

Reasoning About Equations Using Equality

- Strand:** Patterns, Functions, Algebra
- Topic:** Using reasoning to assess the equality in closed and open arithmetic equations
- Primary SOL:** 4.16 The student will recognize and demonstrate the meaning of equality in an equation.
- Related SOL:** 4.4a

Materials:

- Partners Explore Equality and Equations activity sheet (attached)

Vocabulary

equal symbol (=), equality, equation, expression, not equal symbol (\neq), number sentence, open number sentence, reasoning

Student/Teacher Actions: What should students be doing? What should teachers be doing?

1. Activate students’ prior knowledge about the equal sign. Formally assess what students already know and what misconceptions they have. Ask students to open their notebooks to a clean sheet of paper and draw lines to divide the sheet into thirds as shown below. Create a poster of the same graphic organizer for the room. This activity should take no more than 10 minutes; it is not a teaching opportunity but simply a data-collection activity.

Equality and the Equal Sign (=)

I know that-	I wonder if-	I learned that-

- a. Let students know they will be creating a graphic organizer about equality and the equal sign; that is, what it means and how it is used. The organizer is to help them keep track of what they know, what they wonder, and what they learn.
 - The “I know that” column is where they can write things they are pretty sure are correct.
 - The “I wonder if” column is where they can write things they have questions about and also where they can put things they think may be true but they are not sure.
 - The “I learned that” column is where they will return throughout the lesson to write down things they figure out and learn.
- b. Their ideas can be written as phrases and can contain words, numbers, and symbols.
- c. Provide 3–5 minutes for students to work individually to enter information into the first two columns. Circulate around the room to gather information about

- where students are in their understanding. If students ask whether an idea is correct or not correct, let them know that this is time for them to put down what they think, so it can go in the “I wonder” column. Remind them as they go through the lesson they should look for the answer to their question.
- d. Facilitate a whole-class sharing, about 5 minutes, by first asking students to volunteer some things they wrote in the “I know that” column. Record their comments on the class poster. Let students know that you will not be commenting, just recording their ideas to come back to during the lesson. After the first student shares, ask for any different ideas for that column. If a student questions whether something is true, then ask the class if it is OK to put a question mark by the statement so they can think about this during the lesson. Then collect ideas for the “I wonder if” column and remind students before you start that these are ideas to look for during the lesson.
 - e. Inform the students that, during the lesson, they can go back and revise their Equality and the Equal Sign graphic organizer and add what they learned to the third column.
2. Engage students in a number/mathematics talk/mental mathematics activity, to move students from having a single view that the equal sign means the “answer is” to developing understanding that the equal sign denotes the relation between two equal quantities. Write the equations or number sentences provided below on the board, one at the time, for students to consider.
- a. Ask students to do some mental mathematics and *think* about the statement and the expressions on each side of the equal sign, determine whether the equation is *true or false*, and be able to share how they made their decision. In order to push students beyond simply doing the arithmetic on each side when possible, but to think about the equality relationships between the expressions in an equation, ask them to try to come up with two different ways to think about the number sentence.
 - b. Have several students share their strategy and record the strategy on the board. Your feedback should emphasize the relationship of equality in the equation. Help students understand that the equal sign signifies a relation between two quantities. It is sometimes useful to use words that express that relation more directly. For example, “13 is the same as 10 plus 3” or “18 + 7 is the same as 15 + 10.” In other words, when you know you have the same *quantity or amount* on both sides, even without finding the total on each side, you know the two sides are equal.
 - c. After the number talk activity, have students return to the graphic organizer and ask them to reflect, think about what they have learned, and add their ideas in the third column.
 - $9 = 9$ (Answers may vary, but listen for same amount on both sides)
 - $8 + 6 = 7 + 7$ (If take 1 from 8 and add it to 6, you have 7 + 7 on both sides.)

- $13 + 22 = 13 + 21$ (False; adding a different amount to 13 so the total on each side is different. Asks students what symbol would make this expression true [$13 + 22 \neq 13 + 21$])
 - $37 + 28 = 27 + 38$ (If you take 10 from 37 and add it to 28, you have 27 + 38 on both sides)
 - $5 + 5 + 5 + 5 = 4 + 4 + 4 + 4 + 4$ (Realize that four groups of 5 is the same amount as five groups of 4, or $4 \times 5 = 5 \times 4$)
 - $426 \div 9 = 426 \div 6$ (False; fewer groups with 9 in each group than with 6 in each group. Asks students what symbol would make this expression true: $426 \div 9 \neq 426 \div 6$)
 - $26 - 17 = 27 - 18$ (27 is 1 more than 26 so need to take one more away (18 to have the same amount on both sides.)
3. Students will work in ability-alike pairs to complete the tasks on the Partners Explore Equality and Equations activity sheet. Let student know they will continue to work with equality and equations, but instead of determining whether an equation is true or false they are going to be identifying a missing number so that the expression on each side of the equal sign represents the *same amount or quantity*. Write the two open sentences on the board: $5 + \underline{\quad} \times 7 = 5 + 42$ and $27 + 93 = \underline{\quad} + 100$. Ask students to determine what number is missing by reasoning or thinking about how the expression on the left is related to the expression on the right, just as they did with the True/False equations. Give students a few minutes and then ask for the missing numbers. If there is more than one answer for each, write them on the board. Then call on volunteers to explain their reasoning to find the missing number. Record the different strategies students may use.
- a. Distribute the Partners Explore Equality and Equations activity sheet. Review the directions with the students. Set a time limit so that you will have at least 10 minutes at the end of class for a class discussion about the first task and the strategies students used. The first part of the second task can be used for homework and then completed in class the next day.
 - b. Circulate around the room and identify one or two students to share their strategy for each of the six problems. Also, note any misconceptions that you want to address in the closing. If students are having trouble reaching consensus on the correct answer when they are comparing their answers, you may want to allow them to check with a calculator.
 - c. Before the class discussion, write the six problems on the board, leaving space below the problem to make notes during the class discussion.
 - d. Invite students to share one or two strategies for each problem. Once students have shared, you can point out any misconceptions you saw and what students can be alert to so they avoid the mistake caused by the misconception.
4. Provide students with a few minutes to add any ideas they would like to their Equality and Equations graphic organizer. Then collect ideas for the class poster. Also, ask students to help you revisit any question marks in the first or second columns of the poster.

