

Number Patterns: How Do They Grow?

- Strand:** Patterns, Functions, and Algebra
- Topic:** Representing, describing, and recording relationships found in number patterns.
- Primary SOL:** 5.18 The student will identify, describe, create, express, and extend number patterns found in objects, pictures, numbers, and tables.
- Related SOL:** 5.19

Materials

- Toothpicks
- Toothpick Squares activity sheet (attached)
- Cost of Lunches activity sheet (attached)
- How Much Ribbon? activity sheet (attached)
- Create Your Own Pattern activity sheet (attached)

Vocabulary

equation, function machine, growing pattern, input/output machine, number pattern, relationship, rule

Student/Teacher Actions: What should students be doing? What should teachers be doing?

Toothpick Squares

1. Give each student a blank sheet of paper and toothpicks.
 - a. Have students make a square out of the fewest possible toothpicks and represent the results pictorially on the paper.



1 SQUARE

- b. Show students the next figure in the sequence, and ask them to copy it on their paper and draw the next two figures in the sequence. Allow students to share their drawings with each other, and ask them to discuss any patterns they might notice.



2 SQUARES

3 SQUARES

4 SQUARES

- c. Distribute the Toothpick Squares activity sheet. Ask students to complete the input/output table through the fourth row, using the information from the toothpick figures they created or drew.
2. Next, ask students to think about building the sequence of toothpick figures to determine the pattern as an *input/output machine*. Ask: “When 1 goes into the machine, what number comes out?” “When 3 goes in, what number comes out?” “When 4 is the

input, what is the output? How do you know?” Facilitate a discussion that moves beyond just working with the numbers in the output column or “add four each time” to have students think about using the input number to find the output or “multiplying the input by 4.”

- a. Explain to students that the outputs represent a growing pattern. Ask, “*Who thinks they know why?*” Listen for students to say that as the number of squares increases the number of toothpicks increases. Some students may say each time it increases by four toothpicks. Point out that that is one way of thinking that requires looking down the column and uses the previous number to get the new output.
Next, ask students to use another way of thinking about what the output will be based on the input: looking at the rows in the table. Ask students to think about what is happening by using the idea of an input/output machine to complete the remainder of the toothpick table and the questions that follow. Circulate around the room, asking questions for students who may get stuck or have incorrect numbers in the table.
- b. Draw the original table on the board, and ask for volunteers to help you fill in the output for an input of 9 and 15. Ask how using an input/output machine made finding the output for 15 quicker or easier. Share that once they identify the *relationship rule* that describes the pattern, it does not matter how big the input number is—they can quickly find the output.
- c. Next, give students a chance to pair with someone to discuss the answers to questions a, b, and c on their Toothpick Squares activity sheet. After students have time to discuss with a partner, invite students to share with the class. Take some time with question c and show them how to check their rule. Work through a couple of rows to show that if a rule is correct it will work for any input. Ask students to use the rule to find how many toothpicks would be needed to 145 squares. Invite students who used different strategies to share.
- d. If needed, go over how to move from 1) the verbal description of the generalization, to 2) a verbal description for the process to find out how many toothpicks are needed for any number of squares in this pattern, to 3) the symbolic equation for the rule. Say, “*If I have ___ squares, what would I have to do to figure out how many toothpicks would be needed?*” Tell students you will use the letter “s” to stand for the number of squares. Discuss how letters can represent numbers. Lead students to $(s \times 4)$, where s represents any number of squares and $(\times 4)$ is the rule to find out how many toothpicks are needed to build the squares.

Cost of Lunches

3. Have students experiment with a new pattern. Distribute the Cost of Lunches activity sheet. Read the directions: “The fifth grade is going on a field trip, and the teachers need to order bag lunches and a bottle of water for each student and chaperone going on the trip. The teachers need help; they want to find a quick way to find the cost of all lunches, no matter how many people are going on the trip. Each meal with water costs the same amount. Say, “Instead of finding out how many toothpicks it takes to build the

square, let's look at how much money we will need to pay for any number of lunches." Use the input/output table to think about the rows and to discover the pattern, describe the rule for the pattern in words, and find the equation for the rule for any number of lunches. Students may work in pairs to discuss the problem, but each student should complete an activity sheet. Circulate around the room to evaluate who has misconceptions and who seems to be developing a good understanding. Pose questions when you notice a team having difficulties. When everyone has finished, direct each team to join another team to discuss their work. Listen in on the conversations and make note of points you want to address in the whole group debrief.

How Much Ribbon?

