

Properties of Quadrilaterals

Strand: Polygons and Circles

Topic: Exploring quadrilaterals

Primary SOL: G.9 The student will verify and use properties of quadrilaterals to solve problems, including practical problems.

Related SOL: G.2, G.6, G.10

Materials

- Properties of Quadrilaterals: Part 1 activity sheet (attached)
- Properties of Quadrilaterals: Part 2 activity sheet (attached)
- Properties of Quadrilaterals: Part 3 activity sheet (attached)
- Zombie Quadrilateral activity sheet (attached)
- Dynamic geometry software package (Activity can be modified to use paper instead.)

Vocabulary

bisect, diagonal, isosceles trapezoid, parallel, parallelogram, quadrilateral, rectangle, regular polygon, rhombus, right angles, square, trapezoid

Student/Teacher Actions: What should students be doing? What should teachers be doing?

1. Have students work in pairs to the Properties of Quadrilaterals: Part 1 activity sheet. Each student should record his/her own findings. Have students discuss their findings with their partner. Discuss the findings as a whole group. The teacher (or students) should create an anchor chart of the generalizations/properties that partners determined during the activity.
2. Have students work in pairs to complete the chart on Properties of Quadrilaterals: Part 2. Each student should record his/her own findings. Have partners join another team to compare and discuss their answers. Discuss the finding as a whole group.
3. Have students work in pairs or individually to complete the Properties of Quadrilaterals: Part 3 activity sheet. Discuss the findings as a whole group.
4. Have students work individually to complete the Zombie Quadrilateral activity sheet. Discuss findings as a whole group when completed.

Assessment

- **Questions**
 - Sally says she drew a trapezoid with parallel sides that are congruent. Could her drawing be a trapezoid? Explain.
 - What is another name for an equiangular quadrilateral that is not regular?
 - For parallelogram $ABCD$ with diagonals that intersect at E , what do you know about the angles, sides, and diagonals?
 - For parallelogram $ABCD$ with diagonals that intersect at E , why might someone call the segments \overline{AE} , \overline{BE} , \overline{CE} , and \overline{DE} “half-diagonals”?

- **Journal/writing prompts**
 - Have students complete a journal entry summarizing their investigations.
 - List the ways to show a quadrilateral is a parallelogram. Draw a diagram for each.
- **Other Assessments**
 - Show students a quadrilateral $ABCD$ with diagonals that intersect at E . Give them a list of properties, such as $\overline{AB} \cong \overline{CD}$, $\overline{AD} \cong \overline{BC}$, and \overline{BD} bisects \overline{AC} . Students should determine which conditions are sufficient by themselves to prove that $ABCD$ is a parallelogram.
 - Draw two quadrilaterals whose corresponding sides have ratio 1:2 but are not similar.
 - Draw a Venn diagram showing the relationships between parallelograms, rectangles, rhombi, squares, trapezoids, quadrilaterals, and kites.

Extensions and Connections

- Have students complete a Venn diagram showing the relationships among quadrilaterals.
- Have students use two sticks, straws, or linguini as diagonals of quadrilaterals. (They can sketch the quadrilaterals on paper.) Allow students to break or cut the sticks so they are not necessarily congruent. What must be true about the sticks and their intersections for the resulting quadrilateral to be a parallelogram? A rectangle? A rhombus? A square? An isosceles trapezoid?
- Have students explore symmetries for various quadrilaterals. (Which quadrilaterals have point symmetry? Which have line symmetry?)
- Invite a carpenter, builder, or city planner to the class to demonstrate the various job applications that use the current content.
- Have students work in groups to make up their own quizzes, presentations, graphic organizers, or games.

Strategies for Differentiation

- Put the instructions for how to use the geometry software on audio tape for students.
- Create assistive learning tools (e.g., Venn diagrams, mappings, tables) for students.
- Color-code to differentiate various parts of the lesson.
- Have students work in groups to create Venn diagrams to summarize the lesson.
- Enlarge the table in the Properties of Quadrilaterals: Part 2 activity sheet.
- Use a compare-and-contrast strategy to summarize the properties of the quadrilaterals.
- Show a completed table from exercises on an overhead so students can see whether they have completed theirs correctly.
- Color-code using different-colored dots to mark properties of different types of quadrilaterals in the table.
- Let students who have difficulty writing draw diagrams with the properties marked (i.e., congruent and parallel sides) rather than writing out the properties.

