

How Does the Pattern Grow?

Strand:	Patterns, Functions, and Algebra
Topic:	Representing, describing, and extending growing numerical and geometric patterns
Primary SOL:	4.15 The student will identify, describe, create, and extend patterns found in objects, pictures, numbers, and tables.

Materials

- Flowers for Our Teacher activity sheet (attached)
- Counters or tiles for modeling flowers
- Toothpick Worm activity sheet (attached)
- Toothpicks for modeling the worm

Vocabulary

extend, growing pattern, input, output, pattern of change, rule

Student/Teacher Actions: What should students be doing? What should teachers be doing?

1. Students will work in pairs to complete a task that reviews third-grade concepts about growing patterns and extends their learning to describing and writing a rule for the growing pattern. Distribute the Flowers for Our Teacher activity sheet and counters or tiles to build the flowers. The task has multiple entry points so that students of all levels will be able to work on the task.
 - a. Direct students to read the instructions on the handout to themselves and underline words they are not sure about. Then have a volunteer read the directions out loud. Ask students whether there are words they want to talk about. Review words that you suspect some students may not be sure about, such as bouquet, petal, and geometric shape pattern. Avoid saying too much about what makes the sequence of bouquets a pattern, because this is what students will be investigating.
 - b. Inform students they can use the manipulative you provide to build a concrete model of the flower bouquets. You may want to invite a student demonstrate one way to model a flower. Remind students they are to discuss the tasks and questions with their partner, but they must each write the decisions on their own activity sheet. Provide adequate time for students to explore and talk with their partner.
 - c. Circulate around the room to support students with scaffolding questions rather than providing answers. For all representations—concrete, pictorial, list, and table—the following questions bring out the important concepts regarding growing patterns and early function thinking. As students begin to describe the pattern in words, you should ask questions to help them move to the next step of generalization and then writing a rule during the whole-class discussion.
 - What pattern(s) do you notice?

- How does the pattern grow?
 - What changes as you move from one bouquet/stage to the next?
 - What are some connections you see among the representations (concrete, pictorial, numerical list, and table)?
 - What is something that you see more clearly in each representation? What is something that is not shown as clearly by each representation?
 - Are the representations an efficient way to determine how many petals are required for a larger bouquet, such as ones with 10, 15 or even more flowers?
- d. Before the whole-class discussion, ask each pair of students to join another pair. You may want to use what you learned while walking around to assign the pairings of partner groups and provide suggestions for what students should look for in others' work. In pairs, students should compare their responses and discuss any differences of opinion in order to reach a consensus about an appropriate response. Remind students that they must be able to provide evidence to justify their responses. Continue to circulate around the room and make note of any misconceptions to address, important concepts to highlight, and connections you want to bring out during the whole-class discussion.
- e. The whole class discussion is the time you to summarize important ideas such as pattern of change, clarify misconceptions, and emphasize connections among representations. This will be more meaningful to students if you can use students' work and words to help make your points.
- f. The last step in the discussion is to move to writing the rule. Call on several students who described the rule in words and write their words on the board. Then, if any students wrote anything with numbers and symbols, call on them to share and record their responses. Using student words such as "the number of petals is the number of flowers times 6" or "every time you add a flower you have 6 more petals," ask students whether it matters how many or how few flowers are in the bouquet for their description of the rule to be true. Validate students' thinking; let them know they have described the rule in words that is true for any number of flowers. Now the class will take the next step and examine how mathematicians use symbols and numbers to reduce the number of words.
- Ask students whether they decided the task was a growing-patterns task or a repeating-patterns task. This idea has been addressed in Grade 3, so facilitate a brief discussion and clarify that it is a growing pattern because the number of petals is 6, 12, 18, 24, and the numbers grow by six each time.
 - The next step in getting to the mathematician's rule is to write the students' statements using an equal sign and operation symbols following the order of the student's description of the rule. That is, "number of petals = number of flowers \times 6" or "number of petals = 6 + 6 + 6 + 6 +" Then ask which might be the most helpful for finding the petals for any number of flowers. Validate the second description and

ask, “How would I know how many sixes to add, and how long it would take to add up, say, 35 sixes?” Some students should bring out that “number of flowers \times 6 “is the multiplication statement for the repeated addition and is much quicker to compute.

