Rational Expressions

Strand: Expressions and Operations

Topic: Performing operations with linear and quadratic rational expressions

Primary SOL: AII.1 The student will

a) add, subtract, multiply, divide, and simplify rational algebraic expressions.

Related SOL: AII.1c

Materials

- Simplifying Rational Expressions activity sheet (attached)
- Performing Operations on Rational Expressions activity sheet (attached)
- Simplifying Rational Expressions (Extension) (attached)
- Graphing utility

Vocabulary

binomial, common factors, factor, least common multiple (LCM), linear, monomial, polynomial, quadratic, quotient, rational expression, simplify, undefined

Student/Teacher Actions: What should students be doing? What should teachers be doing?
Activity 1; Time: 90 minutes

1. Write the following rational expressions across the top of the board: $x^2 - 4$, $x^2 + 4x + 4$, $x^2 - 4x + 4$, and $x^4 - 2x^2$. Have volunteers come up and factor the expressions. Next, ask six different volunteers to come up and create fractions from the factored expressions, e.g., $\frac{(x+2)(x-2)}{(x+2)(x+2)}$. Have pairs of students discuss what can be done to simplify the created fractions. Write the created fractions on the side of the board; they will be revisited.
2. Review simplifying fractions. Stress that a fraction can be simplified only if the numerator and denominator have common factors (*factors*: numbers and/or variables that are being multiplied). For example, $\frac{18}{24}$ can be simplified because $\frac{18}{24} = \frac{6 \times 3}{6 \times 4}$, which shows that there is a common factor of 6. To simplify $\frac{18}{24}$, we divide by $\frac{6}{6}$ (an expression equal to 1), which results in $\frac{3}{4}$. (Note: You might also show this by writing the prime factorization of each number and crossing out like factors.)
3. Refer back to the rational expressions written across the top of the board and the fractions that were created by the students from the factored expressions. Stress again that we can simplify only when the numerator and denominator have common factors. (Note: It is important that students understand that addends cannot be simplified; only

factors can be simplified.) Have a student read one of the fractions aloud (e.g., "*x* plus two, times *x* minus two, divided by *x* plus two, times *x* plus two.") Now, ask the student to repeat this, and ask the rest of the class to count how many times the student says the word *times*. Have another student read the numerator only, and ask the class how many times the word *times* was said. After they answer "once," ask how many factors are separated by multiplication. Explain that when simplifying $\frac{(x+2)(x-2)}{(x+2)(x+2)}$, neither the *x*'s nor the 2s can be simplified by themselves. Only the factors (numbers and/or variables that are being multiplied) can be simplified, and those factors must be identical. Thus, we cannot divide out the $x + 2$ with the $x - 2$.

4. Have students work in groups to simplify an expression similar to the one shown below. Let them write their solution on a whiteboard or poster paper for everyone to see. Debrief the activity in the light of the students' output with the following guide questions in mind.

$$\frac{4x^2(x-5)}{4x(x^2-5)}$$

   a. Why can 4 be divided out but not 5?
   b. Which of the following, x^2 or x, can be simplified? Why?
   c. How would you change $x^2 - 5$ so that you can divide out the factor $(x - 5)$?

5. Distribute copies of the Simplifying Rational Expressions activity sheet, and have students work in pairs to complete it. Instruct partners to alternate reading the factored expressions aloud. Stress that they should focus on how many expressions are being multiplied. These factors may be simplifiable.

Activity 2; Time: 90 minutes

1. Review with students the multiplication and division of fractions. Focus on the fact that we can simplify by dividing out common factors in a numerator and a denominator.
2. Provide several examples to show that the method of multiplying and dividing fractions is the same for multiplying and dividing rational expressions. Pose the following questions to check for students' conceptual understanding of the process.
   - *"Which of the following solutions is correct? What makes the other two solutions wrong?"*

     a) $\frac{x^2-x-6}{x^2+8x+6} \cdot \frac{x+2}{x-3} = \frac{(x+2)(x-3)}{(x+2)(x+6)} \cdot \frac{x+2}{x-3} = x+6$

     b) $\frac{x^2-x-6}{x^2+8x+6} \cdot \frac{x+2}{x-3} = \frac{(x+2)(x-3)}{(x+2)(x+6)} \cdot \frac{x+2}{x-3} = \frac{1}{x+6}$

     c) $\frac{x^2-x-6}{x^2+8x+6} \cdot \frac{x+2}{x-3} = \frac{(x+2)(x-3)}{(x+2)(x+6)} \cdot \frac{x+2}{x-3} = \frac{x+2}{x+6}$

