Transformation Investigation

Strand:	Equations and Inequalities
Topic:	Investing the components of the equation of a line
Primary SOL:	A.6 The student will
	c) graph linear equations in two variables.
Related SOL:	A.6a, A.7d

Materials

- Graphing utilities
- Graph paper
- Optional Transformation Investigation Student Activity Sheet

Vocabulary

transformation, translation, reflection, slope, slope-intercept form, y*-intercept* (earlier grades)
x-intercept(A.6)
parent function, function families (A.7)

Student/Teacher Actions: What should students be doing? What should teachers be doing?

Note: In this activity, students will graph linear equations of the form $y = mx + b$ and investigate transformations in the parent function $y = x$ as m and b change.

1. Distribute graph paper. On their graphing utilities, have students use an *x*-axis labeled from −10 to 10 and a *y*-axis labeled from −6 to 6. Ask students to sketch a graph of the parent function $y = x$. Inform students that the parent function will be used to make comparisons and generalizations throughout this investigation, so they may want to graph it on a separate sheet of graph paper and keep it to the side.

2. Have students sketch a graph for each of the following equations

    $$y_1 = x + 1 \qquad y_2 = x + 4 \qquad y_3 = x - 1 \qquad y_4 = x - 3$$

    Direct students to record data in a table, such as the one below, and answer the following questions:

    - What effect does changing b have on the parent function $y = x$?
    - What generalizations can you make about the transformation seen when you change the y-intercept of a function?

	y	y_1	y_2	y_3	y_4
y-intercept					
Slope					

3. Have students sketch a graph for each of the following equations:

    $$y_1 = 2x \qquad y_2 = \tfrac{1}{2}x \qquad y_3 = -5x \qquad y_4 = -\tfrac{2}{3}x$$

    Direct students to record data in a table and then answer the following questions:

- Compare the data for y_1, y_2, y_3, y_4 to the data for the parent function. What effect(s) does changing the slope have on the parent function?
- What generalizations can you make about the transformation seen in a graph when you change the slope of a function?

4. Students should become familiar with describing the transformations of linear functions. The following (adapted from the 2016 VDOE Algebra I Vocabulary Word Wall Cards) generalize these transformations:

Linear Function
(Transformational Graphing)
Translation
$$g(x) = x + b$$
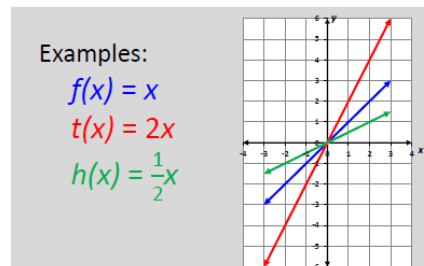
Examples:
$f(x) = x$
$t(x) = x + 4$
$h(x) = x - 2$
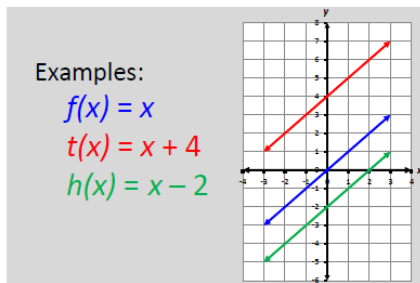
Vertical translation of the parent function,
$f(x) = x$

Linear Function
(Transformational Graphing)
Vertical Dilation ($m > 0$)
$$g(x) = mx$$

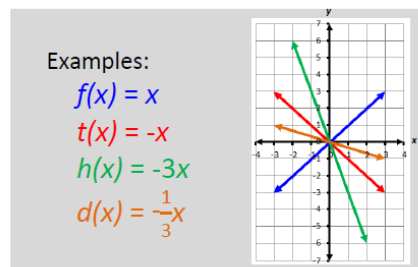Examples:
$f(x) = x$
$t(x) = 2x$
$h(x) = \frac{1}{2}x$

Vertical dilation (stretch or compression) of the parent function, $f(x) = x$

Linear Function
(Transformational Graphing)
Vertical Dilation/Reflection ($m < 0$)
$$g(x) = mx$$