4. Next, let students investigate a different situation that generates a growing pattern. Distribute the How Much Ribbon? activity sheet. In this scenario, the art teacher is going to have students make bows that require 2 feet of ribbon each. Provide some general directions and encouragement and allow student teams to work on the task. Circulate around the room to evaluate who has misconceptions and who seems to be developing a good understanding. Pose questions when you notice a team having difficulties. When everyone has finished, direct each team to join another team to discuss their work, when their answers differ they need to reconcile the differences. Listen in on the conversations and make note of points you want to address in the whole-group debrief.

Create Your Own Pattern

5. To close out the lesson, encourage students to stretch their thinking and determine whether they can identify and describe a situation that generates an output that is growing pattern. Working as a team, students can use the Create Your Own Pattern activity sheet to create a story based on a situation and use the situation to get the table started. Then have teams trade activity sheets so they can try to finish each other's table and answer the questions. When everyone is finished, trade back so the team who wrote the story gets their paper back to review the work of the other team and determine if their story was clear. Collect the papers to review as a formative assessment to help you determine what instructional steps you need to take next. Also, identify the work you want to share the following day to highlight the important ideas and perhaps to revisit trouble spots.

Assessment

- **Questions**
 - How does an input/output machine help to think about what is going on in an input/output table?
 - Jacob buys packages of soda. One package contains 6 cans of soda. Three packages contain 18 cans of soda. Create an input/output table with a title and with the input and output columns labeled. Then fill in the table and find the output for 9 and 11 packages. Next, find the number of packages (input) for 90 cans of soda (output). What is the rule for any number of packages in words? In an equation?
 - Name some different situations that would result in growing of patterns.

- **Journal/writing prompts**

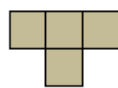
- How can an input/output table help us find two different patterns based on relationships in the table? Which one is most helpful in determining outputs for a given input?
- How can the table help to find the output for any number in a pattern based on a relationship in the table? Explain and give an example.
- Explain the difference between a repeating pattern and a growing pattern. Give an example of each.

- **Other Assessments**

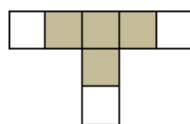
- If the rule is subtract \$0.65, what might the numerical pattern in the input/output table be? Write a situation to go with this pattern.
- Think about a real-world context, such as grocery shopping, the lunch room, etc., and identify a situation that involves a relationship that is a growing pattern. Create a story problem based on the situation.
- Ask students to create a pattern that decreases and write a situation.

Extensions and Connections (for all students)

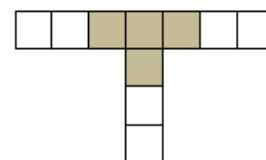
- Keith says if a pattern is increasing, the rule will be either addition or multiplication. If the pattern is decreasing, the rule will be either subtraction or division. Explain his thinking and give examples.
- Have students work in pairs. They should each write a rule and give it to their partner. The partner then uses numbers to create a pattern using that rule. Use the Create Your Own Pattern activity sheet.
- Students who need a challenge may enjoy working on a problem that requires some flexibility in thinking about relationships. Have students use colored tiles to form a letter of the alphabet. For example, the letter T can be constructed with three tiles across the top and three tiles going down the center. Have students record this in an input/output table with “stage numbers” in the first column and the number of tiles added to stage 1 to get other stages in the second column. Next, have students continue the sequence of figure by adding tiles as shown below, and have them record the number of tiles added as stage 2 in the input/output table. Show students Stage 3 and let them fill the stage number and the number of tiles added.



Stage 1



Stage 2



Stage 3

- Have students continue to add and record the stage and number of tiles for five more stages, up to stage 8. Then ask the students to write an explanation in words to describe how the change occurs. Listen for responses such as “subtract