   - *"When dividing rational expressions, why do you invert the second rational expressions (divisor) and multiply?"*
   - *"Why do the rules for simplifying fractions apply to rational expressions?"*

3. Review the addition and subtraction of fractions (rational numbers) with like and unlike denominators. Remind students that the reason the least common multiple (LCM) of 6 and 8 is 24 because 6 = 3 · 2 and 8 = 2 · 2 · 2. The least common multiple must include one of every distinct factor from each of 6 and 8; therefore, the LCM = 3 · 2 · 2 · 2, which is 24.
4. Have students find the LCM for several sets of numbers.
5. Using the same method of factoring and multiplying distinct factors to determine the LCM, transition into finding the LCM of variable expressions. Advise students to put parentheses around pairs of addends as a reminder that the entire expression is a factor.
6. Show examples of adding and subtracting rational expressions with
    - like monomial denominators; and
    - like polynomial denominators.
7. Then, show examples of addition and subtraction of rational expressions with
    - unlike monomial denominators; and
    - unlike polynomial denominators.
8. Remind students that if the value of the denominator changes by a factor, the value of the numerator must change by the same factor in order to preserve the equality of the expression.
9. Have students work in pairs to complete the Performing Operations on Rational Expressions activity sheet one problem at a time. Observe students' work, check for accuracy of procedure, and provide immediate feedback.

Activity 3; Time: 90 minutes

1. Distribute copies of the Rational Expressions (Extension) activity sheet. Have students work in small groups to discuss which operation(s) each expression contains and decide which steps must be taken to simplify each expression. Have groups share ideas with the entire class.
2. Demonstrate two methods for simplifying problems such as No. 2 on the handout.
    - Method 1: Multiply the numerator and denominator by the LCM of all fractions within the complex expression. Then, simplify the resulting rational expression.

$$\frac{x(2x+1)}{x(2x+1)} \cdot \frac{\frac{1}{x} - \frac{1}{2x+1}}{\frac{4x}{2x+1}} = \frac{(2x+1)-x}{4x^2} = \frac{x+1}{4x^2}$$

    - Method 2: Simplify the numerator of the complex fraction; simplify the denominator of the complex fraction; then, simplify the resulting rational expression.

$$\frac{\frac{1}{x} - \frac{1}{2x+1}}{\frac{4x}{2x+1}} = \frac{\frac{2x+1}{x(2x+1)} - \frac{x}{x(2x+1)}}{\frac{4x}{2x+1}} = \frac{\frac{x+1}{x(2x+1)}}{\frac{4x}{2x+1}} = \frac{x+1}{x(2x+1)} \div \frac{4x}{2x+1} = \frac{x+1}{x\cancel{(2x+1)}} \cdot \frac{\cancel{2x+1}}{4x} = \frac{x+1}{4x^2}$$

Assessment

- **Questions**
  - Why must all rational expressions be factored before they can be simplified?
  - How can you simplify $\frac{x-y}{y-x}$?
  - How can you determine the common denominator when adding rational expressions?
  - In a fraction, why must we change the value of the numerator when we change the value of the denominator?
  - Why are common denominators required for addition and subtraction but not for multiplication and division?
- **Journal/Writing Prompts**
  - Compare simplifying and performing operations on rational expressions to simplifying and performing operations on nonvariable fractions.
  - Explain why the expression $\frac{x^2+6}{x^2+3}$, which seemingly can be simplified, actually cannot be simplified.

