Complex Numbers

Strand: Expressions and Operations

Topic: Performing complex number arithmetic

Primary SOL: AII.2 The student will perform operations on complex numbers and express the results in simplest form using patterns of the powers of *i*.

Related SOL: AII.1; AII.3

Materials

- Vocabulary Study activity sheet (attached)
- Student Learning Log for Vocabulary Study activity sheet (attached)
- Powers of *i* Table activity sheet (attached)
- Complex Numbers activity sheet (attached)
- Connecting Complex Numbers to Science activity sheet (attached)
- Activity sheets 1-4 (attached)

Vocabulary

associative property, closed, *commutative property, complex numbers, distributive property, integers, irrational numbers, natural numbers (counting numbers), pure imaginary numbers, rational numbers, real numbers, whole numbers*

Student/Teacher Actions: What should students be doing? What should teachers be doing?
Time: 90 minutes

1. Distribute copies of the Vocabulary Study activity sheet to students. Review with students the vocabulary for this lesson using the VDOE Algebra II Vocabulary Cards. Then, facilitate a discussion using the term *real numbers*. Display the Vocabulary Study activity sheet for students on an interactive board, a document camera, or a chart re-created on a whiteboard or chalkboard. Share the definition of *real numbers* while completing the activity sheet. Then, ask students to provide ideas/information for examples, non-examples, and characteristics for the term *real numbers*.
2. Have students work in pairs to complete the Vocabulary Study sheet for one or more terms assigned. Remind them to research characteristics, non-examples, and examples for each assigned term as needed, using their textbooks and electronic devices (i.e., computers, laptops, tablets, or other smart device). Exemplars of the Vocabulary Study sheets can be posted in the room for students to review and study. As students research their assigned terms and complete their sheets, teachers can assess students' competency by completing the Student Learning Log for Vocabulary Study activity sheet. Alternatively, have student groups self-evaluate their understanding of the vocabulary from the Vocabulary Study sheet.
3. Distribute copies of the Powers of *i* Table activity sheet to students. Guide students in evaluating $i^0, i^1, \dots i^4$ by expansion. Then, use the think-pair-share strategy to have students discover the pattern for powers of *i*. Finally, have student pairs complete the

table. (Think-pair-share involves students thinking about the problem individually, then pairing with another student to discuss ideas and, finally, sharing conclusions with the class.)

4. Distribute copies of the Complex Numbers activity sheet to students. While pairs of students work together, walk around and check for understanding, asking leading questions as necessary. After they complete the handout, have each pair compare their answers with answers obtained by another pair group, discussing any discrepancies and arriving at a consensus for each answer. Finally, review the answers with the whole class.

Assessment

- **Questions**
  - How can you quickly evaluate i raised to any power? Share/explain how you arrived at your solution.
  - After performing operations with complex numbers, what determines the expression is in its simplest form?
  - What are imaginary numbers? What purpose do they serve?
  - What is the relationship between algebraic properties and simplifying complex numbers?
- **Journal/writing prompts**
  - Explain why the product of conjugates is always a real and rational number.
  - Construct a diagram to explain the relationship between real and complex numbers.
- **Other Assessments**
  - Gallery walk for diagrams
  - Partner interviews using the Powers of *i* Table

Extensions and Connections

- Activity Sheet 5: Connecting Complex Numbers to Science

  Teacher notes

  1. Before handing out the Connecting Complex Numbers to Science activity sheet, write the following three terms on the board – *amps*, *voltage*, and *ohms*. Ask: *"What is the connection between the three terms?" "In what class/course might these terms be used?"*
  2. Share with students that scientists and engineers use complex numbers when calculating the flow of electricity through circuits. Circuits are the building blocks of electronics, such as radios, cellphones, televisions, appliances, etc. Ask, *"How do electrical engineers determine that a circuit can handle the expected electrical load?"*
  3. Alternating current (AC) is used in the majority of electronics and complex machines, because it provides a more efficient power source. Therefore, electrical engineers and electricians need to be able to calculate voltage, current, and impedance. Ask: *"What does impedance mean?"* It is the amount of resistance to AC in a current or an electronic component.