Examples:
$f(x) = x$
$t(x) = -x$
$h(x) = -3x$
$d(x) = -\frac{1}{3}x$

Vertical dilation (stretch or compression) with a reflection of $f(x) = x$

5. Have students sketch a graph for each of the following equations. You can use a graphing utility such as https://www.desmos.com/calculator to graph linear equations. Students and teachers can find out more about graphing using the Desmos graphing calculator at http://learn.desmos.com/graphing .

$$y_1 = 2x \qquad y_3 = -2x \qquad y_2 = \frac{2}{5}x \qquad y_4 = -\frac{2}{5}x$$

Direct students to record data in a table and then answer the following questions:

- What generalizations can you make about the transformation seen when you graph functions with opposite slopes?

Assessment

- **Questions**
  - When the slope of a line is +1, what is the result of changing the y-intercept?
  - When the slope (m) of a line is greater than 1, what is the effect on the parent function $y = x$?
  - When the slope of a line is less than 1 but greater than zero, what is the effect on the parent function $y = x$?
  - When the slope of a line is −1, what transformation is seen in relation to the parent function $y = x$?

- **Journal/Writing Prompts**
  - Compare and contrast the behaviors of the functions $y = x - 2$ and $y = -2x$ in relation to $y = x$.

- **Extensions and Connections (for all students)**
  - Ask students how the graph of the parent function, $y = x$, would be transformed when graphing the function $y = -x + 2$.

Strategies for Differentiation

- Review vocabulary taught at earlier grades, if needed.
- Encourage the use of graphing calculators, graph paper, or dry-erase boards with a grid for students to see the transformations.
- Use a demonstration tool (e.g., document camera or digital display) to illustrate procedures in the graphing utility.
- Use different colors for the parent functions and comparison functions.
- Provide steps to follow if students are using a graphing utility.
- Provide copies of the table for students to use for recording information from each set of functions.
- Have students answer all generalization questions individually, in small groups, or in a large group, depending on the needs of the students.
- Have students work in groups of four, with each student graphing a separate function. Then, students can come together as a group to make comparisons between their graphs and the graph of the parent function.

Note: The following pages are intended for classroom use for students as a visual aid to learning.

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Name: _____ Date: _____

Transformation Investigation – Activity Sheet

1. Sketch a graph for *y = x.* (consider using a regular black lead pencil)
2. Sketch a graph for each of the following equations – use the graphs attached and tables with each graph. (consider using different colored pencils to create each graph)

$$y_1 = x + 1 \qquad y_2 = x + 4 \qquad y_3 = x - 1 \qquad y_4 = x - 3$$

3. Complete the table below with the *y*-intercept and slopes for each equation.

	y	y_1	y_2	y_3	y_4
y-intercept					
Slope					

- What effect does changing b have on the parent function $y = x$?

    _____

    _____

- What generalizations can you make about the transformation seen when you change the *y*-intercept of a function?

    _____

    _____

4. Sketch a graph for each of the following equations (consider using different colored pencils) – use the graphs and attached tables:

$$y_1 = 2x \qquad y_2 = \frac{1}{2}x \qquad y_3 = -5x \qquad y_4 = -\frac{2}{3}x$$

Record data in the table and then answer the following questions:

	y	y_1	y_2	y_3	y_4
y-intercept					
Slope					

- Compare the data for y_1, y_2, y_3, y_4 to the data for the parent function. What effect(s) does changing the slope have on the parent function?

    _____

    _____

- What generalizations can you make about the transformation seen in a graph when you change the slope of a function?

    _____

    _____

6. Sketch a graph for each of the following equations. Go to www.desmos.com/testing to graph each linear equation. We will do this together… First graph $y = x$, then:

$$y_1 = 2x \qquad y_3 = -2x \qquad y_2 = \frac{2}{5}x \qquad y_4 = -\frac{2}{5}x$$

Record data in a table and then answer the following questions:

	y	y_1	y_2	y_3	y_4
y-intercept					
Slope					

- What generalizations can you make about the transformation created when you graph two functions with opposite slopes?

