Logic and Conditional Statements

Strand: Reasoning, Lines, and Transformations

Topic: Investigating symbolic form while working with conditional statements

Primary SOL: G.1 The student will use deductive reasoning to construct and judge the validity of a logical argument consisting of a set of premises and a conclusion. This will include
a) identifying the converse, inverse, and contrapositive of a conditional statement; and
b) translating a short verbal argument into symbolic form.

Related SOL: G.2a, G.6, G.7, G.8, G.9

Materials

- Vocabulary and Symbols activity sheet (attached)
- Vocabulary and Symbols (Teacher's Reference) (attached)
- Logic and Conditional Statements, Part 1, activity sheet (attached)
- Logic and Conditional Statements (Teacher's Reference) (attached)
- Logic and Conditional Statements, Part 2, activity sheet (attached)
- Flash cards (attached)
- Logic and Conditional Statements handout (attached)

Vocabulary

biconditional, conclusion, conditional statement, conjecture, contrapositive, converse, disprove, hypothesis, inverse, logical argument, negate, negation, proof, prove, verify, Venn diagram

Student/Teacher Actions: What should students be doing? What should teachers be doing?

1. Distribute the Vocabulary and Symbols activity sheet, to review the basic vocabulary included in this activity.
2. Distribute the Logic and Conditional Statements, Part 1, activity sheet, and work through the examples with students.
3. Distribute the Logic and Conditional Statements, Part 2, activity sheet, and have students work in pairs or small groups to complete the problems.
4. Have students discuss findings with their partners.
5. Discuss findings as a whole group. Distribute the Logic and Conditional Statements handout, and review it with students. Have students use it for quick reference.
6. Use the Flash Cards and the Logic and Conditional Statements handout as a quick review and a check for understanding.

Assessment

- **Questions**
  - What is the inverse of the converse of $p \rightarrow q$? Use symbols *p, q,* $\rightarrow$, and ~ and mathematics vocabulary to answer this question.
  - What conclusion can you draw, using all of the following statements?
    $\sim q \rightarrow s \qquad t \rightarrow \sim r \qquad q \rightarrow t \qquad u \rightarrow \sim s$

- **Journal/Writing Prompts**
  - Draw a Venn diagram of the two statements: "No reptiles have fur" and "All snakes are reptiles." Then, draw a logical conclusion, if possible. *(Note: While Venn diagrams are not a focus of this standard, they may be used as representations.)*
  - Write a converse-inverse-contrapositive poem by writing an "if …, then" statement followed by its converse, inverse, and contrapositive. Create additional stanzas using related "if …, then" statements.
  - Read one of Laura Numeroff's books, such as *If You Give a Mouse a Cookie,* to the class. Discuss it as an extended syllogism or logical chain, and have students write a story that is a logical chain of syllogisms.
- **Other**
  - Have students work in pairs to evaluate strategies.
  - Use activity sheets to help assess student understanding.

Extensions and Connections (for all students)

- Have students investigate Lewis Carroll's logic puzzles.
- Have students solve the logic puzzle from J.K. Rowling's *Harry Potter and the Sorcerer's Stone,* 1998, p. 285.
- Invite a politician or political analyst to visit the class. Ask the guest speaker to explain the relationships among facts, trends, and educated guesses.
- Have students use presentation software to present vocabulary.

Strategies for Differentiation

- Some students may find the included "if …, then" statements to be difficult. Provide alternate "if …, then" statements for use on the Logic and Conditional Statements activity sheets:
  a. If we win the game today, then we go to the state championship.
  b. If it is the weekend, then we do not go to school.
  c. A triangle has three sides.
  d. Complementary angles measure 90°.
- Have students write their own conditional statement in "if …, then" form. Take four slips of paper. Write "If" on one slip, "then" on another, the hypothesis on the third, and the conclusion on the fourth. Flip the hypothesis (top to bottom), and write the negation of the hypothesis on the back. Do the same for the conclusion. Use these slips to illustrate converse, inverse, and contrapositive. This can also be done as a whole-group activity,

with students holding the slips in front of the class. When introducing symbols, label the hypothesis, conclusion, and negation statements with *p*, ~*p, q,* and ~*q.*

- Use three slips of paper, as above, labeled with *p,* $\rightarrow$, and *q* to illustrate the converse, inverse, and contrapositive of conditional statements using symbols. (Write ~*p* on the back of *p* and ~*q* on the back of *q*, flipping from top to bottom.)
- Have students write an "if …, then" statement of their choosing on an index card. Write the converse, inverse, and contrapositive on the back of the card. Check that all statements generated by students are correct. In the next class, use these cards to have students quiz each other in pairs, and then trade cards with another pairs of students.
- Have students use highlighters to mark the hypothesis and conclusion in conditional statements.
- Provide students with the reference sheets on colored paper. (Flash Cards and Conditional Statements)

Note: The following pages are intended for classroom use for students as a visual aid to learning.

