What are the Chances?

Strand: Probability and Statistics

Topic: Investigating and describing experimental and theoretical probabilities

Primary SOL: 7.8 The student will
- a) determine the theoretical and experimental probabilities of an event; and
- b) investigate and describe the difference between the experimental probability and theoretical probability of an event.

Related SOL: 7.1

Materials

- Coins
- Number cubes
- What Are the Chances? Part 1 activity sheet (attached)
- What Are the Chances? Part 2 activity sheet (attached)
- Probability Summary Sheet (attached)
- Probability Summary Sheet: Answer Key (attached)
- Calculators

Vocabulary

certain event, equally likely, Fundamental Counting Principle, impossible event, outcome, probability, sample space, simple event, tree diagram (earlier grades)

experimental probability, law of large numbers, theoretical probability (7.8)

Student/Teacher Actions: What should students be doing? What should teachers be doing?

1. Ask students to create a list of events that are likely to occur and another list of events that are not likely to occur. Use the lists to illustrate that the probability of an event occurring is a ratio between zero and 1. Discuss events that are not likely to occur or have a probability close to zero. Discuss events that are likely to occur or have a probability close to 1.
2. Show students a coin, and ask what the possible outcomes are when a coin is flipped (i.e., heads, tails). Ask students what the chances of flipping heads are. Write their responses on the board, and discuss the different representations (i.e., 1/2, 0.50, 50 percent). Have students explain their responses. (This should include a discussion of the formula they used for finding probability.)

$$\frac{\text{number of possible favorable outcomes}}{\text{total number of possible outcomes}}$$

3. Show students a number cube, and ask what the possible outcomes are when a number cube is rolled (i.e., 1, 2, 3, 4, 5, 6). Ask students what the chances of rolling a 3 are. Write their responses on the board and discuss the different representations (i.e., 1/6, approximately 0.17, and approximately 17 percent). Students may use a calculator as

the convert the fractions, decimals, and percents. Have students explain their responses.

4. Ask students whether they think the theoretical probability for heads will hold true if we flip a coin 10 times. Demonstrate this and record the results. Ask students what the probability of flipping heads was. Discuss whether this was the same or different from the theoretical probability they already established. Explain that the probability they got after flipping the coin 10 times is called experimental probability, which results from calculating probability using the results of an experiment. Discuss how this differs from theoretical probability. Share with students how experimental probabilities are calculated: $\frac{\text{number of times desired outcomes occur}}{\text{number of trials in the experiment}}$
5. Continue the experiment for 10 more rounds and recalculate the probabilities using a total of 20. Discuss that, as the number of trials increases, the experimental probability gets closer to the theoretical probability (Law of Large Numbers).
6. Distribute the What Are the Chances? Part 1 activity sheet. Explain that students will individually evaluate each game of chance. They will first explore the theoretical probability for each game of chance and then perform the experiment 10 times. For each trial, they will record the actual outcome and state whether it matches the original calculations.
7. When students have finished their experiments, have them answer questions 1 and 2 on What Are the Chances? Part 2. Discuss the results as a class. Students should note that their experimental probabilities did not match their theoretical ones well. Discuss the importance of sample size with students, and have them identify situations in which sample size would be important. Ask students to determine how they could get a better sample size with their games of chance. Combine the class data for experimental probability in order to fill in the chart for question 3.
8. Use the class data to complete question 4. Have students complete questions 4 and 5 and discuss their responses. Students should see that the more trials that are performed, the closer their experimental probability will be to the theoretical probability (Law of Large Numbers).
9. Distribute the summary sheet and ask students to fill in questions 1 through 5 based on the group conversations. Complete the four problems at the bottom of the page and discuss the final outcomes.

Assessment

- **Questions**
  - Why is it useful to know about probability?
  - What is the difference between the theoretical and experimental probability of an event?
  - How does the experimental probability of an event change as the number of trials increases?

- **Journal/writing prompts**
  - Write a paragraph to explain the Law of Large Numbers to someone who is unfamiliar with the term.
  - Write an explanation of the two types of probability for someone who has never heard of them.
  - How are experimental and theoretical probabilities alike? How are they different?

- **Other Assessments**
  - Have students design their own experiment and compare theoretical to experimental results.
  - Read students a list of events, and have them decide whether they represent theoretical or experimental probability.

Extensions and Connections

- Have students create their own games of chance and have classmates determine whether the games are fair, using what they know about theoretical and experimental probability.
- Set up stations with spinners and playing cards and have students explore the theoretical and experimental probability of different events, such as spinning a certain color or number or choosing cards by color, suit, or number and suit.

Strategies for Differentiation

- Allow students to use online versions of the manipulatives in the lesson to explore theoretical and experimental probability.
- Have each game with directions and questions on a separate piece of paper so students only have to focus on one activity at a time.
- Print the What Are the Chances? activity sheet using a landscape format so there is more room for students to write.
- Review previous vocabulary and preteach new essential vocabulary to some students before introducing the lesson.
- Divide students into small groups of 3–4 for activities, or assign each student a partner to work with.

Note: The following pages are intended for classroom use for students as a visual aid to learning.

What Are the Chances? Part 1

Directions: Calculate the theoretical probability for the given event. For each game of chance, perform the experiment 10 times. For each trial, record the actual outcome in the "Result" row.

