

Arc Length and Area of a Sector

Strand: Polygons and Circles

Topic: Investigating arcs and areas

Primary SOL: G.11 The student will solve problems, including practical problems, by applying properties of circles. This will include determining
c) arc length; and
d) areas of a sector.

Related SOL: G.14d

Materials

- Problem Solving with Circles activity sheet (attached)
- Cake Problem activity sheet (attached)
- Graphing utility
- Demonstration circle (document camera, overhead, projector, whiteboard, or other)
- Duct tape

Vocabulary

arc, arc angle, arc length, arc measure, area of a circle, area of a sector, central angle, chord, circle, circumference, diameter, distance, formula, height, length, pi (π), proportional, Pythagorean Theorem, radius, sector, semicircle, straight segment

Student/Teacher Actions: What should students be doing? What should teachers be doing?

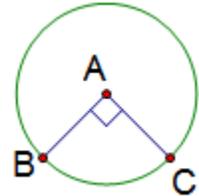
1. Have students use their graphing utility to find the lengths of three different ramps that in-line skaters might use on the Problem Solving with Circles activity sheet.
2. Review the formula for finding the circumference of a circle, if necessary.
3. Discuss how to find the lengths of various arcs of a circle.
4. Have students use the Pythagorean Theorem to find the length of the first ramp.
5. Hold a class discussion on the following questions:
 - What makes one ramp better than another?
 - Which ramp is the safest? Why?
 - Which construction is more challenging? Why?
6. Explain the Cake Problem, as shown on the Cake Problem activity sheet, to students without handing out the sheets. Make sure everyone understands the problem.
7. Display a 10-inch diameter demonstration circle (use a document camera, overhead, whiteboard, or other) to model the cake, and ask a volunteer to make an estimate of the placement of the cut that solves the problem.
8. Distribute the Cake Problem activity sheet, and have students work in small groups to solve the problems. Each student should record the solutions on his/her own activity sheet.
9. Have each group present their solutions to the class. Discuss the variations of solutions.

- Return to the original display of the cake with the estimated placement of the cut, and draw the correct solution. How close was the initial estimate?

Assessment

• Questions

- What is the difference between arc length and arc measure?
- GeoPizza sells 12-inch and 16-inch diameter pizzas. They cut the 12-inch pizza into six slices and the 16-inch pizza into eight slices. Would you get more pizza selecting three slices of the 12-inch pizza or two slices of the 16-inch pizza? Explain.
- The length of a minor arc BC of a circle A is 16π centimeters and $\overline{AB} \perp \overline{AC}$. Find the radius of the circle. Justify your answer.



• Journal/writing prompts

- Have students complete a journal entry summarizing one of the activities completed in this lesson.
- Write a practical problem and solution using arc length or area of a sector.
- Describe a formula that could be used to find arc length. Use mathematical vocabulary to explain what each unknown represents.

• Other Assessments

- Use the group presentations to assess comprehension.
- Have students design a ramp and compute its length.

Extensions and Connections

- Have students investigate how the Cake Problem changes if a different number of slices is requested.
- Have students find the steepness of the various ramps and use this data to determine the level of difficulty of each ramp.
- Use *The Librarian Who Measured the Earth*, by Kathryn Lasky and Kevin Hawkes, to investigate Eratosthenes' method for estimating the circumference of the Earth.
- Search the web for other descriptions of Eratosthenes' method of estimating the circumference of the Earth.
- Have students do a think-pair-share in the classroom. Show various pictures of ramps with their measurements. Have students work in pairs to determine which ramps are better for a beginning skater to use. Have them share their results with the class.
- A school's running track is formed by two parallel segments joined on the ends by two semicircles. Each straight segment is l meters long, and each semicircle is d meters in diameter. Write a formula for finding the distance, D , around the track.
- Arrange for students to make a trip to the school's baseball diamond, and have them use a tape measure to measure the dimensions of infield. Have them find the area of the infield to determine the amount of dirt needed to cover the infield.
- Arrange for students to visit a local skating park, take measurements of the ramps and ramp heights, and draw sketches. Have students determine which ramps are the safest.

Mathematics Instructional Plan – Geometry

- *Around the World in 80 Days*, by Jules Verne, tells the tale of a voyage around the world by rail and steamer. If an 80-day voyage around the world follows the equator (it didn't in the book), about how long should the voyagers allow to travel from Quito, Ecuador, to Libreville, Gabon?
- Research transportation for this part of the voyage via land and water. (No travel by air!)

