Types of Variation

Strand: Statistics

Topic: Exploring variations

Primary SOL: AII.10 The student will represent and solve problems, including practical problems, involving inverse variation, joint variation, and a combination of direct and inverse variations.

Related SOL: AII.1a

Materials

- Proportional Relationships activity sheet (attached)
- Variation Examples activity sheet (attached)
- Direct, Inverse, and Joint Variations activity sheet (attached)
- Graphing utility

Vocabulary

combined variation, constant of proportionality, direct proportion, direct variation, inverse variation, joint variation, proportion, proportional, ratio, rate

Student/Teacher Actions: What should students be doing? What should teachers be doing?

Time: 90 minutes

1. Review direct variation and the generalized model used to describe a direct variation ($y = kx$). Distribute the Proportional Relationships activity sheet. Provide instruction related to inverse variation, joint variation, and a combination of direct and inverse variations. Include examples of finding the equation from a variation statement and of creating a variation statement from an equation. Show how to obtain the constant of variation, using given data. Some examples of data include the force of springs and the length they are stretched, the speed of an object in free fall and time, and the average speed of a trip and the time it takes to travel.
2. Distribute the Variation Examples activity sheet. Have students complete the activity. (*Note: This will help students transition to confidently translating the various variation types.*)
3. Distribute the Direct, Inverse, and Joint Variations activity sheet. Allow students to work collaboratively on Part 1, using prior knowledge of other disciplines to fill in the blanks. Have students work individually to complete the activity in whole or in part. (*Note: You may choose to use parts of this handout for assessment or for another activity.*)

Assessment

- **Questions**
    - Direct variation, inverse variation, and joint variation are names of specific types of mathematical models. What patterns in given data would lead you to choose one of these to model the data?
    - What are some formulas used in practical, everyday activities that are examples of direct, inverse, and joint variation?

- **Journal/writing prompts**
    - Explain the information you use to determine whether a variation exists.
    - In your own words, explain what is meant by a "constant of variation."
- **Other Assessments**
    - Use part or all of the Direct, Inverse, and Joint Variations activity sheet for assessment.
    - Have students find examples of variation that could be used in a career they are interested in pursuing.

Extensions and Connections

- Have students look at half-lives and decay to correspond with the Earth Science curriculum.
- Ask students to research average vehicle stopping distance and the effect of speed.

Strategies for Differentiation

- Have students create and use flash cards, each with a variation type listed on one side and several examples of that type on the other.
- Provide students with sentence frames of the exact language they should use as they describe the variation relationships listed in the AII.10 standard.
- Create an "I Have. Who Has?" game to provide students with additional practice verbalizing variation relationships.

Note: The following pages are intended for classroom use for students as a visual aid to learning.

Proportional Relationships

Identify the slope of following proportional relationships.

1.

x	y
0	0
6	15
10	25
12	30

2. The distance a car travels at 60 mph is proportional to the time driven.

3. The interest earned at a rate of 5 percent with a savings account is proportional to the principal amount in the account.

4.

KWH	Electric Bill ($)
200	24.00
500	60.00
750	90.00
1104	132.48

5.

Hours Worked	Pay ($)
2	15.00
5	37.50
10	75.00
40	300.00

Variation Examples

Type of Variation	Examples of Variation Statements	Corresponding Equations
Direct	1. G varies directly as *t* squared. 2. 3.	1. $G = kt^2$ 2. 3.
Inverse	1. 2. 3.	1. 2. 3.
Joint	1. 2. 3.	1. 2. 3.
Combined	1. 2. 3.	1. 2. 3.