Note: The following pages are intended for classroom use for students as a visual aid to learning.

Virginia Department of Education © 2018

Properties of Quadrilaterals: Part 1

Name _____ Date _____

Complete each of the following tasks or questions.

1. Draw a parallelogram, using dynamic geometry software.
 - a. Draw a segment, and label the endpoints A and B .
 - b. Draw a point not on the segment, and label the point C .
 - c. Construct a line through C , parallel to \overline{AB} .
 - d. Draw segment \overline{AC} .
 - e. Construct a line through B , parallel to \overline{AC} .
 - f. Label the point where the two lines intersect as D .
 - g. Construct the sides of parallelogram $ABCD$. Hide the parallel lines.
 - h. You have now formed parallelogram $ABCD$. Gently move one of the points, and notice how the parallelogram changes.
2. Measure the length of each segment, and record your findings here.
 $\overline{AB} =$ _____, $\overline{BC} =$ _____, $\overline{CD} =$ _____, $\overline{DA} =$ _____
3. What do you notice? Now, gently move one of the points, and notice what happens to the segment lengths. What generalization can you make about the sides of a parallelogram?
4. Measure each angle, and record your findings here.
 $m\angle A =$ _____, $m\angle B =$ _____, $m\angle C =$ _____, $m\angle D =$ _____
5. What do you notice? Now, gently move one of the points, and notice what happens to the angle measures. What generalization can you make about the angles of a parallelogram?
6. Draw and measure the diagonals. Label their point of intersection as E . Measure the lengths \overline{AE} and \overline{ED} . What do you notice? Measure the lengths \overline{BE} and \overline{EC} . What do you notice?
7. What generalizations can you make about the properties of a parallelogram?

8. A *rectangle* is a parallelogram with four right angles. How would you prove that a parallelogram is a rectangle? Draw a rectangle using dynamic geometry software, and explain how you prove that it is a rectangle. Record your findings here. Measure all angles, all sides, and all diagonals (and even the angles formed by the diagonals) to help with your reasoning.

9. A *rhombus* is a parallelogram with all sides congruent. How would you prove that a parallelogram is a rhombus? Draw a rhombus using dynamic geometry software, and explain how you prove that it is a rhombus. Record your findings here. Measure all angles, all sides, and all diagonals (and even the angles formed by the diagonals) to help with your reasoning.

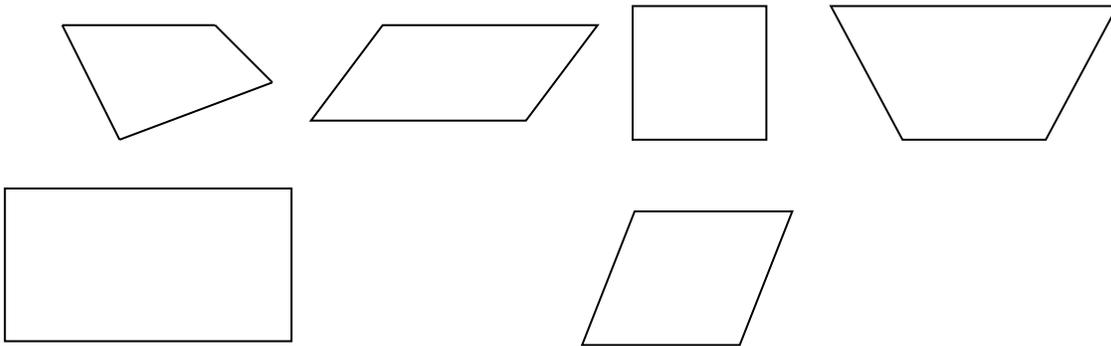
10. A *square* is a parallelogram with four right angles and four congruent sides. How would you prove that a parallelogram is a square? Draw a square, using dynamic geometry software, and explain how you prove that it is a square. Record your findings here. Measure all angles, all sides, and all diagonals (and even the angles formed by the diagonals) to help with your reasoning.

