Slope

Strand: Reasoning, Lines, and Transformations

Topic: Exploring slope, including slopes of parallel and perpendicular lines

Primary SOL: G.3 The student will solve problems involving symmetry and transformation. This will include
a) investigating and using formulas for finding distance, midpoint, and slope; and
b) applying slope to verify and determine whether lines are parallel or perpendicular.

Related SOL: G.3d, G.6, G.7, G.9

Materials

- Slope of a Line activity sheet (attached)
- Slopes of Parallel and Perpendicular Lines activity sheet (attached)
- Slopes of Parallel and Perpendicular Lines Equations of Lines activity sheet (attached)
- Card Set: Equations
- Graph paper
- Dynamic geometry software
- Desmos (free online graphing software and graphing calculator) www.desmos.com

Vocabulary

distance, horizontal, midpoint, negative reciprocal, parallel, perpendicular, reciprocal, rise, run, slope, vertical

Student/Teacher Actions: What should students be doing? What should teachers be doing?

1. Distribute the Slope of a Line activity sheet, and have students work in small groups to complete it. Each student should record his/her own findings. Have students discuss findings with their partners. Discuss findings as a whole group.
2. Distribute the Slopes of Parallel and Perpendicular Lines activity sheet, and have students work in small groups to complete it. This activity can be done on graph paper if software is not available. Each student should record his/her own findings. Have students discuss findings with their partners. Discuss findings as a whole group.
3. Using $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ with the following coordinates of A (–3, 2), B (3, –4), C (4, 1), and D (–3, –6), demonstrate that the product of slopes of perpendicular lines is −1, and the slopes of perpendicular lines are negative reciprocals.
4. Distribute the Slopes of Parallel and Perpendicular Equations of Lines activity sheet. Have students work in groups of two or three. Each group should receive a copy of the Card Set: Equations (already cut up). Introduce the activity by explaining that groups must find pairs of equation cards to place in rows within the appropriate table, indicating the equations form either parallel or perpendicular pairs of lines.

5. Using technology to complete Slopes of Parallel and Perpendicular Equations of Lines activity sheet requires a computer with an internet connection. Have students use the internet-based graphing utility Desmos, found at www.desmos.com. Have students click on the red button to "Start Graphing." Click the wrench for settings, deselect minor gridlines, and change the step to 1. With Desmos, students can enter the equation of the line in slope-intercept form or in standard form. Have students enter one equation in Desmos and then compare another card to the first equation one at a time. As students type in the equation and see the graph of the line, students should point out several characteristics of each line (i.e., whether the slope is positive, negative, zero, or undefined; the *x*- and *y*-intercepts; and whether the lines intersect). Students will select pairs of equation cards to place in rows within the appropriate table, indicating the equations are either parallel or perpendicular.

Assessment

- **Questions**
  - What is the slope of a line passing through the two points D (−2, 3) and E (6, −3) (draw the line on a coordinate plane)?
  - What is the graph of a line passing through F (3, 1) parallel to $\overleftrightarrow{DE}$ (from the question above)? Show your work, and explain how you drew this line.
  - What is the graph of a line passing through F (3, 1) perpendicular to $\overleftrightarrow{DE}$ (from the question above). Show your work, and explain how you drew this line.
  - A store needs to install a ramp for people in wheelchairs. The ratio of the slope of this ramp (height:length) must be no more than $\frac{1}{12}$. If the ramp must reach a height of 28 inches, how long must the ramp be?
  - Line segment $\overline{SP}$ has the equation y = 2x + 3. If $\overline{SP}$ is one side of a rectangle named PQRS, find the equations of the line segments forming the other three sides of rectangle PQRS.
  - How are the slopes of horizontal and vertical lines related?

- **Journal/writing prompts**
  - Have students complete a journal entry summarizing the Slopes of Parallel and Perpendicular Lines activity.
  - Describe how to find the slope of a line when given the graph of the line.
  - Describe how to determine whether a line has a positive or negative slope.
  - Describe how to find the slope of a line perpendicular to a line with given slope.

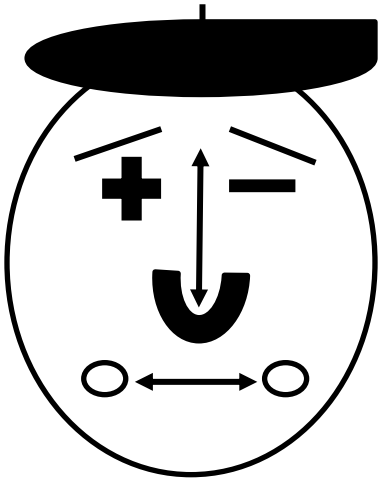
- **Other Assessments**
  - Have each group present their findings to the class from the activities completed.
  - Write the definition and formula for slope using the points P (a, b) and Q (c, d).
  - Have groups of students write test questions to use for assessment purposes.

Extensions and Connections

- Give students the coordinates of the vertices of quadrilaterals, and have them use slope to determine whether the quadrilaterals are parallelograms, rectangles, or neither.
- Have students explore tangent lines and circles in the coordinate plane.

Strategies for Differentiation

- Use graph paper and a straightedge to complete the activity.
- Review vocabulary such as *horizontal* and *vertical*.
- Link slopes of perpendicular lines to algebraic properties.
- Use linking cubes to illustrate the idea of slope. Place one cube, and then place two cubes, one on top of the other, next to the first cube. Place three cubes, one on top of the other, next to the two cubes. Continue with the pattern. Lay a ruler on the stacks. Discuss the slope of the ruler. Repeat with different arrangements of cube stacks (e.g., 1-1-2-2-3-3-4-4; 1-3-5-7-9), and compare the slopes.
- Have each student review slope vocabulary by creating the slope man (positive, negative, undefined, and zero slope).



