



Mathematics Standards of Learning

Curriculum Framework 2009

Trigonometry

Board of Education
Commonwealth of Virginia

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by the

Virginia Department of Education

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The 2009 *Mathematics Curriculum Framework* can be found in PDF and Microsoft Word file formats on the Virginia Department of Education's Web site at <http://www.doe.virginia.gov>.

Virginia Mathematics Standards of Learning Curriculum Framework 2009

Introduction

The 2009 *Mathematics Standards of Learning Curriculum Framework* is a companion document to the 2009 *Mathematics Standards of Learning* and amplifies the *Mathematics Standards of Learning* by defining the content knowledge, skills, and understandings that are measured by the Standards of Learning assessments. The Curriculum Framework provides additional guidance to school divisions and their teachers as they develop an instructional program appropriate for their students. It assists teachers in their lesson planning by identifying essential understandings, defining essential content knowledge, and describing the intellectual skills students need to use. This supplemental framework delineates in greater specificity the content that all teachers should teach and all students should learn.

Each topic in the *Mathematics Standards of Learning Curriculum Framework* is developed around the Standards of Learning. The format of the Curriculum Framework facilitates teacher planning by identifying the key concepts, knowledge and skills that should be the focus of instruction for each standard. The Curriculum Framework is divided into two columns: Essential Understandings and Essential Knowledge and Skills. The purpose of each column is explained below.

Essential Understandings

This section delineates the key concepts, ideas and mathematical relationships that all students should grasp to demonstrate an understanding of the Standards of Learning.

Essential Knowledge and Skills

Each standard is expanded in the Essential Knowledge and Skills column. What each student should know and be able to do in each standard is outlined. This is not meant to be an exhaustive list nor a list that limits what is taught in the classroom. It is meant to be the key knowledge and skills that define the standard.

The Curriculum Framework serves as a guide for Standards of Learning assessment development. Assessment items may not and should not be a verbatim reflection of the information presented in the Curriculum Framework. Students are expected to continue to apply knowledge and skills from Standards of Learning presented in previous grades as they build mathematical expertise.

TOPIC: TRIANGULAR AND CIRCULAR TRIGONOMETRIC FUNCTIONS

TRIGONOMETRY STANDARD T.1

The student, given a point other than the origin on the terminal side of the angle, will use the definitions of the six trigonometric functions to find the sine, cosine, tangent, cotangent, secant, and cosecant of the angle in standard position. Trigonometric functions defined on the unit circle will be related to trigonometric functions defined in right triangles.

ESSENTIAL UNDERSTANDINGS

- Triangular trigonometric function definitions are related to circular trigonometric function definitions.
- Both degrees and radians are units for measuring angles.
- Drawing an angle in standard position will force the terminal side to lie in a specific quadrant.
- A point on the terminal side of an angle determines a reference triangle from which the values of the six trigonometric functions may be derived.

ESSENTIAL KNOWLEDGE AND SKILLS

The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to

- Define the six triangular trigonometric functions of an angle in a right triangle.
- Define the six circular trigonometric functions of an angle in standard position.
- Make the connection between the triangular and circular trigonometric functions.
- Recognize and draw an angle in standard position.
- Show how a point on the terminal side of an angle determines a reference triangle.

TOPIC: TRIANGULAR AND CIRCULAR TRIGONOMETRIC FUNCTIONS

**TRIGONOMETRY
STANDARD T.2**

The student, given the value of one trigonometric function, will find the values of the other trigonometric functions, using the definitions and properties of the trigonometric functions.

ESSENTIAL UNDERSTANDINGS

- If one trigonometric function value is known, then a triangle can be formed to use in finding the other five trigonometric function values.
- Knowledge of the unit circle is a useful tool for finding all six trigonometric values for special angles.

ESSENTIAL KNOWLEDGE AND SKILLS

The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to

- Given one trigonometric function value, find the other five trigonometric function values.
- Develop the unit circle, using both degrees and radians.
- Solve problems, using the circular function definitions and the properties of the unit circle.
- Recognize the connections between the coordinates of points on a unit circle and
 - coordinate geometry;
 - cosine and sine values; and
 - lengths of sides of special right triangles (30° - 60° - 90° and 45° - 45° - 90°).

TOPIC: TRIANGULAR AND CIRCULAR TRIGONOMETRIC FUNCTIONS

**TRIGONOMETRY
STANDARD T.3**

The student will find, without the aid of a calculator, the values of the trigonometric functions of the special angles and their related angles as found in the unit circle. This will include converting angle measures from radians to degrees and vice versa.

ESSENTIAL UNDERSTANDINGS

- Special angles are widely used in mathematics.
- Unit circle properties will allow special angle and related angle trigonometric values to be found without the aid of a calculator.
- Degrees and radians are units of angle measure.
- A radian is the measure of the central angle that is determined by an arc whose length is the same as the radius of the circle.

ESSENTIAL KNOWLEDGE AND SKILLS

The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to

- Find trigonometric function values of special angles and their related angles in both degrees and radians.
- Apply the properties of the unit circle without using a calculator.
- Use a conversion factor to convert from radians to degrees and vice versa without using a calculator.

TOPIC: INVERSE TRIGONOMETRIC FUNCTIONS

**TRIGONOMETRY
STANDARD T.4**

The student will find, with the aid of a calculator, the value of any trigonometric function and inverse trigonometric function.