11. A *trapezoid* is a quadrilateral with exactly one pair of parallel sides. Why can't you prove that a trapezoid is a parallelogram? Draw a trapezoid, using dynamic geometry software. Measure all sides and angles of the trapezoid. What do you notice? How can you prove that a trapezoid is an isosceles trapezoid? What do you notice about angles, sides, and diagonals?

12. Which quadrilaterals have:

Mathematics Instructional Plan – Geometry

- a) four right angles?
- b) exactly one pair of parallel sides?
- c) two pair of opposite sides congruent?
- d) four congruent sides?
- e) two pair of opposite sides parallel?
- f) no sides congruent?
- g) perpendicular diagonals?
- h) opposite angles congruent?
- i) diagonals bisect each other?
- j) Four congruent angles?



12. Using your knowledge of quadrilaterals, investigate the types of symmetry (point symmetry, line symmetry, or no symmetry) the quadrilaterals shown above might have. Justify your answers.

Point symmetry: When a figure can be mapped onto itself by a rotation of 180 degrees.

Line symmetry: When a figure can be mapped onto itself by a reflection over a line.

Properties of Quadrilaterals: Part 2

Name _____ Date _____

Complete the following table using a “D” (for definition) for any property that is a part of the definition of the polygon and checking off all the other polygons that have each property. (See the first row for an example.)

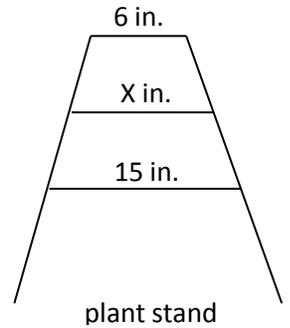
	Properties	Quadrilateral	Parallelogram	Rectangle	Rhombus	Square	Trapezoid	Isosceles Trapezoid
Definitions	Polygon with four sides	D	✓	✓	✓	✓	✓	✓
	Both pairs of opposite sides must be \parallel .							
	All angles must be right angles.							
	All sides must be \cong .							
	Exactly one pair of opposite sides is \parallel .							
	Exactly one pair of opposite sides is \cong .							
	Two pair of adjacent sides must be \cong .							
Properties	Both pairs of opposite sides must be \cong .							
	Both pairs of opposite angles must be \cong .							
	All angles must be \cong .							
	Diagonals must bisect each other.							
	Diagonals must be \cong .							
	Diagonals must be \perp .							
	Diagonals bisect opposite angles.							
	Consecutive angles must be supplementary.							
What is POSSIBLE?	All sides may be \cong .							
	Both pairs of opposite angles may be \cong .							
	All angles may be \cong .							
	All angles may be right angles.							
	Diagonals may be \cong .							
	Diagonals may bisect each other.							
	Diagonals may be \perp .							

Properties of Quadrilaterals: Part 3

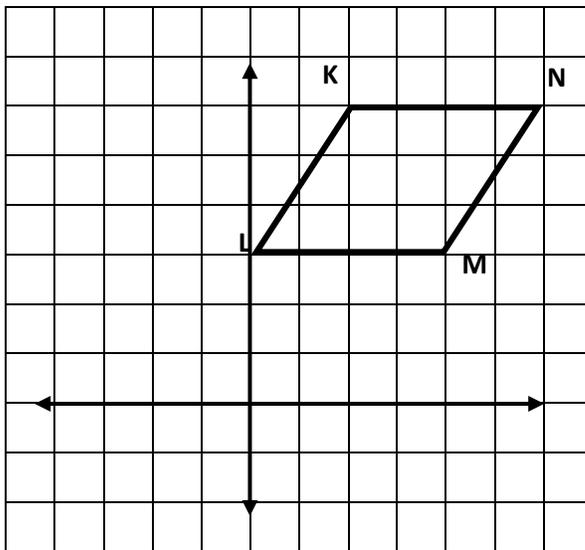
Name _____ Date _____

Use your knowledge of quadrilaterals to solve the following problems.

1. You want to build a plant stand with three equally spaced circular shelves. You want the top shelf to have a diameter of 6 inches and the bottom shelf to have a diameter of 15 inches. The diagram at the right shows a vertical cross-section of the plant stand. What is the length of the middle shelf?