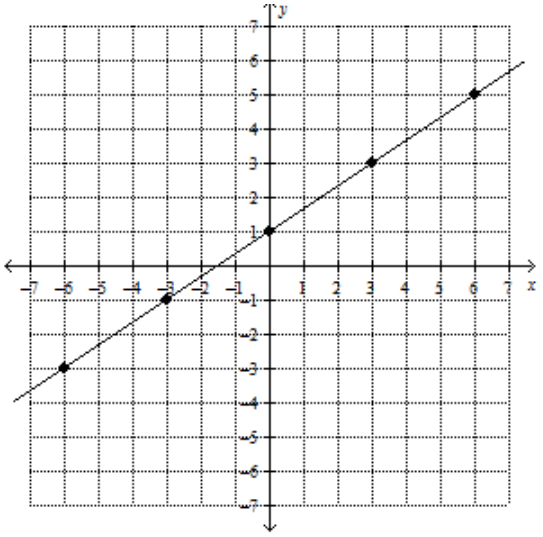
Note: The following pages are intended for classroom use for students as a visual aid to learning.

Slope of a Line

Name _____ **Date** _____

1. Select two points on the graph of the line below, and label one *P* and the other *Q*.

2. Draw a vertical path followed by a horizontal path, from *P* to *Q*.



3. What is the vertical distance? _____

4. What is the horizontal distance? _____

5. What is the ratio of the vertical distance to the horizontal distance? _____

6. What term describes this ratio?

7. The formula for finding the slope of the line through two points is $m = \frac{y_2 - y_1}{x_2 - x_1}$. Use this formula and the points you used above to compute the slope of the line. Do you get the same answer?

8. Find three other points on the line through *R* (−2, 4) with the same slope as the line above. Graph the line. Does it appear that the two lines are parallel, perpendicular, or neither?

9. Find three other points on the line through *R* (−2, 4) with slope $-\frac{2}{3}$. Graph the line.

10. Use a corner of a piece of paper to check the angle formed by the two lines. What does that angle appear to be? Does it appear that the two lines are parallel, perpendicular, or neither?

11. Multiply the slopes of the two lines together. What is the product?

Slopes of Parallel and Perpendicular Lines

Name _____ **Date** _____

If available, use a dynamic geometry software package to graph the following lines on the same coordinate plane, and answer the questions. You will need to show a grid or coordinate plane.

Part 1

1. Open a new sketch, and show the grid or coordinate plane.
2. Plot two points with integer coordinates that do not form a horizontal or vertical line, and name them *A* and *B*. Draw the line $\overline{AB}$.
3. Use the slope formula to compute the slope of $\overline{AB}$.
4. Use the dynamic geometry software package to measure the slope of $\overline{AB}$. Do the answers to 2 and 3 agree? If not, explain why they are different.
5. Select a point not on $\overline{AB}$, and construct a line through the point, parallel to $\overline{AB}$. Label two points on this line *C* and *D*.
6. Use the dynamic geometry software package to measure the slope of $\overline{CD}$.
7. Gently move the lines to change their slopes, and notice how the slopes are related.
8. What can you conclude about the slopes of parallel lines?
9. Select a point not on $\overline{AB}$, and construct a line through the point, perpendicular to $\overline{AB}$. Label two points on this line *E* and *F*.
10. Use the dynamic geometry software package to measure the slope of $\overline{EF}$.
11. Use the dynamic geometry software package to compute the product of the slopes of $\overline{AB}$ and $\overline{EF}$. What do you notice?
12. Gently move the lines to change their slopes (without making either line horizontal), and notice what happens to the product of the slopes.
13. Adjust the lines until one is horizontal and one is vertical. What do you notice about the slopes of the lines and the product of the slopes?
14. What can you conclude about the product of the slopes of perpendicular lines?
15. Name your file as directed by your teacher.

Part 2

1. Open a new sketch. Graph lines *k*, *l, m,* and *n* as described below.
2. Line *k* contains the points (4, 0) and (0, 2).
3. Line *l* contains the points (2, −2) and (−4, 1).
4. Line *m* contains the points (3, 3) and (0, −3).
5. Line *n* contains the points (0, 3) and (−2, 2).
6. What point appears to be the intersection of line *k* and line *m*?
7. What is the slope of each line? Explain how you found your answers.

8. Which lines are parallel? Explain.

9. Which lines are perpendicular? Explain.

10. Draw a line *p* that is parallel to line *m*. What is the slope of line *p*?

11. Which line or lines are perpendicular to line *p*?

12. Is it possible to draw a line parallel to line *m* that is *not* perpendicular to line *k*? Explain.

13. Name your file as directed by your teacher.

Slopes of Parallel and Perpendicular Equations of Lines

These lines are parallel	

These lines are perpendicular	

Card Set: Equations

Print on card stock and cut out.

$y = 4(x + 1)$	$-2y - 10 + 2x = 0$
$x - y = -2$	$x + 5 + y = 0$
$y = \frac{4}{3}x - 1$	$3x + 20 = -4y$
$y = -x + 3$	$-15 - x = -5y$
$2x - y = 1$	$0 = 5y - x$
$x + 2y = -8$	$2y + 8 = 3x$
$4x - 3y = 9$	$4y = x + 3$
$y + 4x + 6 = 0$	$8x - 6y = 14$