4. Say: Let's explore this relationship by calculating total current and voltage. We will also investigate impedance, which is in total opposition to current flow in a circuit. While the science may appear intimidating, the mathematics in this activity is basic computation involving complex numbers.
5. Teacher will distribute the Connecting Complex Numbers to Science activity sheet. Students can work individually, in pairs/threes, or in small groups to complete the activity.
6. The teacher should circulate around the room to assist students as needed. Pose facilitating questions to working groups:
   - *"Which mathematical operation is used to calculate total?"*
   - *"What does each variable represent in the given formula?"*
   - *"What is the process for calculating impedance?"* Remember to total/sum the values of each type of circuit component – resistor, inductor, or capacitor.
   - *"When dividing imaginary numbers, what do you need to remember about the denominator?" "Does the word 'conjugate' come to mind?"*

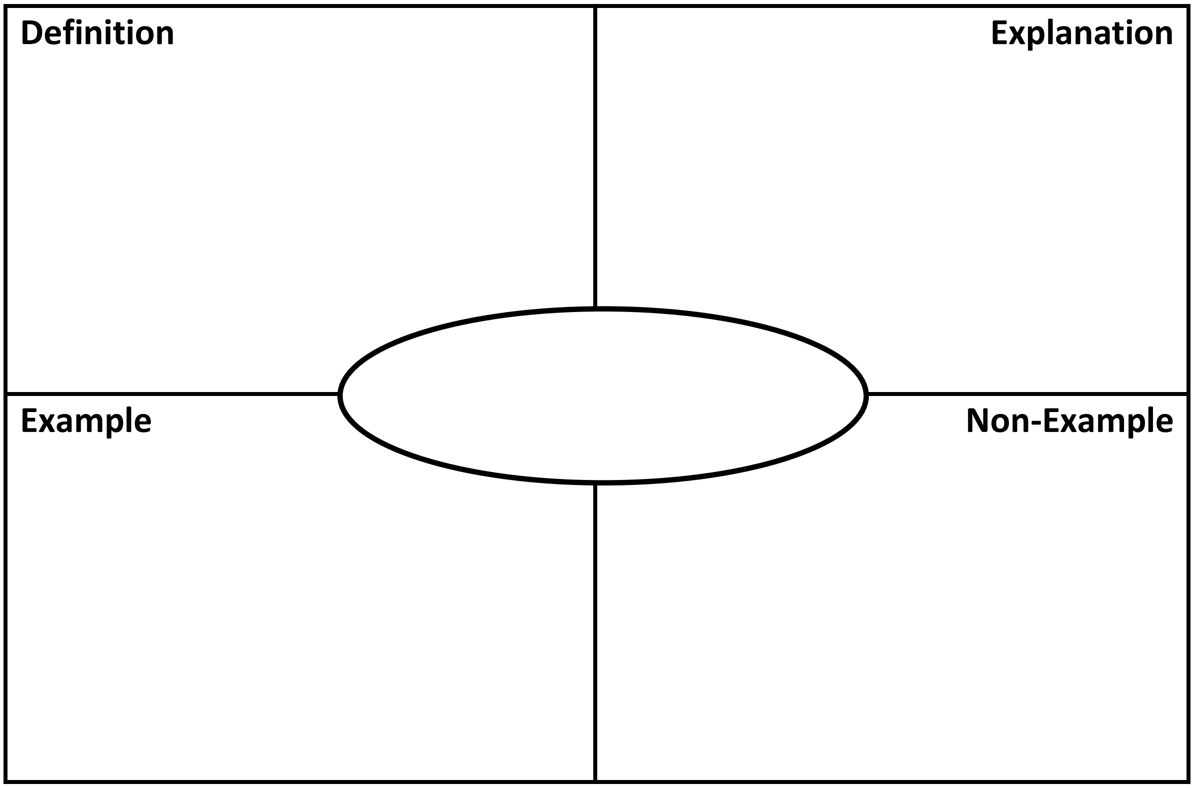
- Support students in sharing their solutions with the class or another group. Be sure that students understand the application of the concept to another content area (science).

