Factors, Zeros, and Solutions

Strand: Functions

Topic: Exploring relationships among factors, zeros, and solutions

Primary SOL: AII.8 The student will investigate and describe the relationships among solutions of an equation, zeros of a function, *x*-intercepts of a graph, and factors of a polynomial expression.

Related SOL: AII.1d, AII.4b, AII.7b

Materials

- Zeros and Factors activity sheet (attached)
- Matching Cards activity sheet (attached)
- Sorting Challenge activity sheet (attached)
- Graphing utility
- Colored paper
- Graph paper

Vocabulary

factor, fundamental theorem of algebra, imaginary numbers, multiplicity, polynomial, rate of change, real numbers, root, slope, solution, x-intercept, y-intercept, zeros of a function

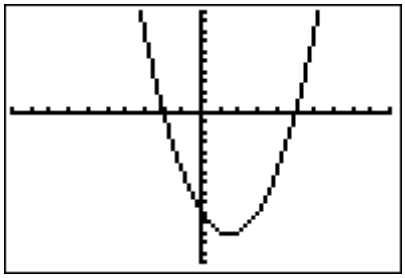
Student/Teacher Actions: What should students be doing? What should teachers be doing?

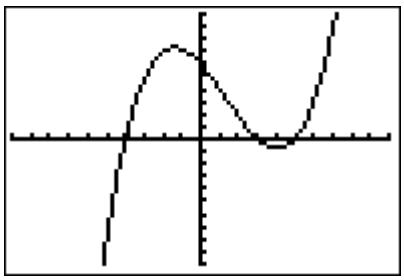
Time: 90 minutes

1. Have students solve the equation $3x + 5 = 11$. Then, have students set the left side of the equation equal to zero. Tell students to replace zero with y and consider the line $y = 3x - 6$. Instruct them to graph the line on graph paper. Review the terms *slope*, *x-intercept*, and *y-intercept*. Then, have them perform the same procedure (i.e., solve, set left side equal to zero, replace zero with y, and graph) for the equation $-2x + 6 = 4$. They should notice that in both cases, the x-intercept is the same as the solution to the equation.
2. Students explored factoring of zeros in Algebra I. Use the following practice problem to review: $y = x^2 + 4x + 3$. Facilitate a discussion with students using the following questions:
    - *"What is the first step in graphing the given line?"*
    - *"How do you find the factors of the line?"*
    - *"What role does zero play in graphing the line?"*
    - *"In how many 'places' did this line cross (intersect) the x-axis?"*
    - *"When the function crosses the x-axis, what is the value of y?"*
    - *"What is the significance of the x-intercepts of the graphs?"*
3. Review the term *zeros* (SOL AII.7d). Remind students that zeros of a function are the x-intercepts of the graph of the function. Be sure students are able to make the connection between the facts that a zero of a function is the value of x that makes $f(x) = 0$ and that a point on the graph where $f(x) = 0$ is an x-intercept.

4. Have students graph $y = (x - 3)(x + 2)$ using a graphing utility. Have them investigate the *x*-intercepts and discuss with partners what the *x*-intercepts can tell us about the equation $y = (x - 3)(x + 2)$. Next, have them replace *y* with zero and solve the equation.
5. Present the following equation for students to solve $2x^2 - 10 = 8x$, using an algebraic method of factoring, the quadratic formula, or completing the square. Monitor student work, then select three students (one using each of the three methods) to share their problem-solving approach with the class. Using think-write-pair-share, ask students to write and discuss the rationale for their selected method for finding the solution with a partner/neighbor.
6. After reviewing the three problem-solving approaches shared by students, provide additional modeling as needed for factoring, the quadratic formula, or completing the square.
7. Have students refer to the previous example and look at the equation written in factored form: $0 = 2(x + 1)(x - 5)$. Discuss the relationship between the factors of the equation and the zeros of the function using a graphing utility.
8. Review the fundamental theorem of algebra and its corollary, which states that a polynomial function of degree *n* will have exactly *n* zeros, including multiplicities. Emphasize that although a perfect-square trinomial function such as $y = x^2 + 8x + 16$ has only one zero, it is still a quadratic function due to multiplicities of zeros.
9. Distribute the Zeros and Factors activity sheet, and have students complete it.
10. Have students complete the Matching Game activity sheet. Facilitate the creation of a foldable graphic organizer to assist students with sorting the cards into three categories (graph, equation, and zeros). Distribute a piece of colored paper along with the matching cards (precut and randomized since cards are grouped in solutions). Guide students to take a colored sheet of paper first and turn the paper to the landscape position. Then, demonstrate how to fold the paper into three equal sections. Guide students to label the first column “Graph,” the second column “Equation,” and the third column “Zeros.”
11. Distribute the Sorting Challenge activity sheet, making sure the cards are precut and randomized before they are given to students. This activity challenges students to investigate and describe the relationships among solutions of an equation, zeros of a function, *x*-intercepts of a graph, and factors of a polynomial expression.

Assessment





- **Questions**
  - What are the zeros of the function graphed at right?
  - What are possible factors of the function graphed at right?
  - For the equation $y = (x-6)(x+2)$, what are the zeros of the function?
  - What are the solutions to the equation $x^2 - 4x - 12 = 0$?
- **Journal/writing prompts**
  - Explain why knowing the zeros of a function is not enough to define the function.
  - In your own words, state the fundamental theorem of algebra. How is this theorem important when graphing a function, using the zeros of the function?
- **Other Assessments**
  - Select students to work in pairs. Present the partners with a polynomial function. Each pair will find the zeros, x-intercepts, and factors of their function. Once the teacher provides feedback on their responses, each pair will set the function equal to zero and find the solution.
  - Create a set of polynomial functions on index cards. Give each student an index card and have him/her graph the function using a graphing utility. Students should be able to identify the zeros, x-intercepts, and factors of the given function.