_____

_____

Assessment

- **Questions**
  o When the slope of a line is +1, what is the result of changing the y-intercept?

  _____

  o When the slope (m) of a line is greater than 1, what is the effect on the parent function $y = x$?

  _____

  o When the slope of a line is less than 1 but greater than zero, what is the effect on the parent function $y = x$?

  _____

  _____

  o When the slope of a line is −1, what transformation is seen in relation to the parent function $y = x$?
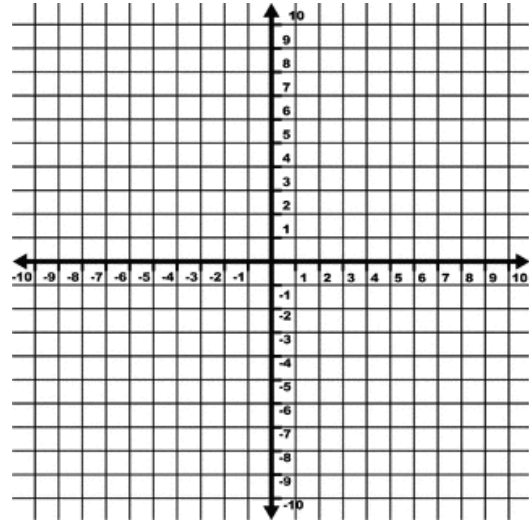
  _____

  _____

  o Compare and contrast the behaviors of the functions $y = x - 2$ and $y = -2x$ in relation to $y = x$.

  _____

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  o How would the graph of the parent function, $y = x$, be transformed when graphing the function $y = -x + 2$.
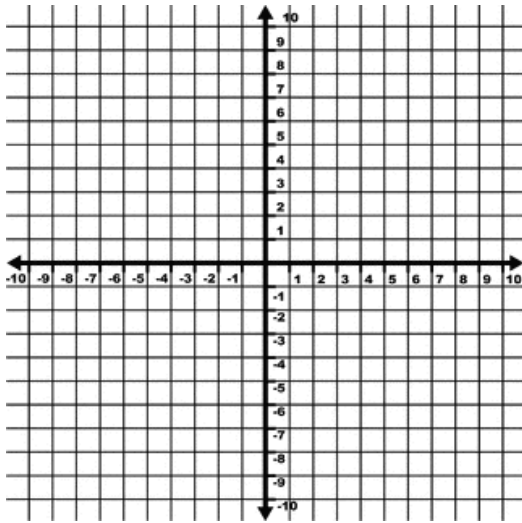
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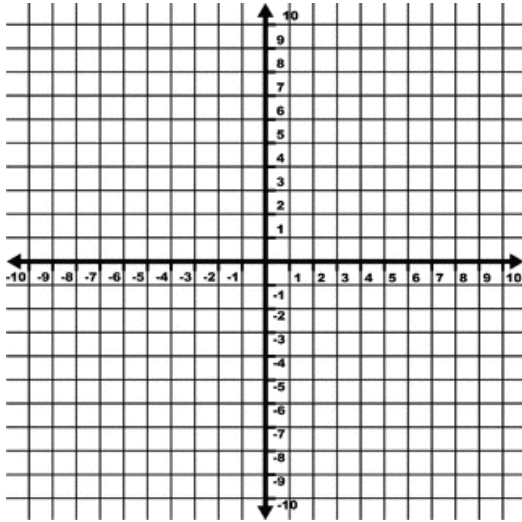
x	y