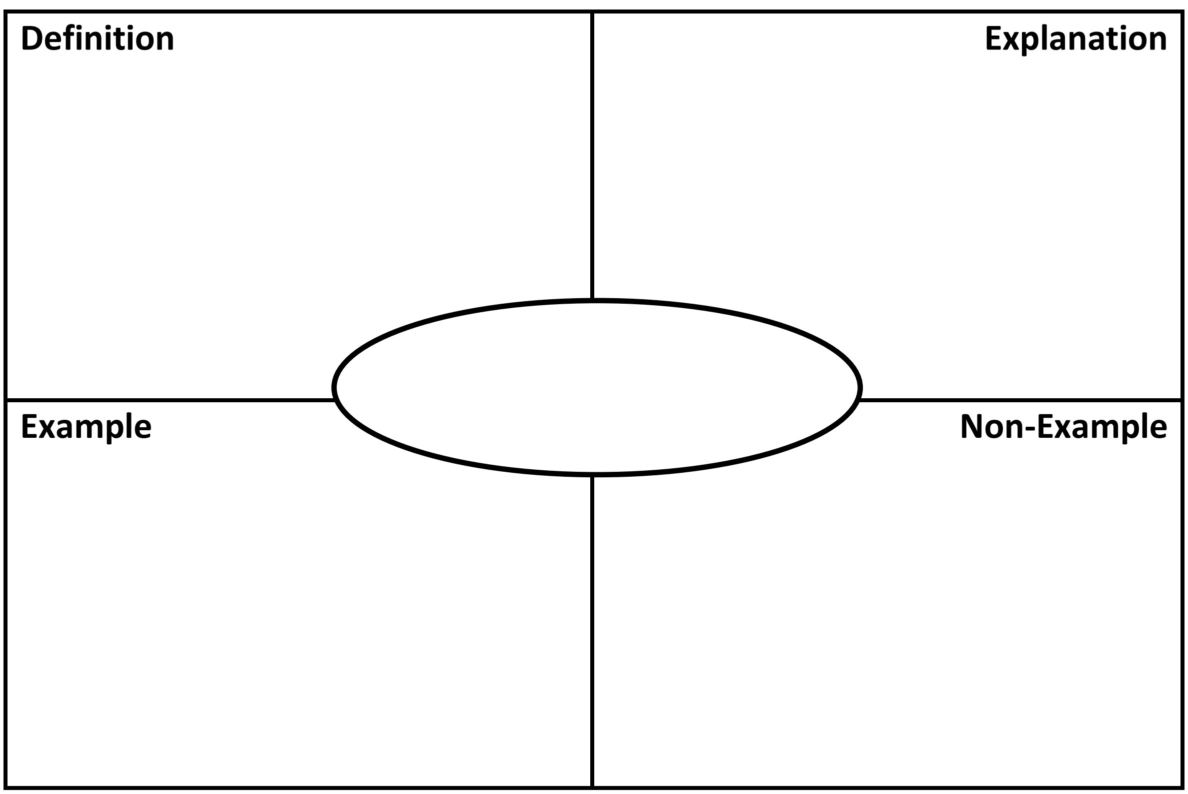
Strategies for Differentiation

- Give students the option of another graphic organizer with examples of each operation.
- Have students create and use flash cards or interactive notes of properties and sets of real numbers.
- Provide students with extra practice, including variables within the complex numbers.
- Show students how to do complex arithmetic on a graphing utility.
- Show similarities with complex arithmetic and polynomial arithmetic.
- Allow students to present proofs of complex operations orally.
- Allow students to use a talking graphing utility to do calculations with complex numbers.

Note: The following pages are intended for classroom use for students as a visual aid to learning.

Vocabulary Study





Student Learning Log for Vocabulary Study

Basic Understanding: I can relate the vocabulary to this mathematics lesson.

Proficient: I can connect the vocabulary to this lesson and to other mathematical concepts.

