absolute value, square root, cube root, rational, polynomial, exponential, and logarithmic) families and convert between graphic and symbolic forms of functions. Use a transformational approach to graphing. Use graphing calculators as a tool to investigate the shapes and**

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	behaviors of these functions. **(SOL AII.6)** **MPE.19 Graph linear equations and linear inequalities in two variables, including** **a) determining the slope of a line when given an equation of the line, the graph of the line, or two points on the line; describing slope as rate of change and determine if it is positive, negative, zero, or undefined; and** **b) writing the equation of a line when given the graph of the line, two points on the line, or the slope and a point on the line.** **(SOL A.6)**
11. Explain why the *x*-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.	**MPE.17 Determine optimal values in problem situations by identifying constraints and using linear programming techniques.** **(SOL AFDA.5)** **MPE.26 Solve, algebraically and graphically,** **a) absolute value equations and inequalities;** **b) quadratic equations over the set of complex numbers;** **c) equations containing rational algebraic expressions; and** **d) equations containing radical expressions.** **Use graphing calculators for solving and for confirming the algebraic solutions.** **(SOL AII.4)**
12. Graph the solutions to a linear inequality in two variables as a half plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.	**MPE.19 Graph linear equations and linear inequalities in two variables, including** **a) determining the slope of a line when given an equation of the line, the graph of the line, or two points on the line; describing slope as rate of change and determine if it is positive, negative, zero, or undefined; and**

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	b) writing the equation of a line when given the graph of the line, two points on the line, or the slope and a point on the line. (SOL A.6) **MPE.26 Solve, algebraically and graphically,** **a) absolute value equations and inequalities;** **b) quadratic equations over the set of complex numbers;** **c) equations containing rational algebraic expressions; and** **d) equations containing radical expressions.** **Use graphing calculators for solving and for confirming the algebraic solutions.** **(SOL AII.4)**

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Interpreting Functions	
Understand the concept of a function and use function notation	
1. Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If *f* is a function and *x* is an element of its domain, then *f*(*x*) denotes the output of *f* corresponding to the input *x*. The graph of *f* is the graph of the equation *y* = *f*(*x*).	**MPE.16 Investigate and analyze functions (linear, quadratic, exponential, and logarithmic families) algebraically and graphically. Key concepts include** **a) continuity; (SOL AFDA.1)** **b) local and absolute maxima and minima; (SOL AFDA.1)** **c) domain and range, including limited and discontinuous domains and ranges;** **d) zeros;** **e) x- and y-intercepts;** **f) intervals in which a function is increasing or decreasing;** **g) asymptotes;** **h) end behavior;** **i) inverse of a function;** **j) composition of multiple functions;** **k) finding the values of a function for elements in its domain; (SOL A.7) and** **l) making connections between and among multiple representations of functions including concrete, verbal, numeric, graphic, and algebraic. (SOL A.7)** **(SOL AII.7)**
2. Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.	
3. Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. *For example, the Fibonacci sequence is defined recursively by f(0) = f(1) = 1, f(n+1) = f(n) + f(n-1) for n ≥ 1.*	**MPE.10 Investigate and apply the properties of arithmetic and geometric sequences and series to solve real-world problems, including writing the first *n* terms, finding the *n*th term, and evaluating summation formulas. Notation will include Σ and a_n. (SOL AII.2)**