x	y

x	y
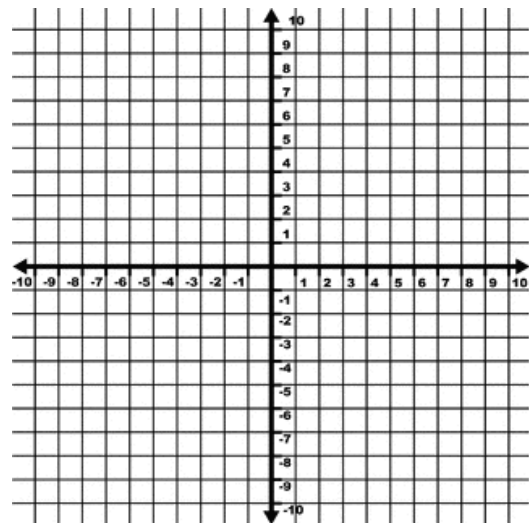
x	y
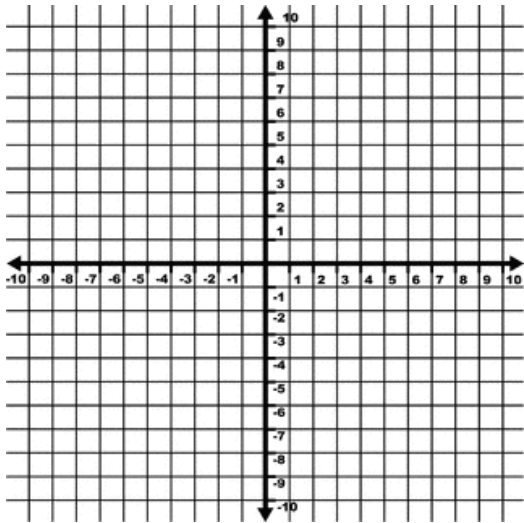
x	y
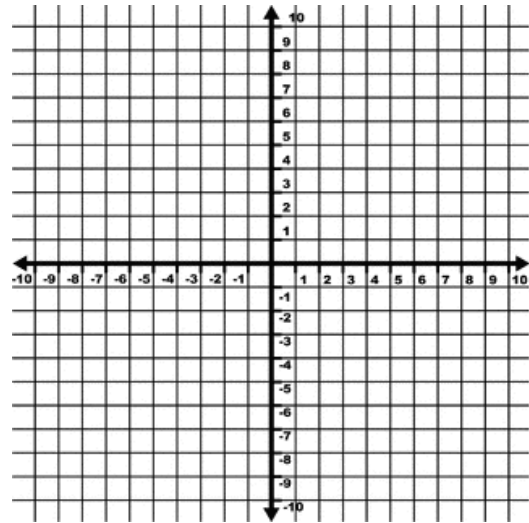
x	y
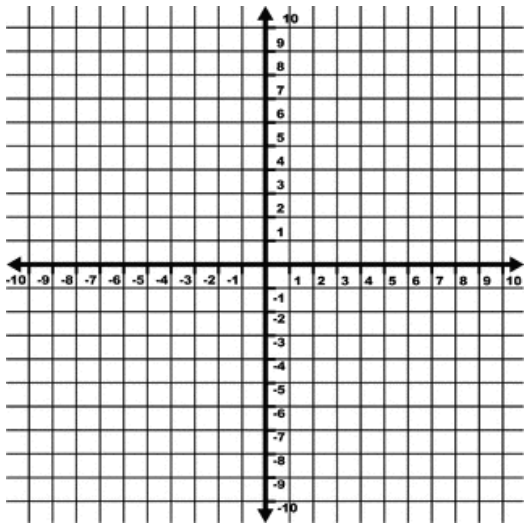
