Exponential Growth and Decay

Strand: Algebra and Functions

Topic: Exploring Exponential Models

Primary SOL: AFDA.3 The student will collect and analyze data, determine the equation of the curve of best fit in order to make predictions, and solve practical problems using models of linear, quadratic, and exponential functions.

Related SOL: AFDA.1, AFDA.2

Materials

- Paper Folding activity sheet (attached)
- Area of Smallest Section activity sheet (attached)
- M&M Decay activity sheet (attached)
- Paper
- Graph paper
- Candies (fun-size packaging)
- Cups
- Paper towels
- Graphing utility
- Poster paper/markers

Vocabulary

domain, doubling time, end behavior, exponential decay, exponential growth, exponential regression, half-life, model, scatterplot, range, x-intercept, y-intercept

Student/Teacher Actions: What should students be doing? What should teachers be doing?

1. Divide the class into three groups, and assign the groups to one of three learning stations, each with a different activity sheet. Provide each group with poster paper with a table and a graph. Ask each group to post their output on the wall.
2. Let each group move from station to station and have them copy the data table, create a scatterplot on their graphing utility, and find the exponential regression equation.
3. When they return to their seats, ask them to share and discuss their answers to the questions after each activity.
4. Debrief the activity using the questions in the assessment as guide.

Assessment

- **Questions**
  - Using the regression models of the three activities, what do the numbers in the equation represent?
  - How do you recognize whether the equation models exponential growth or exponential decay?

  - Using the data, the scatterplot, or the equation from the Area of Smallest Section Activity, how many trials will it take for the area to be half the original area? (Discuss the concept of half-life.)
  - What will happen to the remaining area of the paper, if you continue on performing the experiment? (Discuss the end behavior of the function.)
  - Using the data, scatterplot, or the equation from M&M Decay activity station, how many trials will it take for the number of M&M candies remaining to be half the original amount of M&M candies?
  - What will happen to the number of M&M candies if you continue repeating trials in the experiment? (Discuss the end behavior of the function.)
  - Using the data, scatterplot, or the equation from the Paper Folding Activity, how many trials will it take for the number of sections to double? (Discuss the doubling time.)
  - What are the practical domain and practical range of the three exponential function models?

- **Journal/writing prompts (include a minimum of two)**
  - Summarize the differences between exponential growth and an exponential decay. Give examples of quantities that grow exponentially and quantities that decay exponentially.
  - Is it possible for each of the exponential models to have an x-intercept? Explain your answer in the context of the activity/activities. What does the y-intercept of each equation represent in the activity/activities?

- **Other Assessments (include informal assessment ideas)**
  - Suppose you buy a new phone on January 1 for $500. Every month, the value of the phone depreciates by 10 percent. This means that the value of the phone is reduced by $50 (10 percent of $500) after the first month.
    - Create a data table for 12 months.
    - Make a scatterplot of the data.
    - Describe how the value of the phone changes over time.
    - Find the exponential regression equation.
    - Use the equation to predict the value of the phone after two years.

Extensions and Connections (for all students)

- **Assign students to read "Step 1: Change Your Life with One Calculation: How Compound Interest from Investing Grows Your Money Faster than Anything Else,"** by The Motley Fool, March 23, 2016 **(https://www.fool.com/how-to-invest/thirteen-steps/step-1-change-your-life-with-one-calculation.aspx).** Find a regression equation for each of the following:
  a. Years and Savings Account
  b. Years and Money Market Fund
  c. Years and Certificate of Deposit
  d. Years and Stock Market

Do you think this scenario is possible in a practical situation? Why or why not?

Strategies for Differentiation

- Each activity can be done for one class period, and the whole class will do the same activity instead of learning stations. Give more time on the discussion of the lesson.
- Use www.desmos.com for creating tables and scatterplots instead of poster paper. Project the output for the whole class to see.
- Use vocabulary cards for related vocabulary listed above.
- Group students by mixed ability levels and assign roles (reader, recorder, etc.) to each member of the group.

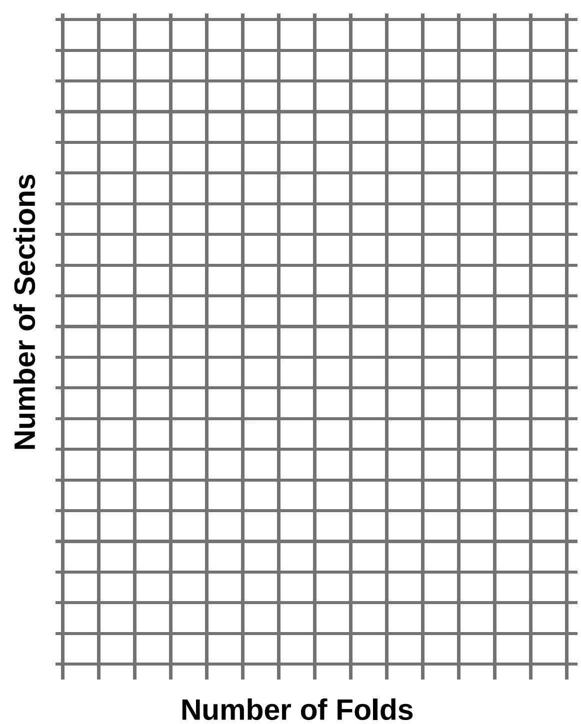
Note: The following pages are intended for classroom use for students as a visual aid to learning.

Paper Folding

Instructions:

1. Repeat folding an 8.5” x 11” sheet of paper in half and determine the number of sections the paper has after each fold.
2. Record your data in the table below and continue folding in half until it becomes too difficult to fold the paper.
3. Then make a scatterplot of your data.