Assessment

- **Questions**

- Why do you think that you cannot write number sentences that look like that?
- Do you agree or disagree with Student A? Why?
- Why do you believe this equation is true?
- Why do you believe this equation is false?
- What do you do when there is more than one number that follows the equal sign?
- How do you know that the number that you put in the box makes the number sentence true?
- How do you know the amounts on each side are the same?
- Is there any other way to figure out whether the sentence is true?
- Can you complete the equation, for example, $4 \times 9 = \underline{\quad} \times 6$ with doing any calculations?

- **Journal/writing prompts**

- Is this a good definition of the equal sign? Circle good or not good and explain your answer.
 - The equal sign means two amounts are the same. Good Not good
 - The equal sign means what the answer is. Good Not good
- The teacher wrote the equation $8 + 4 = \square + 5$ on the board. Mason said the missing number was 12. Is Mason correct or incorrect? What reasoning do you think Mason was using?
- Write a definition of equality and provide examples.

- **Other Assessments (include informal assessment ideas)**

- Rescue Zoo has two elephant pens with six elephants in each one. The elephants are too crowded so they are building two new pens. How many elephants will they put in each pen when the new ones are finished? (Equal number in each pen.) Write an equation to solve this problem. $\underline{\quad} \times \underline{\quad} = \underline{\quad} \times \underline{\quad}$
- For $36 = \underline{\quad}$, write three equations. Use two different operations for each equation.

Extensions and Connections (for all students)

- Evaluate true/false equations and solve for the missing number or numbers in open number sentence written with fractions or decimals.
- Apply the idea of writing number sentences to solve the following contextual problems.
 - Patricia and Daniel are neighbors playing outside. They both like oranges, so each went back to their house to get oranges. Patricia brought out 6 oranges and Daniel brought out 3 oranges. Each ran back to get more. Patricia brought out 4 more oranges and Daniel brought out 3 more oranges. Daniel ran back again and brought out 4 more oranges. A friend

came over to play with them, so Patricia gave 6 oranges to their friend and Daniel gave the friend 3 oranges. Now, after sharing with the friend, do you think Daniel and Patricia have the same number of oranges? Justify your answer.

Strategies for Differentiation

- Work through this four-minute video, "[Demonstrating Equality through a Balance Scale](https://www.youtube.com/watch?v=z7P3gVkSVEI)," with students who need more experience in using a balance scale and working with small numbers to understand equality. URL: <https://www.youtube.com/watch?v=z7P3gVkSVEI>
- Work one-on-one with students using beans and a balance scale to develop what it means to be a true sentence and a false sentence using numbers 20 or less. For example, model a "true" equation, such as $1 + 3 = 2 + 2$, highlighting the fact that the amount on both sides is the same or equal to each other. Each expression is equal to 4. Model a "false" equation, such as $1 + 3 = 3 + 2$, highlighting the fact that both sides are not the same and not equal to each other. Four is not equal to 5. This is false (not true). Then, provide students with number sentences to determine whether they are true or false. Ask students to predict first and then test their prediction with a balance scale.
- Revise the handout to use smaller numbers.

Note: The following pages are intended for classroom use for students as a visual aid to learning.

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Partners Explore Equality and Equations

Directions: Decide who will work in the left column and who will work in the right column. The goal is to find the missing number as quickly as you can that will make the equation true; that is, the value of the expression on the left is the same as the value of the expression on the right side of the equal sign. You will time yourselves. Start at the same time, and when each person finishes, write down the time you finished.

- When both people are finished, compare your papers. If you filled in different numbers, you must discuss the problems and reach agreement on the correct missing number.
- You get 1 point for each problem that is correct and 3 points for finishing first.

$46 + 77 = 44 + \underline{\quad}$	$46 + 77 = 44 + \underline{\quad}$	
$16 - \underline{\quad} = 17 - 8$	$16 - \underline{\quad} = 17 - 8$	
$56 \div 8 = 7 \times \underline{\quad}$	$56 \div 8 = 7 \times \underline{\quad}$	
$\underline{\quad} + 330 = 126 + 327$	$\underline{\quad} + 330 = 126 + 327$	
$46 + 28 = 27 + 50 - \underline{\quad}$	$46 + 28 = 27 + 50 - \underline{\quad}$	
$9 \times 8 = 9 \times 10 - \underline{\quad}$	$9 \times 8 = 9 \times 10 - \underline{\quad}$	
Time finished:	Time finished:	
Points for correct problem:	Points for correct problem:	

Write five equations similar to the ones above for you partner to solve by finding a missing number. Your partner must come up with five equations for you to solve. Trade papers and solve the equations. When both people are finished, compare your papers. If you filled in different numbers, you must discuss the problems and reach agreement on the correct missing number. You get 1 point for each problem that is correct.

1.
2.
3.
4.
5.

Points Earned: _____