Game 1: Flip a coin

Theoretical probability of flipping a heads:

fraction _________ decimal _________ percent _________

Theoretical probability of flipping a tails:

fraction _________ decimal _________ percent _________

Trial	1	2	3	4	5	6	7	8	9	10
Result: Heads or Tails										

Experimental probability of flipping a heads:

fraction _________ decimal _________ percent _________

Experimental probability of flipping a tails:

fraction _________ decimal _________ percent _________

Game 2: Roll a Number Cube

Theoretical probability of rolling a 1:

fraction _________ decimal _________ percent _________

Theoretical probability of rolling a 6:

fraction _________ decimal _________ percent _________

Trial	1	2	3	4	5	6	7	8	9	10
Result: 1,2,3,4,5 or 6										

Experimental probability of rolling a 1:

fraction _________ decimal _________ percent _________

Experimental probability of rolling a 6:

fraction _________ decimal _________ percent _________

What Are the Chances? Part 2

Comparing Experimental and Theoretical Probabilities

1. Complete the table below with the theoretical probability for each event. Then use the results from your experiments to calculate the experimental probability for each event.

Game of Chance	**Event**	**Theoretical Probability**	**Experimental Probability**
Flip a Coin	Tails		
Roll a Number Cube	4		

2. Use the data you collected to determine when the theoretical and experimental probabilities were the closest. Discuss your individual results below.

3. Collect and record the data from the entire class for each game of chance.

Game of Chance	**Event**	**Class Experimental Probability**
Flip a Coin	Tails	$\frac{\text{Number of tails in class}}{\text{Total number of coin flips}}$
Roll a Number Cube	4	$\frac{\text{Number of times 4 was rolled}}{\text{Total number of class rolls}}$

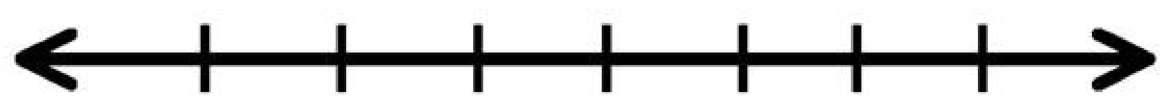
4. Are the experimental probabilities different in numbers 1 and 3? Why, or why not?

5. What do you think would happen if more trials were conducted?

Probability Summary Sheet

Probability is the _________________ that an event will occur.
The probability of an event occurring is a ratio between ____ and ____.

PIGS CAN FLY	SELECTING AN EVEN NUMBER FROM 1 TO 10	THE SUN WILL COME UP TOMORROW
0	**0.5**	**1**



Probability can be written as a _____________, _____________ or _____________.

The **THEORETICAL PROBABILITY** of an event is the ___________ probability and can be determined with a ratio.

$$\frac{\textbf{number of possible favorable outcomes}}{\textbf{total number of possible outcomes}}$$

1. In a box of 24 crayons, where there is only one of each color, what is the probability that you will select a red crayon?

2. What is the probability of selecting a crayon that is not white?

The **EXPERIMENTAL PROBABILITY** of an event is determined by carrying out an _____________.

$$\frac{\textbf{number of times desired outcomes occur}}{\textbf{number of trials in the experiment}}$$

Student Survey

Red	I I I I
Blue	I I I
Purple	I I I I

3. What is the probability that a classmate will like blue?

4. What is the probability that a classmate will *not* like purple?

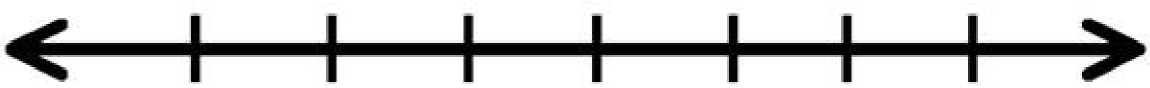
In experimental probability, as the number of trials increases, the experimental probability gets closer to the theoretical probability.

This is called the ______________ of ______________ ______________.

Probability Summary Sheet: Answer Key

Probability is the __likelihood___ that an event will occur.
The probability of an event occurring is a ratio between _0_ and _1_.

PIGS CAN FLY	SELECTING AN EVEN NUMBER FROM 1 TO 10	THE SUN WILL COME UP TOMORROW
0	**0.5**	**1**



Probability can be written as a __fraction__, _decimal_ or _percent__.

The **THEORETICAL PROBABILITY** of an event is the _expected_ probability and can be determined with a ratio.

$$\frac{\textbf{number of possible favorable outcomes}}{\textbf{total number of possible outcomes}}$$

1. In a box of 24 crayons, where there is only one of each color, what is the probability you will select a red crayon? $\frac{1}{24}$

2. What is the probability of selecting a crayon that is not white? $\frac{23}{24}$

The **EXPERIMENTAL PROBABILITY** of an event is determined by carrying out an _experiment_.

$$\frac{\textbf{number of times desired outcomes occur}}{\textbf{number of trials in the experiment}}$$

Student Survey

Red	I I I I
Blue	I I I
Purple	I I I I

3. What is the probability that a classmate will like blue? $\frac{3}{11}$

4. What is the probability that a classmate will NOT like purple? $\frac{7}{11}$

In experimental probability, as the number of trials increases, the experimental probability gets closer to the theoretical probability.

This is called the ____Law______ of ___Large___ ___Numbers____.