2. Prove that the quadrilateral shown below on the grid is a parallelogram. Show all work.

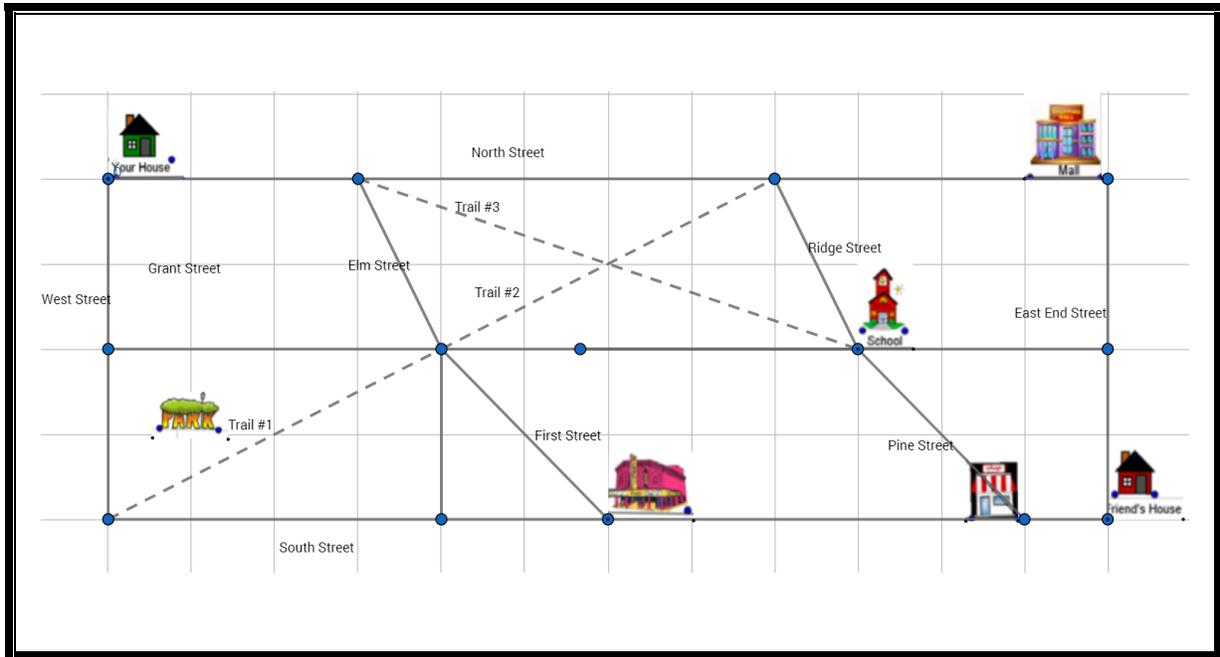


Zombie Quadrilateral

Name _____ Date _____

Breaking News! It has been reported that the principal of your school has been infected by the zombie virus. The entire town is at risk for a zombie outbreak.

Your principal was last seen chasing joggers in the park. You and your friends need to find the fastest routes to safety. You find a map of your town in a kitchen drawer. Use the map to determine which routes you and your friends should take for each situation.



1. You are at the school and need to get to safety. You have the choice of running down Pine Street or up Ridge Street. Which is a shorter distance? What does this tell you about Elm Street and First Street? Explain your answer.

2. You are finally safe at your house when you hear the breaking news that the zombies are heading toward your friend's house. Your friend wants to run to your house because he is afraid to be home alone. He thinks it is safer to run on the roads, but you say it is a shorter distance to run a straight diagonal from each house. Who is correct? What is the difference in distance? What does this tell you about diagonals in a quadrilateral? Justify your answer.

3. Betty is at the school and wants to join you and your friend in the safety of your house. She is not sure how to get to your house and her phone is dead, so she cannot use GPS. Zombies have been reported walking down Main Street by the park. Describe two different routes Betty can take that do not include her travelling down the part of Main Street above the park. What is the distance of each route?

4. Billy escaped zombie joggers in the park and is at West Street and South Street. He wants to run to the mall where other non-infected townspeople have gathered for safety. He has two options:
 - a.) Run through the woods up Trail 1 and Trail 2, then head east on North Street.
 - b.) Run east on South Street, then head north up East End Street. Which is the shortest route? Justify your reasoning mathematically and with words.