Number of Folds	**Number of Sections**
0	
1	
2	
3	
4	
5	
6	
7	



4. Using your graphing utility, determine the mathematical model that represents this data:

   y = _____________

5. Explain in words what the mathematical model means.

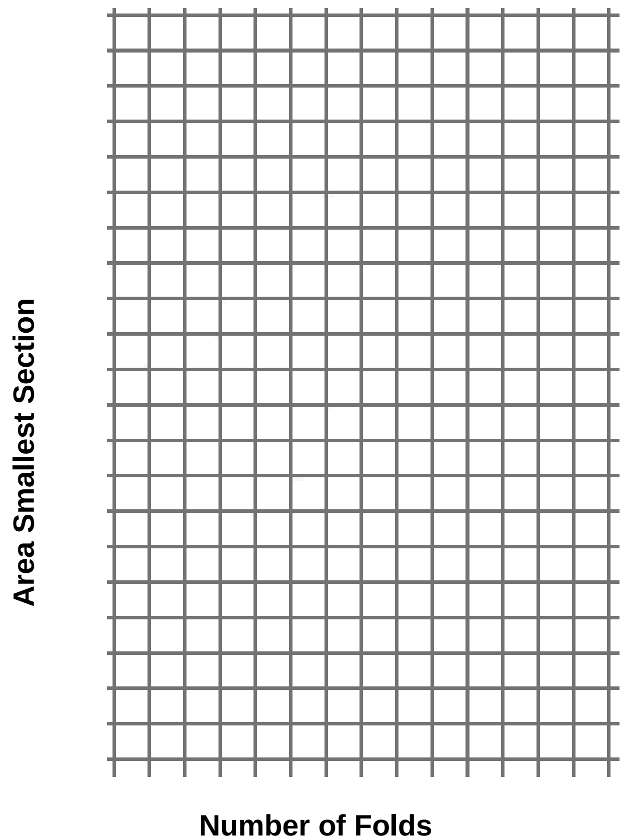
6. What might be different if you tried this experiment with wax paper or tissue paper? This is an example of exponential growth. The thickness of the paper grows very rapidly with each fold. To get an idea of this incredible growth, consider the following:
   - At seven folds, the sheet of paper is as thick as a notebook;
   - At 17 folds, the sheet of paper would be taller than the average house;

- At 20 folds, the sheet of paper would be thick enough to extend a quarter of the way up the Sears Tower in Chicago; and
- At 30 folds, the sheet of paper would cross the outer limits of the atmosphere.

Area of Smallest Section

1. Determine the area of the original sheet of paper and record it on the table when the number of folds is zero.
2. Fold the piece of paper in half. Determine the area of the paper after you have made a fold, and record your data in the table.
3. Repeat the process, and determine the area of the paper after each fold until you have completed the table below.
4. Make a scatterplot of your data.

Number of Folds	**Area of Smallest Section**
0	
1	
2	
3	
4	
5	
6	
7	
8	



10. Using your graphing utility, determine the mathematical model that represents this data:

    y = _______________.

    This is an example of exponential decay.

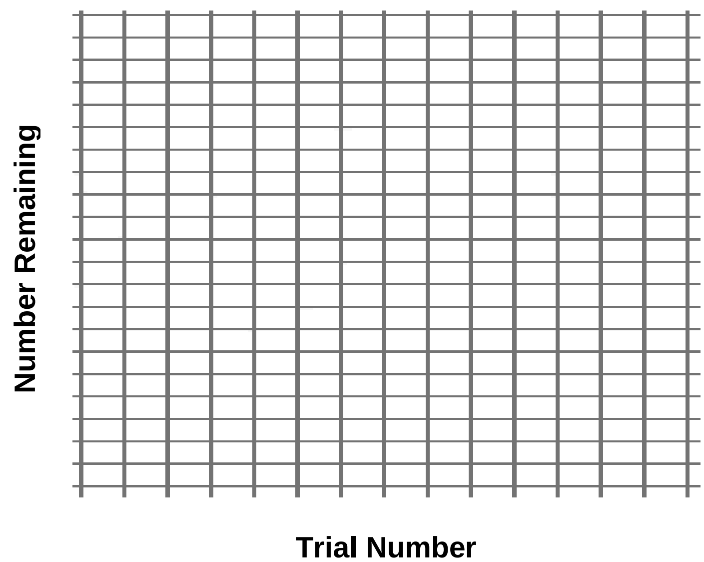
11. Explain what each part of the mathematical model means.

12. What would be the area of the smallest section of the piece of paper if you were able to fold it 10 times?

M&M Decay

1. Empty your bag of M&M candies onto the table and count the candies. Record the number on the table indicating it as the initial amount.
2. Place the M&M candies in a cup and mix them well. Pour them out on the desk, count the number that display an “m,” and place them back in the cup. The others may be eaten or removed. Record the number of M&M candies that show an “m” in your data table as the number of M&M candies remaining after trial 1.
3. Then repeat the procedure. Continue until the number of M&M candies remaining is less than five, but greater than zero.
4. Use a graphing utility to make a scatterplot of your data. Copy your scatterplot onto the grid below. Then use the graphing utility to find the curve of best fit and graph the equation. Sketch in the curve and write your equation.

Number of M&Ms	
Trial Number	**Number of M&Ms Remaining**
0 (initial amount)	
1	
2	
3	
4	
5	
6	
7	
8	



Equation:_________________

5. In your model, ***y = a(b)*** ^{x}, what value do you have for ***a***? To what does ***a*** seem to relate when you consider your data? When x = 0, what is your function value? Compare this to the values in your data table.

6. What is the value for ***b*** in your exponential model? Explain the significance of this value and how it relates to your data.

7. If you started with 40 M&M candies, how many trials do you think it would take before the number of M&M candies was between five and zero? What equation would model this new, initial value?