How Many Triangles?

Strand: Triangles

Topic: Investigating the triangle inequality theorem

Primary SOL: G.5 The student, given information concerning the lengths of sides and/or measures of angles in triangles, will solve problems, including practical problems. This will include
a) ordering the sides by length, given the angle measures;
b) ordering the angles by degree measure, given the side lengths;
c) determining whether a triangle exists; and
d) determining the range in which the length of the third side must lie.

Related SOL G.8a, G.10

Materials

- How Many Triangles? (Part 1) activity sheet (attached)
- How Many Triangles? (Part 2) activity sheet (attached)
- Pipe cleaners or narrow strips of paper
- Rulers
- Markers
- Compasses
- Protractors

Vocabulary

angle, corresponding, inequality, interval, isosceles triangle, opposite angle, opposite side, range of the length of the third side of a triangle, scalene triangle, segment, side

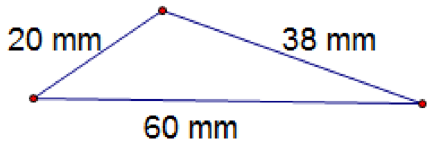
Student/Teacher Actions: What should students be doing? What should teachers be doing?

1. Distribute pipe cleaners or narrow strips of paper, markers, compasses, protractors, and the How Many Triangles? (Part 1) activity sheet. Have students work in pairs to complete the activity. Each student should record his/her own findings. Have students discuss the findings with their partners. Discuss the findings as a whole group.
2. Distribute How Many Triangles? (Part 2) activity sheet, and have students work in pairs to complete it. Each student should record his/her own findings. Have students discuss the findings with their partners. Discuss the findings as a whole group.

Assessment

- **Questions**
  - Can you create a triangle using any three straight sticks? Explain.
  - The Browns are driving toward Richmond, Virginia. They see a sign that reads, "Charlottesville 75 miles, Petersburg 35 miles." Ashley comments that she did not think Charlottesville and Petersburg were only 40 miles apart. Explain why the distance between the two cities does not have to be 40 miles.

- What is wrong with the diagram shown below? Explain.



- The base of an isosceles triangle measures 12 inches. What do you know about the length of the two legs? Be as specific as possible, and explain your reasoning.
- The longest side of a triangle measures 10 cm, and the shortest side measures 4 cm. What is the range for the length of the third side? (Hint: Could the third side measure 12 cm?) Be as specific as possible and explain your reasoning.
- ΔABC has the following angle measures: $m\angle A = 60$, $m\angle B = x - 2$, and $m\angle C = x + 2$. List the sides in order from longest to shortest. Explain your reasoning.
- Why is the longest side of a right triangle always the hypotenuse?

- **Journal/Writing Prompts**
  - Explain, in your mathematics journal, how to tell whether you can make a triangle with side lengths *a* cm, *b* cm and *c* cm.
  - Explain, in your mathematics journal, how to determine the order of the lengths of the sides of a triangle, given the angle measures.
  - Explain, in your mathematics journal, how to determine the order of the measures of the angles of a triangle, given the lengths of the sides.
  - Explain how, in an isosceles triangle, you can determine by looking at the angle measures whether the base is longer or the legs are longer.
  - Explain why the hypotenuse of a right triangle must be the longest side.
  - Explain what happens when trying to form a triangle end-to-end with two segments whose sum equals the length of the third segment.
- **Other Assessments**
  - You have a 15-foot piece of narrow pipe and will cut it twice to form three pieces. Each piece must measure a whole number of feet (i.e., 1 foot or 4 feet, not 4½ feet). How many ways can you cut it so that the pieces form a triangle end-to-end?
  - Have students draw a diagram to illustrate the rules about corresponding (largest or smallest) angles being opposite corresponding (longest or shortest) sides.
  - Give a student a piece of dry linguini and have them break it into three pieces that show why you cannot always form a triangle using three segments placed end to end.

Extensions and Connections (for all students)

- Connect this lesson to the converse of the Pythagorean theorem.

- Give students the coordinates of the vertices of a triangle, and have them order the angles by their measures.
- Connect this lesson to the shortest distance between a point and a line or a plane.

Strategies for Differentiation

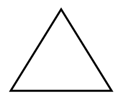
- Allow students to use an enlarged activity sheet so that they have enough room to work and visualize what they are doing.
- Have students do a similar activity using dynamic geometry software or an applet found on the internet.
- Have students color-code the sides and angles of the triangles. (Mark a side and the vertex of the opposite angle with the same color.)
- Have students use a folding carpenter's ruler to explore these ideas.
- Create a fill-in-the-blank outline of the key concepts, and have students fill in the blanks.