Extensions and Connections

- Write in expanded form the equation of a function that has a leading coefficient of 3 and zeros that are −3 and 4.
- Write the equation of a cubic function that has only two distinct real zeros.
- After students do the matching activity, have them create their own similar matching activity cards. Assign them double zeros, pairs of distinct zeros, and sets of three zeros, and have them create a six-card matching set for each.

Strategies for Differentiation

- Create sets of matching cards for students to match functional “solutions to,” zeros, and factors.
- Have students use interactive whiteboards to match zeros, equations, and graphs.
- Use vocabulary cards for related vocabulary listed above.

Note: The following pages are intended for classroom use for students as a visual aid to learning.

Zeros and Factors

Match each given a polynomial function in factored form and state the zeros of the function.

_____	1.	$y = (x-2)(x+8)$	a. (4,0) and (−6,0)
_____	2.	$y = (x+4)(x+6)$	b. (2,0) and (−8,0)
_____	3.	$y = (x+4)(x-6)$	c. (−2,0) and (8,0)
_____	4.	$y = (x+2)(x+8)$	d. (−4,0) and (−6,0)
_____	5.	$y = (x+2)(x-8)$	e. (2,0) and (8,0)
_____	6.	$y = (x+4)(x+6)$	f. (4,0) and (6,0)
_____	7.	$y = (x-2)(x-8)$	g. (−2,0) and (−8,0)
_____	8.	$y = (x+6)(x-4)$	h. (6,0) and (−4,0)

Below are the zeros for a given function. Write the equation of a polynomial function in factored form for each set of zeros.

9. −3, 9

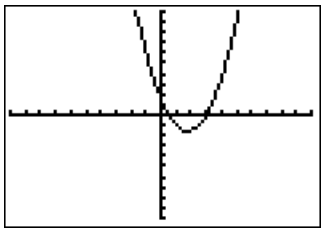
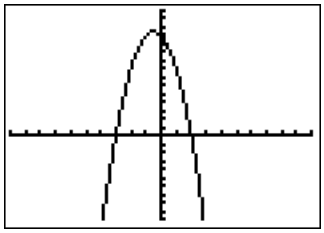
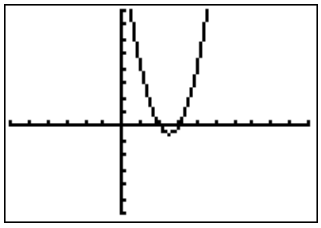
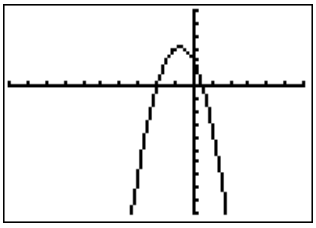
10. 7, 4

11. −1, 2, 4

12. −5, 0, 3

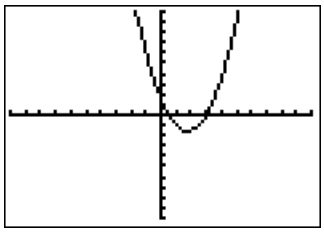
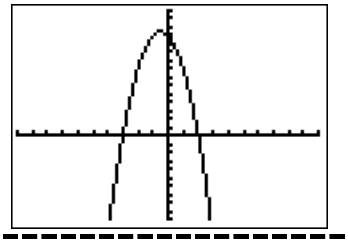
Matching Cards

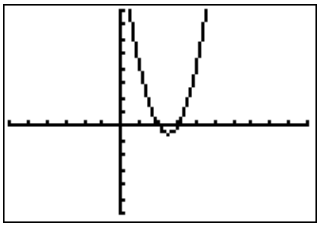
Copy cards on cardstock and cut out.

Factored form $y = (2x-1)(x-3)$	Solutions and zeros $\left\{\frac{1}{2}, 3\right\}$
Graph 	Factor Form (x+2)(x-3)
Solutions and zeros {-3,2}	Graph 
Factored form $y = 2(x-2)(x-3)$	Solutions and zeros {2,3}
Graph 	Graph 
Factored Form $y = (2x-1)(x+2)$	Solutions and zeros $\left\{-2, \frac{1}{2}\right\}$

Sorting Challenge

Copy cards on cardstock and cut out.

Equation $2x^2 = 7x - 3$	Solutions and zeros $\left\{\frac{1}{2}, 3\right\}$
Function $y = 2x^2 - 7x + 3$	*x*-intercepts $\left(\frac{1}{2}, 0\right)$ and (3,0)
Factored form $y = (2x - 1)(x - 3)$	Graph 
Equation $x^2 + x - 6 = 0$	Solutions and zeros {-3,2}
Function $y = -2x^2 - 2x + 12$	*x*-intercepts (-3,0) and (2,0)
Factored form $y = -2(x + 3)(x - 2)$	Graph 

Equation $X^2 = 5x - 6$	Solutions and zeros {2,3}
Function $y = 2x^2 - 10x + 12$	*x*-intercepts (2,0) and (3,0)
Factored form $y = 2(x-2)(x-3)$	Graph 
Equation $x^2 = 1 - \frac{3}{2}x$	Solutions and zeros $\left\{-2, \frac{1}{2}\right\}$
Function $y = -2x^2 - 3x + 2$	*x*-intercepts $\left(\frac{1}{2}, 0\right)$ and $(-2,0)$
Factored form $y = (2x-1)(x+2)$	Graph