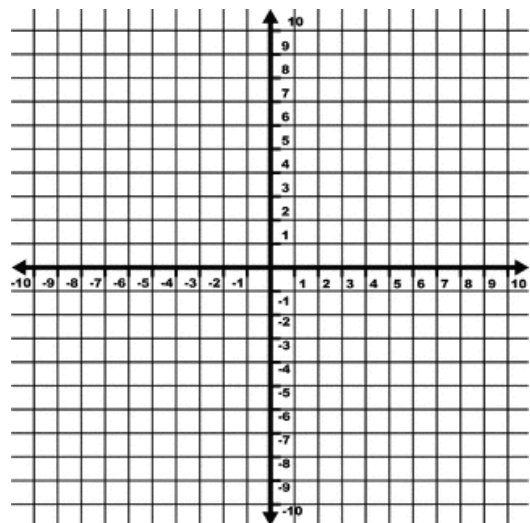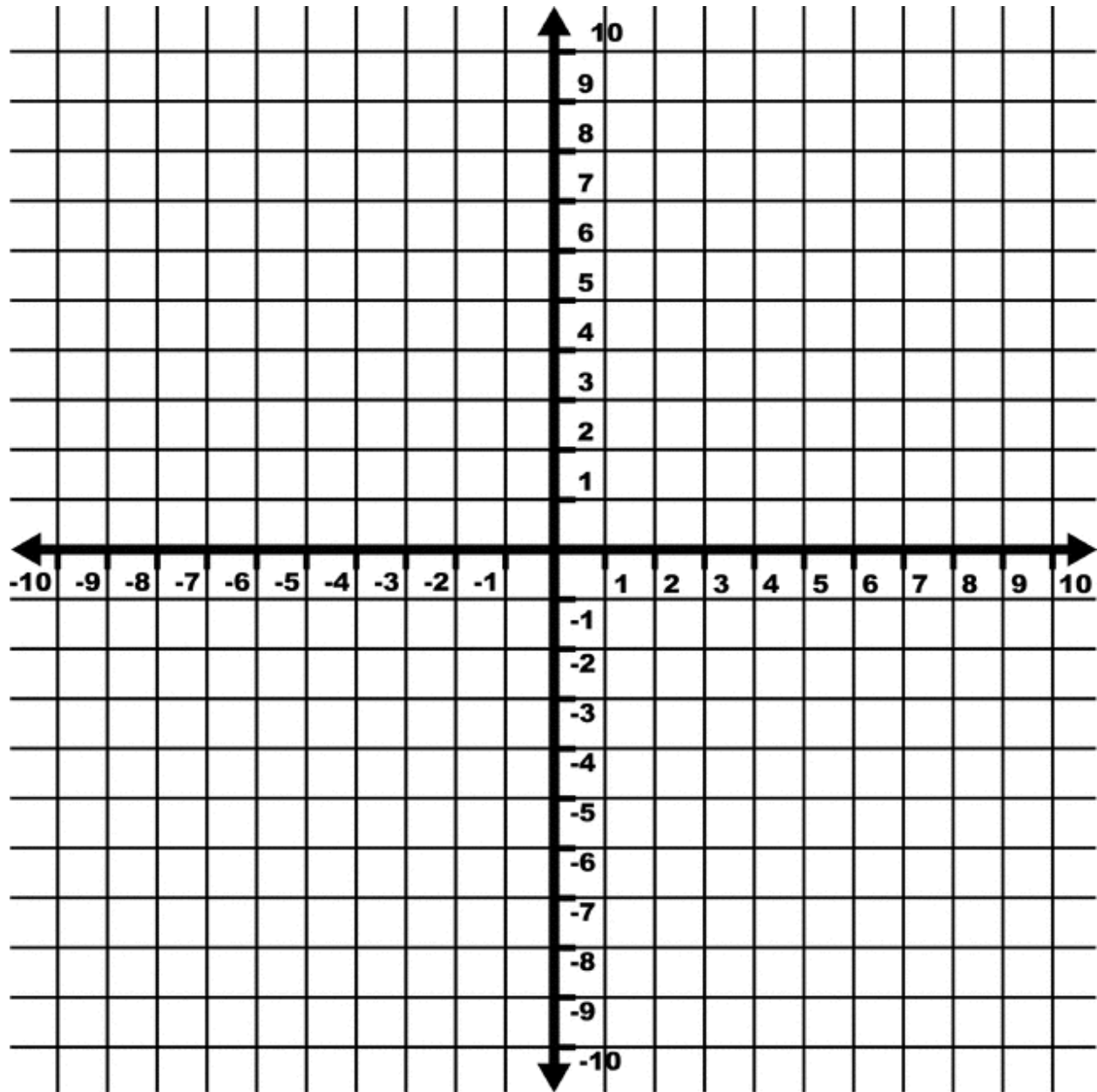| x	y

x	y

Use this graph for sketching the parent function, f(x) = x or y = x.

x	y