Direct, Inverse, and Joint Variations

Part I

Identify each of the following statements as a direct, inverse, or joint variation by filling in the blank with the words *directly, inversely,* or *jointly.*

1. Volume of a gas, *V*, at constant temperature varies _____ with its pressure, *P*.
2. Intensity of sound varies _____ with distance away from the object creating the sound.
3. The weight of a body varies _____ with the square of the distance it is from the center of the earth.
4. The power of an electrical circuit varies _____ as the resistance and current.
5. The heat loss through a glass window of a house on a cold day varies _____ as the difference between the inside and outside temperatures and the area of the window. The heat loss varies _____ as the thickness of the window glass.
6. The amount of sales tax paid varies _____ as the total of the goods purchased.
7. The time to complete a job varies _____ as the number of workers working.
8. To balance a seesaw, the distance a person is from the pivot is _____ proportional to his/her weight.
9. The intensity of a light varies _____as the square of the distance from the light source.
10. The time it takes to complete a specific trip varies _____ as the speed of travel.
11. The cost of gas on a trip varies _____ with the length of the trip.
12. The length of a spring varies _____ with the force applied to it.
13. The number of congruent marbles that fits into a box is _____ proportional to the cube of the radius of each marble.
14. The number of people invited to dinner varies _____ as the amount of space each guest has at the table.
15. The number of people invited to dinner varies _____ as the number of pieces of silverware used.
16. The time it takes to harvest a crop varies _____ with the number of people assisting in the harvest.
17. The time it takes a runner to complete a lap on the track varies _____ as the speed of the runner.
18. The cost of a cake varies _____as the cake's thickness and the square of the radius.
19. The number of calories burned during exercise varies _____ with the time spent performing the exercise.
20. The power generated by a windmill is _____ proportional to the cube of the wind speed.

Part II

Write the equation being described by each of the following statements.

21. The volume, *v*, of a balloon is directly proportional to the cube of the balloon's radius, *r*.

22. The number, *n*, of grapefruit that can fit into a box is inversely proportional to the cube of the diameter, *d*, of each grapefruit.

23. The time, *t*, that a plane spends on the runway varies inversely as the take-off speed.

24. The weight, *w*, that a column of a bridge can support varies directly as the fourth power of its diameter, *d*, and inversely as the square of its length, *l*.

25. The radiation, *r*, from the decay of plutonium is directly proportional to the mass, *m*, of the sample tested and inversely proportional to the square of the distance, *d*, from the detector to the sample.

26. The lift of an airplane wing, *L*, is directly proportional to the square of the velocity, *v*.

27. The velocity, *v*, of a satellite in orbit around Earth is inversely proportional to the orbit radius, *r*.

28. The intensity of a sound, *I*, varies inversely with the square of the distance, *d*, from the source.

29. The speed of an object, *s*, varies directly with the square of the time, *t*, since it has been dropped.

Part III

Write an equation for and solve each of the following word problems.

30. The cost, *c*, in cents, of lighting a 100-watt bulb varies directly as the time, *t*, in hours, that the light is on. The cost of using the bulb for 1,000 hours is $0.15. Determine the cost of using the bulb for 2,400 hours.

31. The power, *P*, in watts of an electrical circuit varies jointly as the resistance, *R*, and the square of the current, *C*. For a 240-watt refrigerator that draws a current of 2 amperes, the resistance is 60 ohms. What is the resistance of a 600-watt microwave oven that draws a current of 5 amperes?

32. The force needed to keep a car from skidding on a curve varies directly as the weight of the car and the square of the speed and inversely as the radius of the curve. Suppose a 3,960-lb. force is required to keep a 2,200-lb. car traveling at 30 mph from skidding on a curve of radius 500 feet. How much force is required to keep a 3,000-lb. car traveling at 45 mph from skidding on a curve of radius 400 feet?

Part IV

Write a general equation for each of the following relationships, and sketch it:

33. *Y* varies jointly as *W* and *X.*

34. *Y* varies directly as the square of *X.*

35. *Y* varies inversely as *X.*

36. *Y* varies inversely as the square of *X.*

Part V

Write a variation statement for each of the following models in which *k* is the constant of variation.

37. $V = \frac{k}{t^2}$

38. $R = klwh$

39. $G = \frac{kp_1p_2}{d}$

40. $S = k(b_1 + b_2)h$