ESSENTIAL UNDERSTANDINGS

ESSENTIAL KNOWLEDGE AND SKILLS

- The trigonometric function values of any angle can be found by using a calculator.
- The inverse trigonometric functions can be used to find angle measures whose trigonometric function values are known.
- Calculations of inverse trigonometric function values can be related to the triangular definitions of the trigonometric functions.

The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to

- Use a calculator to find the trigonometric function values of any angle in either degrees or radians.
- Define inverse trigonometric functions.
- Find angle measures by using the inverse trigonometric functions when the trigonometric function values are given.

TOPIC: TRIGONOMETRIC IDENTITIES**TRIGONOMETRY
STANDARD T.5**

The student will verify basic trigonometric identities and make substitutions, using the basic identities.

ESSENTIAL UNDERSTANDINGS

- Trigonometric identities can be used to simplify trigonometric expressions, equations, or identities.
- Trigonometric identity substitutions can help solve trigonometric equations, verify another identity, or simplify trigonometric expressions.

ESSENTIAL KNOWLEDGE AND SKILLS

The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to

- Use trigonometric identities to make algebraic substitutions to simplify and verify trigonometric identities. The basic trigonometric identities include
 - reciprocal identities;
 - Pythagorean identities;
 - sum and difference identities;
 - double-angle identities; and
 - half-angle identities.

TOPIC: TRIGONOMETRIC EQUATIONS, GRAPHS, AND PRACTICAL PROBLEMS

TRIGONOMETRY STANDARD T.6

The student, given one of the six trigonometric functions in standard form, will

- a) state the domain and the range of the function;
- b) determine the amplitude, period, phase shift, vertical shift, and asymptotes;
- c) sketch the graph of the function by using transformations for at least a two-period interval; and
- d) investigate the effect of changing the parameters in a trigonometric function on the graph of the function.

ESSENTIAL UNDERSTANDINGS

- The domain and range of a trigonometric function determine the scales of the axes for the graph of the trigonometric function.
- The amplitude, period, phase shift, and vertical shift are important characteristics of the graph of a trigonometric function, and each has a specific purpose in applications using trigonometric equations.
- The graph of a trigonometric function can be used to display information about the periodic behavior of a real-world situation, such as wave motion or the motion of a Ferris wheel.

ESSENTIAL KNOWLEDGE AND SKILLS

The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to

- Determine the amplitude, period, phase shift, and vertical shift of a trigonometric function from the equation of the function and from the graph of the function.
- Describe the effect of changing A , B , C , or D in the standard form of a trigonometric equation {e.g., $y = A \sin (Bx + C) + D$ or $y = A \cos [B(x + C)] + D$ }.
- State the domain and the range of a function written in standard form {e.g., $y = A \sin (Bx + C) + D$ or $y = A \cos [B(x + C)] + D$ }.
- Sketch the graph of a function written in standard form {e.g., $y = A \sin (Bx + C) + D$ or $y = A \cos [B(x + C)] + D$ } by using transformations for at least one period or one cycle.

TOPIC: INVERSE TRIGONOMETRIC FUNCTIONS

**TRIGONOMETRY
STANDARD T.7**

The student will identify the domain and range of the inverse trigonometric functions and recognize the graphs of these functions. Restrictions on the domains of the inverse trigonometric functions will be included.

ESSENTIAL UNDERSTANDINGS

- Restrictions on the domains of some inverse trigonometric functions exist.

ESSENTIAL KNOWLEDGE AND SKILLS

The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to

- Find the domain and range of the inverse trigonometric functions.
- Use the restrictions on the domains of the inverse trigonometric functions in finding the values of the inverse trigonometric functions.
- Identify the graphs of the inverse trigonometric functions.

TOPIC: TRIGONOMETRIC EQUATIONS, GRAPHS, AND PRACTICAL PROBLEMS

**TRIGONOMETRY
STANDARD T.8**

The student will solve trigonometric equations that include both infinite solutions and restricted domain solutions and solve basic trigonometric inequalities.

ESSENTIAL UNDERSTANDINGS

- Solutions for trigonometric equations will depend on the domains.
- A calculator can be used to find the solution of a trigonometric equation as the points of intersection of the graphs when one side of the equation is entered in the calculator as Y_1 and the other side is entered as Y_2 .

ESSENTIAL KNOWLEDGE AND SKILLS

The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to

- Solve trigonometric equations with restricted domains algebraically and by using a graphing utility.
- Solve trigonometric equations with infinite solutions algebraically and by using a graphing utility.
- Check for reasonableness of results, and verify algebraic solutions, using a graphing utility.

TOPIC: TRIGONOMETRIC EQUATIONS, GRAPHS, AND PRACTICAL PROBLEMS

**TRIGONOMETRY
STANDARD T.9**

The student will identify, create, and solve real-world problems involving triangles. Techniques will include using the trigonometric functions, the Pythagorean Theorem, the Law of Sines, and the Law of Cosines.

ESSENTIAL UNDERSTANDINGS

- A real-world problem may be solved by using one of a variety of techniques associated with triangles.

ESSENTIAL KNOWLEDGE AND SKILLS

The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to

- Write a real-world problem involving triangles.
- Solve real-world problems involving triangles.
- Use the trigonometric functions, Pythagorean Theorem, Law of Sines, and Law of Cosines to solve real-world problems.
- Use the trigonometric functions to model real-world situations.
- Identify a solution technique that could be used with a given problem.
- Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems.[†]

[†]Revised March 2011