- 1 from the stage number and then multiply by 3 to find how many tiles are added at that stage.
- b. Next, ask them to use their word description to find the output when the stage number is 57. Tell them to keep working on their rule until they can find output, or number of tiles added, for stage number for 57. (57 subtract 1 is 56 and 56 times 3 is 168. Some students may be able to write $(57-1) \times 3$, or even more sophisticated would be $(s-1) \times 3$.)
 - c. So far, the rule only provides information about the number of new tiles added to stage 1 to find any subsequent stage. What we really want is a rule that tells us how many tiles it takes to build each figure at each stage. Ask students to think about what they have learned so far about the change from one figure to another and how would they need to modify the equation so the output is the total number of tiles needed. They need to write an equation for the rule using s for stage number and t for the total number of tiles. Some students may realize that at any stage the total number of tiles is the number of new tiles plus the number of tiles they started with in stage 1 and write this as $t = (s-1) \times 3 + 4$, which is a good step for fifth-graders. You can encourage students to see whether they can simplify this, but do not resort to showing and telling. The value at this level is trying to make connections to things you already know to solve new problems.
 - i. Simplifying this a challenging situation and requires a flexible understanding of multiplication or the distributive property.
 - ii. Using an understanding of multiplication as repeated addition would lead to $t = (s-1) + (s-1) + (s-1) + 4$ and then realizing a total of 3 is being taken away and 4 added, so really only 1 is add and there are three s 's, so we could write this as $t = (s \times 3) + 1$. One of the rules is $t = (s \times 3) + 1$.
 - iii. Using a flexible understanding of the concept of the distributive property, a student might realize that $(s-1) \times 3$ means there are three s 's and then subtract three ones, which would look like $s \times 3 - 1 \times 3$ or $s \times 3 - 3$ but then we have to add 4, so $t = s \times 3 - 3 + 4$ or $t = s \times 3 + 1$.
 - iv. Students may find other rules that will work.
- Have students experiment with joining triangles, hexagons, or other shapes and find a pattern.
 - Guess My Rule: Have students take turns giving the teacher or another student a number to “input” on an input-output table. The teacher or other student will create a one-step rule to write the appropriate “output” number in the table. Do this for several inputs. Next, provide an input number and ask the students/student partner what they think the rule being followed is to determine the output—but do not allow the student to say the rule aloud. The student will prove that he/she knows the rule by being able to give the correct output. When most of the students seem to know the rule, have the students state the rule to the rest of the class.

Strategies for Differentiation

- Provide additional experiences in explaining the pattern and finding missing numbers with numerical growing patterns presented in a sequence, such as 2, 4, 6, 8, or 50, 47, 44,
- Provide additional experiences with building growing shapes to extend a one-step change pattern and then working with the student to discuss the change from the first to the second, the first to the third etc.
- Provide additional work with input/output tables where the student has to fill in missing numbers and identify patterns in the columns and in the rows of the table. For example:

Input	Output
6	9
7	10
8	?
11	?
14	?

Note: The following pages are intended for classroom use for students as a visual aid to learning.

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Toothpick Squares

Follow the teacher’s directions to complete the input/output table for how many toothpicks are needed to build the number of squares. As you build the squares and fill in the table, think about the relationship found in the patterns.

Number of Squares	Number of Toothpicks
Input	Output
1	4
2	8
3	
4	
5	
9	
15	

- a. Describe in words the relationship or how the pattern works to find the number of toothpicks for any number of squares.

- b. Try to represent the description as an equation or rule with numbers and letters when s is the number of squares and t is the number of toothpicks.

- c. Test your rule by choosing the numbers in a row in the table above to see whether it works in your equation.

Cost of Lunches

The fifth grade is going on a field trip, and the teachers need to order bag lunches and a bottle of water for each student and chaperone going on the trip. Each meal costs the same amount. Complete the table and think about the number patterns and relationships that help you fill in the table.

Number of People	Cost
Input	Output
1	\$3
2	\$6
3	
4	
5	
13	
50	
123	

- a. Describe in words the relationship or how the pattern works to find the cost for any number of lunches.

- b. Try to represent the description as an equation or rule with numbers and letters when p is the number of people and c is the cost. _____
- c. Test your rule by choosing the numbers in a row in the table above to see whether it works in your equation.

How Much Ribbon?

The art teacher is having the students make bows to sell at the school’s winter fundraising event. Each bow will require the same amount of ribbon. Fill in the table to show how much ribbon the teacher needs to buy for the number of bows indicated. You can make up your own input numbers for the last two rows. As you complete the table, think about the number patterns and relationships that help you fill in the table.

Number of Bows	Amount of Ribbon Needed
1	2 ft.
2	4 ft.
3	
4	

- a. Describe in words the relationship or how the pattern works to find the amount of money saved after any number of days.

- b. What would the table look like if it was horizontal?

Number of Bows	1	2	3	4				
Amount of Ribbon Needed	2 ft.	4 ft.						

Create Your Own Pattern

Now it is your turn to create a problem for another team to solve. Think about a situation that would generate an input/output growing pattern situation. Write out the story of the situation.

Story: _____

Give your table a title that relates to the situation, and fill in a name for the input and a name for the output. Fill in the first two rows with the input and corresponding output. Stop here. The teacher will tell you when to trade with another person.

Title: _____

?	?
Input	Output

Describe in words the relationship or how the pattern works to find the _____(output) for any _____ (input)?

Try to write an equation or rule using numbers and letters to represent the description of your relationship.