- Underline “the number of petals” and ask whether the number of petals will stay the same or change with the number of flowers in the bouquet. Then underline the number of flowers and ask whether the number of flowers will stay the same or change. Once the class agrees that the number of petals changes and the number of flowers changes, introduce the idea of using letters instead of the phrases to represent petals and flowers.
 - Inform students that mathematicians use letters instead of a phrase or word when something changes. Ask: “What is a good letter for petals?” “What is a good letter for flowers?” Students will likely say p for petals and f for flowers, so you can write the rule $p = f \times 6$ on the board, and then ask whether you could write $p = 6 \times f$. Help students clarify that the order in which two numbers are multiplied does not matter (the commutative property). (Students do not need to recall the name of the property, but they do need to know that order does not matter in multiplication.) If students give other letters, go with the letters the students provide, but let students know that other letters could be used as long as they state what the letters stand for.
 - Now that the students have a rule they can use to find the number of petals for any number of flowers no matter how large, highlight this generalization for students by asking, “How many petals would 41 flowers have?” “How many petals would 543 flowers have?”
 - Ask students to take a few minutes to write in their journals why a rule is helpful when working with patterns.
2. Students will work in pairs to complete the Toothpick Worm activity and continue to develop their use of representations, such as objects, pictures, words, tables, and symbols, to investigate growing geometric patterns. Their investigation will support a deepening understanding of the pattern of change in order to realize the generalization of the pattern in words and as a rule.
- a. Distribute the Toothpick Worm activity sheet and toothpicks.
 - b. Have students read the directions for the task individually and then have someone read it aloud. Clarify the meaning of any words that students need help with. Remind students they need to discuss the task together but they each need to write the team’s solution on their own papers. Resist demonstrating how to perform the task. Allow students to engage in the challenge.
 - c. Circulate around the room and support students by asking questions such as those in 1c. Make note of students’ misconceptions and of students who have strategies you want them to share with the class later.
 - d. Ask each pair of students to join another group. You may want to use what you learned while walking around to assign the pairings of partner groups. In their

groups, students should compare their responses, discuss differences of opinion, and try to reach a consensus about an appropriate response. Allow students to return to their seats and for partners to revise their papers as they wish.

- e. Collect the papers to review as a formative assessment. Determine how you want to provide feedback based on misconceptions that a few students may have that need to be addressed one-on-one or misconceptions that many students have and need to be addressed through a whole-class strategy that uses the students' work.

Assessment

- **Questions**

- What three numbers will come next when you identify and use the pattern to extend the sequence of numbers 15, 18, 21, __, __, __? What is the rule? How would you demonstrate that you have found the rule?
- What are the missing numbers in the following sequence 5, 14, 23, _____, 41, 50, _____, 68 ...? What is the rule? How would you demonstrate that you have found the rule?
- How do you know this is a growing pattern?
- Did you notice how much the total number of toothpicks increases from stage to stage?
- Do you think you have enough toothpicks to complete stage 15? Why or why not?
- Did you notice how much the total number of tiles increases from stage to stage?

- **Journal/writing prompts**

- Compare growing patterns and repeating patterns using words and pictures.
- Have students glue their first four toothpick terms in the Toothpick Worm task from step 1 in their journals to keep as a reference. Have them write about the sequence of terms—what changes from one term to the next and what stays the same.
- Draw a number line, and label on the line the number of petals in the stages of the sequence of flower bouquets (6, 12, 18, and 24). Describe how the visual of the number line can help you determine the pattern.

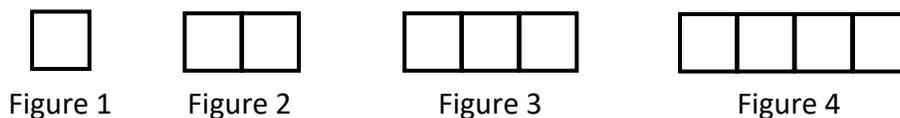
- **Other Assessments**

- If the rule for the pattern is $y = x + 3$, create an input-output table with four rows based on the rule.
- In the flower problem, what is the relationship between the bouquet number and the number of petals added? The number of petals in all?

Extensions and Connections (for all students)

- Have students use a hundreds chart to find and create numerical patterns.
- Have students use a number line to create and extend numerical patterns.
- Have students use a multiplication table to find and describe numerical patterns.