Vocabulary and Symbols

Name _____ **Date** _____

Define each of the following vocabulary terms.

Term	Definition
Conditional Statement	A s_____ that can be w_____ in "if …, then" form.
Hypothesis	The part of a c_____ statement that f_____ "if."
Conclusion	The part of a conditional s_____ that follows the word "then."
Negation	The o_____ of a given s_____ formed by adding or removing the word n_____ from the statement.
Negate	To add or remove the word n_____ from a statement to change its truth value from true to f_____ or from false to t_____ .
Converse	A s_____ formed from a c_____ statement by s_____ the h_____ and the c_____.
Inverse	A s_____ formed from a c_____ statement by n_____ the h_____ and the c_____.
Contrapositive	A s_____ formed from a c_____ statement by s_____ AND n_____ the h_____ and the c_____.
Biconditional	A statement that combines the c_____ and its c_____ when they are both true. It uses the phrase "if and o_____if."

Fill in the meaning of each of the following symbols.

p, q, r, s, t, **etc.**			
→		∨	
~		∴	
∧		↔	

Vocabulary and Symbols (Teacher's Reference)

Define each of the following vocabulary terms.

Term	Definition
Conditional Statement	A statement that can be written in "if …, then" form.
Hypothesis	The part of a conditional statement that follows the word "if."
Conclusion	The part of a conditional statement that follows the word "then."
Negation	The opposite of a given statement formed by adding or removing the word *not* from the statement.
Negate	To add or remove the word *not* from a statement to change its truth value from true to false or from false to true.
Converse	A statement formed from a conditional statement by switching the hypothesis and the conclusion.
Inverse	A statement formed from a conditional statement by negating the hypothesis and the conclusion.
Contrapositive	A statement formed from a conditional statement by switching AND negating the hypothesis and the conclusion.
Biconditional	A statement that combines the conditional and its converse when they are both true. It uses the phrase "if and only if."

Fill in the meaning of each of the following symbols.

***p, q, r, s, t,* etc.**	Meaning: Symbols used to represent statements such as hypotheses and conclusions		
$\rightarrow$	Meaning: if …, then (implies)	$\vee$	Meaning: or
$\sim$	Meaning: not	$\therefore$	Meaning: therefore
$\wedge$	Meaning: and	$\leftrightarrow$	Meaning: if and only if (IFF)

Logic and Conditional Statements, Part 1

Name _____ **Date** _____

Use the following conditional statement to complete 1-11: "If elephants fly, then fish don't swim." Each answer should be a complete sentence, *not* symbols.

1. p is the hypothesis. Write p. _____
2. q is the conclusion. Write q. _____
3. $\sim p$ means "the negation of p." Write $\sim p$. _____
4. $\sim q$ means "the negation of q." Write $\sim q$. _____
5. (Converse) $q \rightarrow p$ means "q implies p" or "If q, then p." Write $q \rightarrow p$.

   _____
6. (inverse) $\sim p \rightarrow \sim q$ means "Not p implies not q" or "If not p, then not q." Write $\sim p \rightarrow \sim q$.

   _____
7. (contrapositive) $\sim q \rightarrow \sim p$ means "Not q implies not p" or "If not q, then not p." Write $\sim q \rightarrow \sim p$. _____
8. $p \wedge q$ means "p and q." Write $p \wedge q$. _____
9. $p \vee q$ means "p or q." Write $p \vee q$. _____
10. $\therefore p$ means "therefore p." Write $\therefore p$. _____
11. $p \leftrightarrow q$ means "p if and only if q." Write $p \leftrightarrow q$. _____

Use the following conditional statement to complete 1-8: "If I win, then you don't lose."