Note: The following pages are intended for classroom use for students as a visual aid to learning.

How Many Triangles? (Part 1)

Name ___________________________________ **Date** ______________________

1. Mark your pipe cleaner or paper strip at 1-inch intervals. Folding only at your marks, try to make triangles with lengths as given in the table, placing them end-to-end. Use a protractor to measure the angles for each triangle. If no triangle can be formed, then write “none” for the measure of each angle. Complete the table.

Side Lengths	**Sketch**	**Triangle? Yes/No**	**Measure of each angle in each triangle formed**
4 in., 4 in., 4 in.		Yes	60°, 60°, 60°
3 in., 5 in., 4 in.			
2 in., 6 in., 4 in.		No	none
1 in., 7 in., 4 in.			
2 in., 5 in., 5 in.			
2 in., 7 in., 3 in.			
3 in., 6 in., 3 in.			

2. Now use your pipe cleaner or strip to form two sides of a triangle with lengths 5 inches and 7 inches.

   a. Could the third side of this triangle with side lengths 5 inches and 7 inches measure …

      1 inch? ______ 2 inches? ______ 2.1 inches? ______ 7 inches? ______

      11 inches? ______ 11.9 inches? ______ 12 inches? ______ 13 inches? ______

   b. Complete: The third side must be greater than ________ inches and less than ________ inches.

3. Draw a scalene triangle. Measure all three angles. Name the vertex of the largest angle *L*, the vertex of the medium angle *M*, and the vertex of the smallest angle *S*. Measure the three sides. Label the longest side *l*, the medium side *m*, and the shortest side *s*. Which side is opposite?

   a. $\angle L$? ________ b. $\angle M$? ________ c. $\angle S$? ________

How Many Triangles?

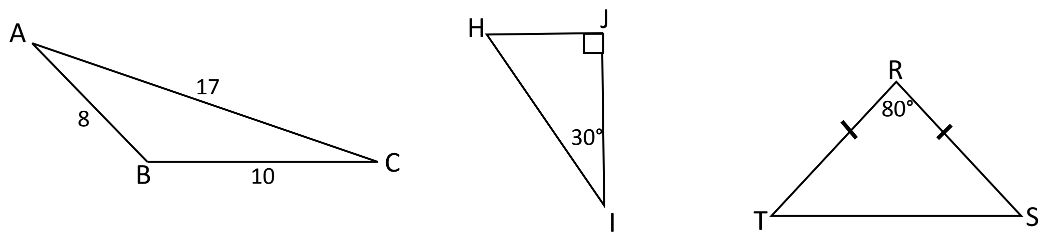
Name ______________________________________ **Date** _______________________

1. Determine whether the following lengths will form a triangle. Briefly explain each answer.

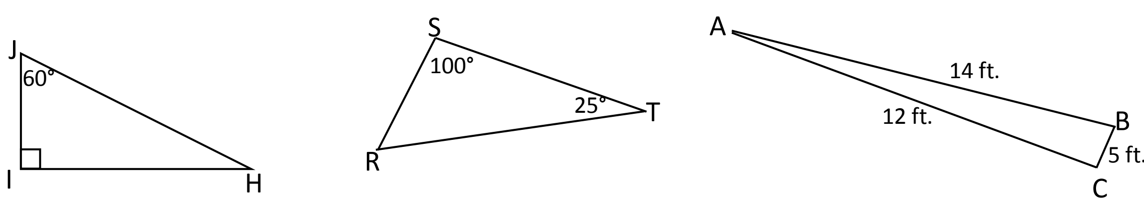
   a. 5 in., 2 in., 8 in. b. 6 cm, 18 cm, 15 cm c. 5 ft., 6 ft., 9 ft.

   d. 7 in., 7 in., 8 in. e. 1.2 mi., 4.0 mi., 1.8 mi. f. 10 mm, 10 mm, 0.001 mm

2. List the sides and angles of each triangle in order from smallest to largest. How do you know?



3. List the sides and angles of each triangle in order from largest to smallest. How do you know?



4. ΔABC has side lengths of 1 inch, $1\frac{7}{8}$ inches, and $2\frac{1}{8}$ inches and angle measures of 90 degrees, 28 degrees, and 62 degrees. Which side is opposite each angle?

5. The lengths of two sides of a triangle are given. Determine the range for the possible lengths of the third side. Explain your reasoning or show work to support your answer.

   a. 5 in., 2 in.   b. 1 cm, 180 cm   c. 15 ft., 10 ft.

   d. 70 in., 70 in.   e. 3 mi., 7 mi.   f. 1.3 mm, 1.6 mm