Mastered: I can apply the vocabulary to many mathematical contexts.

Student/Group Name			
Word 1			
	Basic Understanding	Proficient	Mastered
Definition			
Characteristic			
Example			
Non-Example			
Notes			
Word 2			
	Basic Understanding	Proficient	Mastered
Definition			
Characteristic			
Example			
Non-Example			
Notes			

Powers of *i* Table

i^0	1	i^{12}	
i^1	i	i^{15}	
i^2	−1	i^{21}	
i^3		i^{30}	
i^4		i^{35}	
i^5		i^{40}	
i^6		i^{102}	
i^7		i^{441}	
i^8		i^{1003}	

Complex Numbers

Simplify each of the expressions*:*

1. $\sqrt{-50}$

2. $-\sqrt{-18}$

3. i^{15}

5. i^{50}

7. $i^{13} + i^{23} + i^{40}$

8. $\left(i\sqrt{5}\right)^2$

9. $(3 + 2i) + (9 + 7i) + (-2 - i)$

10. $(5 - 3i) - (-1 + 4i)$

11. $-i(5 + 2i)$

12. $-1i(9 - 2i) - (5 + 7i)$

13. $4i(1 + i) + 3(6 - 2i)$

14. $(3 + i)(2 + i)$

15. $(3 + 5i)(8 - i)$

16. $i(4 - i)(3 + 2i)$

17. $(2 - 3i)^4$

Describe your problem-solving approach for each of the following exercises.

18. $4i(1 + i) + 3(6 - 2i)$

19. $(3 + i)(2 + i)$

20. $(2 - 3i)^4$

Connecting Complex Numbers to Science

Complex numbers are commonly used in electronics when representing voltage, current, and resistance in alternating current (AC).

- Voltage, V, (in volts) is the electrical potential between two points in an electrical circuit.
- Current, I, (in amps) is the rate of flow of electrical charge through a circuit.
- Resistance, R, (in ohms) is the opposition to the flow of current caused by resistors, coils, and capacitors.

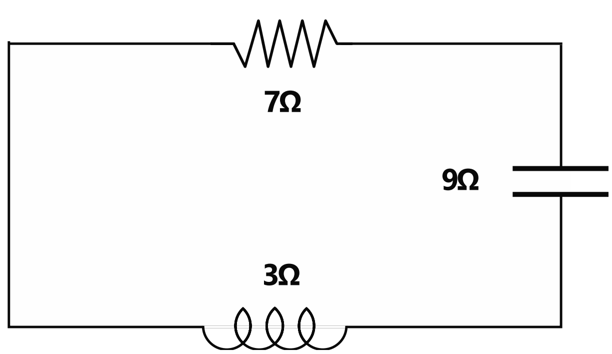
The total impedance in a circuit is represented by a complex number whose real part denotes the opposition to current flow due to resistors ( ) and whose imaginary part represents opposition due to coils ( ) and capacitors ( ). The impedance of a coil/inductor is a positive value. The impedance of a capacitor is a negative value. For example, resistors' impedance (8 ohms) plus coils' and capacitors' impedance (imaginary number) for a total impedance of 8 − 3i:

- 8 ohms: resistors' impedance
- 3 ohms coils' and capacitors' impedance
- Ohm's law: Volts = current • impedance, or *V = I • R*

The total voltage of two circuits in series is found by adding the voltages of the two individual circuits. The relationship between voltage, *V (volts)*; current, *I (amps)*; and resistance, *R (ohms)*; in an AC circuit is given by the formula *V = I • R*.

- Determine the voltage of a circuit where the current (amps) is -5 + 7*i* and resistance (ohms) is 8 – 13*i*.
- Determine the current of a circuit where the voltage is 4 – 6*i* and the resistance is –9 + 14*i*.

Look at the diagram below. Calculate impedance in each series circuit. Remember which components have positive and negative values. Also, inductors/coils and capacitors are imaginary numbers.



Show your work:

How did your knowledge of imaginary numbers support you in calculating voltage, current, and impedance?