- Students can design their own shape patterns and use a chart to record the patterns numerically.
- Students can use what they know about growing patterns to solve problems such as the following:
 - Lisa makes \$7 an hour baby-sitting. She is saving money to go on a trip to the amusement park. She decides she needs at least \$80 for the trip.
 - _____
How much money would Lisa make if she baby-sat for 3 hours?
 - _____
How much money would Lisa make if she baby-sat for 5 hours?
 - _____
How can you determine how many hours she needs to work to make at least \$80?
- Have students find the number of toothpicks for the next four stages and then graph the first eight stages for Toothpick Worm. They should describe what they notice about the graph.
- Have students figure out how many flowers are in a bouquet that has 174 petals and justify their answer.
- Have students explore situations such as the following, where the resulting rule is two steps. Ask them to explore the toothpick pen pattern, represent the pattern in a table, find a rule they can use to find the number of toothpicks to build any number of pens, and demonstrate that the rule is correct.



Strategies for Differentiation

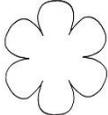
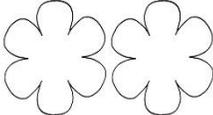
- Provide cutouts of flowers for students to glue on large sheets of paper to create the bouquet of flowers.
- Provide large sheets of paper and glue so that students can glue their toothpick worms to paper.
- Provide centimeter grid paper for students to draw the toothpick worms.
- Have students do a think-pair-share after completing each step of an activity to share what they have discovered or learned.
- Insert additional spacing between questions on the activity sheets for student writing.
- Spend additional time investigating and describing patterns by examining sequences (lists) of numbers.

Note: The following pages are intended for classroom use for students as a visual aid to learning.

Flowers for Our Teacher (Page 1)

The fourth-grade students are making flower bouquets to give to the teachers for Teacher Appreciation Day. Each flower has six petals, and as they were making the bouquets they began thinking about how many petals teachers would get if they received a certain number of flowers. Kaden and Christy started building the following geometric shape pattern to represent bouquets of flowers, and they need some help building the next two bouquets and then representing the petal pattern with numbers.

1. Work with your partner to use counters or tiles to model the first four bouquets of flowers, using the first two bouquets shown below to get started. As you make the flowers for each size bouquet, talk with your partner about what is changing and what stays the same. Is there a pattern in how the bouquets change? Then, draw the bouquets for three flowers and for four flowers. Use concrete models and the pictures below to complete the following questions.

			
1 flower	2 flowers	3 flowers	4 flowers

2. List the number of flower petals for each bouquet of flowers.

____, _____, _____, _____

- a. Describe how you determined the number of petals for each group of flowers.
- b. Would you describe the pattern as a repeating pattern or as a growing pattern? Explain how you decided what type of pattern is shown.

Flowers for Our Teacher (Page 2)

- c. A friend is absent from class, and when you call him after school you want to describe the pattern that you found when moving from one bouquet of flowers to the next bouquet of flowers in the sequence. Record what you will say to your friend on the phone. Remember: He cannot see a picture, so your description needs to help him get a mental picture.

3. Make an input-output table to represent the pattern with the number of flowers in the group as the input and the number of petals as the output.

Input	Output
Number of Flowers	Number of Petals

- a. How many petals would a bouquet of 8 flowers have? _____ Describe how you determined the number of petals.
- b. How many petals would a bouquet of 30 flowers have? _____ Describe how you determined the number of petals.

Toothpick Worm

A toothpick worm grows in a particular way so that it gets longer and longer but not wider. At Stage 1, it has a triangular-shaped head. At Stage 2, a rectangular body section grows from the triangular head as shown below. The worm will add another rectangular body section at each stage of its life. Work with a partner to use toothpicks to build the worm at stages 1–4, one stage at the time. Then follow the directions below and answer the questions.

- Use your toothpicks to build the first two stages of the toothpick worm’s life and then extend the pattern for two more stages. Draw stage 3 and stage 4.



Stage 1



Stage 2

Stage 3

Stage 4

- Examine how the worm changes as it goes through the life stages for any patterns. Describe your worm’s pattern in words, pictures, and numbers.

- Predict and draw what stage 12 would look like without building or drawing any additional stages. Then describe how you decided what to draw.

4.

Create an input-output table for the first four stages that shows the total number of toothpicks used for each stage of the pattern.

Stage Number	Number of Toothpicks

Describe in words the rule for determining how many toothpicks are needed to build a toothpick worm for any stage number.

Write the rule using letters, symbols, numbers, and an equal sign.