Triangular Numbers

Strand: Algebra and Functions

Topic: Determining the curve of best fit for quadratic functions and models

Primary SOL: AFDA.3 The student collect and analyze data, determine the equation of the curve of best fit in order to make predictions, and solve practical problems using models of linear, quadratic, and exponential functions.

Related SOL: AFDA.2, AFDA.4

Materials

- Triangular Numbers activity sheet (attached)
- Graph paper and/or graphing utility
- Manipulatives to build triangular numbers (e.g., counters, marker chips, candies)

Vocabulary

equation, function, linear function, natural numbers, quadratic function, term, triangular number

Student/Teacher Actions: What should students be doing? What should teachers be doing?

Time: 90 minutes

1. Distribute copies of the Triangular Numbers activity sheet.
2. Introduce the problem by defining triangular numbers. Students will complete the first page of the activity sheet. Students can work individually or in small groups.
3. For page 2, facilitate a brainstorming session to gather ideas for attacking the problem. What information is needed in each cell of the table?
4. As the students work in pairs, you might ask the following question to extend their thinking: *Can you see any patterns that might help?*
5. Once students complete page 2, have them share their work with a partner. Each student shares their thinking with their partner.
6. The teacher will introduce the exercises on pages 3-4 by asking, *"What is the relationship between triangular numbers and a quadratic function?"* (The sum of natural numbers is quadratic.) Note: Graphing utilities such as those found at desmos.com (free) may be helpful to students.

Assessment

- **Questions**
  - What is a triangular number?
  - How do you find triangular numbers?
  - What strategies might help you to find the answer?
- **Journal/writing prompts**

- Research Carl Gauss' work with triangular numbers. What did he discover about the patterns triangular numbers create that helped him calculate the solution quickly?
- What is the importance of triangular numbers?

- **Other Assessments**
  - Handshake problem – There a several different versions to this problem available from online resources. The main problem to solve is how many handshakes are completed with a given population. So, if there are 30 students in your classroom, what is the total number of handshakes if each person greeted each other by shaking hands? This problem offers students the opportunity to problem solve using a variety of methods.
  - Create Pascal's Triangle.

Extensions and Connections

- Create Pascal's Triangle and locate triangular numbers.
- Finding triangular numbers on a multiplication table.

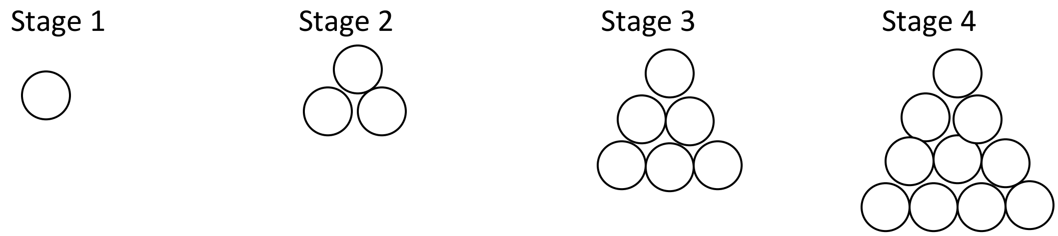
Strategies for Differentiation

- For enrichment, students may use three of the stage (iteration) values and the general form of the quadratic equation to determine the equation of the curve of best fit using systems of equations.
- Create triangular numbers using small dots then connect the outer dots to create triangles.
- Create triangular numbers using any circular manipulative in an equilateral formation. Then transform the triangles to right triangles. This formation will allow students to see the height of the triangle is ***n*** and the length of the triangle is ***n + 1***.
- Use vocabulary cards for related vocabulary listed above.

Note: The following pages are intended for classroom use for students as a visual aid to learning.

Triangular Numbers

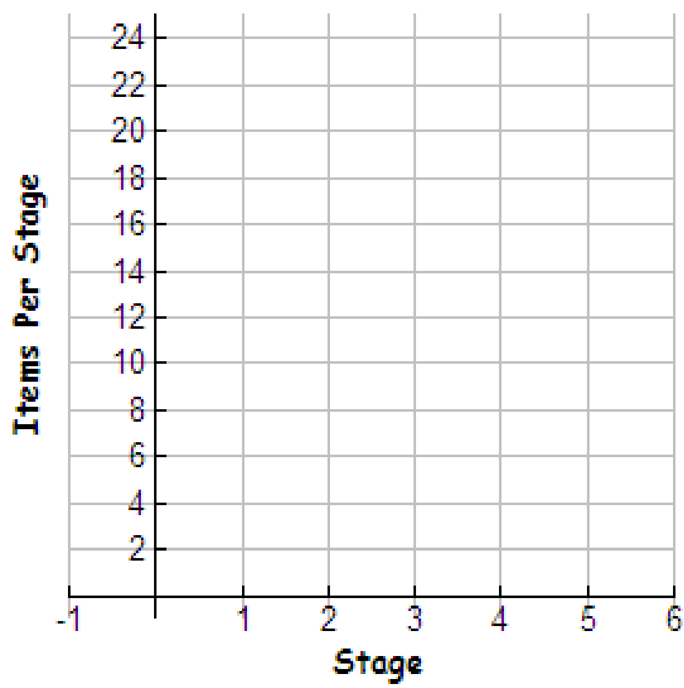
Given the following pattern:



Collecting the Data

1. Complete the table below and plot the points on the coordinate plane.

STAGE	ITEMS PER STAGE
1	
2	
3	
4	
n	



Writing about the Pattern

2. Use Stage 4 above to assist in filling in the following chart by brainstorming methods to determine the number of items at that stage. Each method must be based on that specific stage. Use the illustration to help you in your efforts.

Written explanation	Symbols	Simplify expression

STAGE	ITEMS PER STAGE
1	
2	
3	
4	
n	

3. Determine the difference in the values in the "Items Per Stage." Are value differences the same?

4. If not, continue finding the difference of the last results until the values are constant. How many times did you have to find the difference in values before those values were constant?

Graphing the Data

5. Use a sheet of graph paper or a graphing utility to graph the data (Stage, Items Per Stage).

Using the Data to Predict

6. How many items will there be at the following stages?

Stage	**Items Per Stage**
5	
10	
15	
55	
n	

The general form of the equation that represents ***triangular numbers*** is $\boldsymbol{y} = \boldsymbol{a}\,x^2 + \boldsymbol{b}\,x + \boldsymbol{c}$. The x-value stands for the stage and the y-value stands for the number of items in that stage.

7. Using the stages and information gathered in the table, substitute a stage for all of the x-values and its corresponding y-values (the number of items).

Stage 2 ____________________

Stage 3 ____________________

Stage 4 ____________________

8. Solve the system of equations. Use the simplified equations for Stage 2 and Stage 3. Eliminate the variable ***c***. Solve another system of equations. Use the equations from Stage 3 and Stage 4 and eliminate the variable ***b***, solving for ***a***.

9. Now that you know ***a***, use one of the equations from Nos. 2 or 3 and substitute in ***a***, solving for ***b***.

10. Now you know the values for ***a*** and ***b***. Select one of the original simplified equations from Question 2 and solve for ***c***.

11. Substitute the values found by substituting into the systems into the equation

$\boldsymbol{y} = \boldsymbol{a}x^2 + \boldsymbol{b}x + \boldsymbol{c}$.   ***y* = ________________________**