Extensions and Connections

- Explain to students that there are often restricted values for the variables because the value of a denominator of a fraction cannot be zero. Revisit the handouts students have completed, and call on individuals to give the restricted value(s) for each expression verbally. Tell students to look at not only their answers, but also the original expressions.
- Give students the function $y=\frac{x+2}{x-1}$, and have them consider what the *x*- and *y*-intercepts will be. After they have discussed their thoughts, have them graph the function using a graphing utility.
- Give the function $f(x)=\frac{2x^2-3x-2}{x^2+x-6}$. Let students simplify $f(x)$ and call it $h(x)$. Using a graphing utility, have them graph $f(x)\ and\ h(x)$ simultaneously. Group students and let them answer the following questions. Debrief the activity.
  - Compare the two graphs in terms of their shape, intercepts, zeros, and function values.
  - Using the table of values, evaluate $f(-3)\ and\ h(-3)$. What does the function value signify? How do the graphs look like at $x=-3$?
  - Using the table of values, evaluate $f(2)\ and\ h(2)$. How do the graphs look like at x = 2?
  - Is $f(x)$ the same function as $h(x)$? Justify your answer algebraically, graphically, numerically, and verbally.

Strategies for Differentiation

- Provide a sentence frame for students to read this complex fraction aloud: $\frac{(x+2)(x-2)}{(x+2)(x+2)}$.
- Create a card activity in which students put in correct order cards containing the steps to simplify an expression. Use an interactive whiteboard for this activity.

- Use colored pencils and highlighters to write the same factors in identical colors.
- Use vocabulary cards for related vocabulary listed above.

Note: The following pages are intended for classroom use for students as a visual aid to learning.

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Simplifying Rational Expressions

Simplify the following rational expressions. Select one problem below and explain to your partner the strategy you used in simplifying the expression.

1. $\dfrac{3m}{6m}$

2. $\dfrac{12ab^2}{15a^2}$

3. $\dfrac{(c+2)(c-1)}{c+2}$

4. $\dfrac{r^2(r+3)}{r(r-3)}$

5. $\dfrac{10v+40}{8v+32}$

6. $\dfrac{x-1}{x^2-1}$

7. $\dfrac{d^2+6d+8}{d^2-d-20}$

8. $\dfrac{h^2-9}{h^2+6h+9}$

9. $\dfrac{f^2+2f-8}{2f^2-8}$

10. $\dfrac{m+2}{m^2+8}$

11. $\dfrac{j^2+7j+6}{4j^2+24j}$

12. $\dfrac{2y^2+9y-18}{4y^2-6y}$

Performing Operations on Rational Expressions

Multiply and divide each of the following rational expressions as indicated. Give the result in its simplest form.

1. $\dfrac{8x-32}{x^2(x+1)}\cdot\dfrac{x^2(x^2-x-2)}{4x^2-16x+16}$

2. $\dfrac{w^2+2w-3}{3w^2-w-2}\div\left(w+3\right)$

3. $\dfrac{40mn}{18r^2}\div\dfrac{25m^2n}{27r^2m}$

4. $\dfrac{k^2+5kn-6n^2}{2k^2}\cdot\dfrac{12k}{4k-4n}$

5. $\dfrac{x^2+8x+16}{c^2-6c+9}\div\dfrac{2x+8}{3c-9}$

6. $\dfrac{7x+7y}{y-x}\div\dfrac{21}{4x-4y}$

Add and subtract the following rational expressions and simplify the answers.

7. $\dfrac{5}{a}+\dfrac{3}{a-1}$

8. $\dfrac{3-4n}{n^2+3n-10}-\dfrac{2}{n+5}$

9. $\dfrac{x}{x^2-x-30}-\dfrac{1}{x+5}$

10. $\dfrac{x}{x^2-9}+\dfrac{3}{x\left(x-3\right)}$

11. $\dfrac{3}{p+1}+\dfrac{4}{2p-6}-\dfrac{p^2-5}{p^2-2p-3}$

Simplifying Rational Expressions (Extension)

Simplify the following rational expressions using a method of your choice. Explain the strategy you used in simplifying the expression.

1. $$\frac{\frac{x+3}{x^2-25}}{\frac{x^2-9}{x+5}}$$

2. $$\frac{\frac{1}{x}-\frac{1}{2x+1}}{\frac{4x}{2x+1}}$$

3. $$\frac{\frac{2}{4x+12}}{\frac{4}{2x+6}+\frac{1}{x+3}}$$

4. $$\frac{\frac{4}{x-3}+3}{\frac{4x-1}{x-3}+4}$$

5. $$\frac{\frac{4}{x^2-25}+\frac{2}{x+5}}{\frac{1}{x+5}+\frac{1}{x-5}}$$