1. Write the hypothesis. _____
2. Write the conclusion. _____
3. Negate the hypothesis. _____
4. Negate the conclusion. _____
5. Write the converse. _____
6. Write the inverse. _____
7. Write the contrapositive. _____
8. Write the biconditional. _____

Logic and Conditional Statements, Part 1 (Teacher's Reference)

Use the following conditional statement to answer1-11: "If elephants fly, then fish don't swim." Each answer should be a complete sentence, *not* symbols.

1. p is the hypothesis. Write p. Elephants fly.
2. q is the conclusion. Write q. Fish don't swim.
3. $\sim p$ means "the negation of p." Write $\sim p$. Elephant don't fly.
4. $\sim q$ means "the negation of q." Write $\sim q$. Fish swim.
5. (converse) $q \rightarrow p$ means "q implies p" or "If q, then p." Write $q \rightarrow p$. If fish don't swim, then elephants fly.
6. (inverse) $\sim p \rightarrow \sim q$ means "Not p implies not q" or "If not p, then not q." Write $\sim p \rightarrow \sim q$. If elephants don't fly, then fish swim.
7. (contrapositive) $\sim q \rightarrow \sim p$ means "Not q implies not p" or "If not q, then not p." Write $\sim q \rightarrow \sim p$. If fish swim, then elephants don't fly.
8. $p \wedge q$ means "p and q." Write $p \wedge q$. Elephants fly and fish don't swim.
9. $p \vee q$ means "p or q." Write $p \vee q$. Elephants fly or fish don't swim.
10. $\therefore p$ means "therefore p." Write $\therefore p$. Therefore, elephants fly.
11. $p \leftrightarrow q$ means "p if and only if q." Write $p \leftrightarrow q$. Elephants fly, if and only if fish don't swim.

Use the following conditional statement to answer 1-8: "If I win, then you don't lose."

1. Write the hypothesis. I win.
2. Write the conclusion. I don't lose.
3. Negate the hypothesis. I don't win.
4. Negate the conclusion. I lose.
5. Write the converse. If I don't lose, then I win.
6. Write the inverse. If I don't win, then I lose.
7. Write the contrapositive. If I lose, then I don't win.
8. Write the biconditional. I win, if and only if, I don't lose.

Logic and Conditional Statements, Part 2

Name _____ **Date** _____

1. Write each of the following statements as a conditional statement. Then, circle the hypothesis, and underline the conclusion.

   a. Mark Twain wrote, "If you tell the truth, you don't have to remember anything."

   _____

   b. William Camden wrote, "The early bird catches the worm."

   _____

   c. Helen Keller wrote, "One can never consent to creep when one feels the impulse to soar."

   _____

   d. Mahatma Gandhi wrote, "Freedom is not worth having if it does not include the freedom to make mistakes."

   _____

   e. Benjamin Franklin wrote, "Early to bed and early to rise makes a man healthy, wealthy, and wise."

   _____

2. Write the converse, inverse, and contrapositive for each of the following conditional statements. Determine whether each is true or false.

   a. "If two segments are congruent, then they have the same length."

      Converse: _____

      Inverse: _____

      Contrapositive: _____

      True or false: _____

   b. A rectangle has four sides.

      Converse: _____

      Inverse: _____

      Contrapositive: _____

      True or false: _____

3. Write each of the following statements in symbolic notation:

   Let p represent: you see lightning
   Let q represent: you hear thunder

   a. If you see lightning, then you hear thunder._____

   b. If you hear thunder, then you see lightning._____

   c. If you don't see lightning, then you don't hear thunder._____

   d. If you don't hear thunder, then you don't see lightning._____

4. Write each of the following statements in symbolic notation:

   Let p represent: two planes intersect
   Let q represent: the intersection is a line

   a. If two planes don't intersect, then the intersection is a line._____

   b. If the intersection is not a line, then two planes do not intersect._____

Flash Cards

Conditional Statement	***p* implies *q***
Hypothesis	**"p"**
Conclusion	**"q"**
Biconditional	***If and only if*** **Combines the conditional and its converse when both are true.**
If …, then	**→**
Not	**~**
Converse	**"Switch"** **hypothesis and conclusion**
Inverse	**"Negate"** **Hypothesis and conclusion**
Contrapositive	**"Switch and Negate"**

Logic and Conditional Statements