Strategies for Differentiation

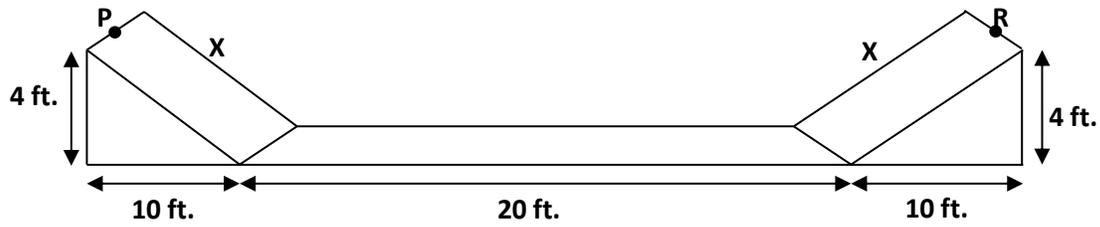
- Have students use a dynamic geometry software package to create and draw the ramps. Have them use the program to measure the distances and the lengths of arcs.
- Have students create a scale model in the classroom with cardboard to replicate the ramps.
- Organize stations to break up and isolate key elements of the lesson. Have students work through the stations in groups to complete the activity.
- Post a mathematics glossary with examples, pictures, and definitions.
- Have students use a graphic organizer to chart information presented in the unit.
- Have students use presentation software to demonstrate their knowledge and understanding of the key terms (e.g., pictures, sound, and motion).
- Have students work in groups to make up their own review materials.
- Teacher may want to provide picture for the word problems 3 and 4.

Note: The following pages are intended for classroom use for students as a visual aid to learning.

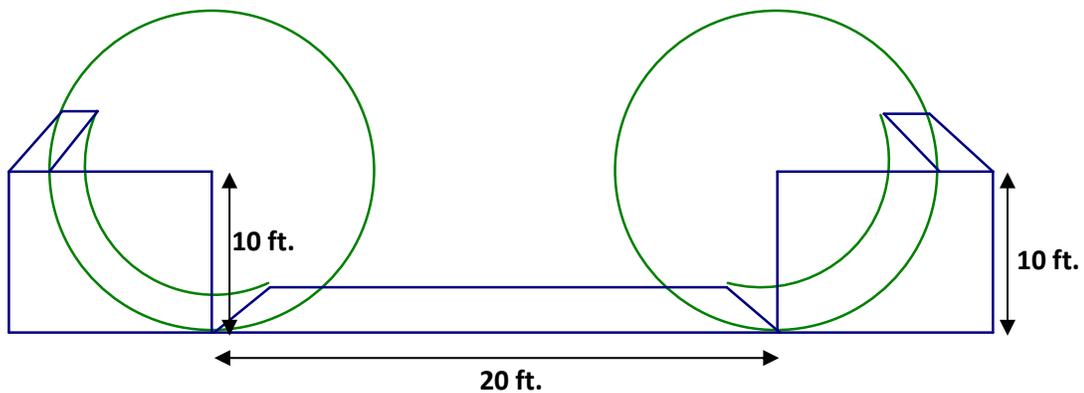
Problem Solving with Circles

Name _____ Date _____

1. Skateboarding has become a popular sport. The Parks Department is thinking of constructing ramps at some of the local playgrounds. The structure has two straight ramps, each of which is 4 feet high and 10 feet long, with a flat space of 20 feet in between. Find the distance a skater travels from the top of one ramp to the top of the other—from point P to point R. (Hint: Use the Pythagorean Theorem.)



2. A half-pipe is formed by two quarter-circle ramps, each of which is 10 feet high, plus flat space 20 feet long between the centers. Find the distance a skater travels from the top of one ramp to the top of the other.



3. Another launch ramp is formed by two arcs, each with a central angle of 60 degrees and a radius of 10 feet. Find the length from the top of one ramp to the top of the other. (Hint: What fractional part of the circle is each arc?)

Mathematics Instructional Plan – Geometry

4. A third ramp has two straight ramps, each of which is 4 feet high and 10 feet long, with a flat space of 20 feet in between. Find the distance a skater travels from the top of one ramp to the top of the other—from point P to point R. (Hint: Use the Pythagorean Theorem.)

Cake Problem

Name _____ Date _____

You have a cake that is 10 inches in diameter. You expect 12 people to share it, so you cut it into 12 equal slices (Figure A).

1. Find the area of each slice of cake.
2. Before you get a chance to serve the cake, 12 more people arrive! So, you decide to cut a concentric circle in the cake so that you will have 24 pieces (Figure B).
3. How far from the center of the cake should the circle cut be made so that all 24 people get the same amount of cake?
4. What is the area of each segment of cake? How much cake will each person receive?

Figure A

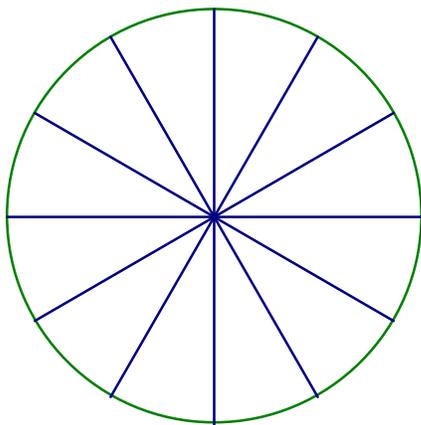


Figure B

