



Mathematics Standards of Learning

Curriculum Framework 2009

Mathematical Analysis

Board of Education
Commonwealth of Virginia

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by the

Virginia Department of Education

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Virginia Mathematics Standards of Learning Curriculum Framework 2009

Introduction

The 2009 *Mathematics Standards of Learning* Curriculum Framework is a companion document to the 2009 *Mathematics Standards of Learning* and amplifies the *Mathematics Standards of Learning* by defining the content knowledge, skills, and understandings that are measured by the Standards of Learning assessments. The Curriculum Framework provides additional guidance to school divisions and their teachers as they develop an instructional program appropriate for their students. It assists teachers in their lesson planning by identifying essential understandings, defining essential content knowledge, and describing the intellectual skills students need to use. This supplemental framework delineates in greater specificity the content that all teachers should teach and all students should learn.

Each topic in the *Mathematics Standards of Learning* Curriculum Framework is developed around the Standards of Learning. The format of the Curriculum Framework facilitates teacher planning by identifying the key concepts, knowledge and skills that should be the focus of instruction for each standard. The Curriculum Framework is divided into two columns: Essential Understandings and Essential Knowledge and Skills. The purpose of each column is explained below.

Essential Understandings

This section delineates the key concepts, ideas and mathematical relationships that all students should grasp to demonstrate an understanding of the Standards of Learning.

Essential Knowledge and Skills

Each standard is expanded in the Essential Knowledge and Skills column. What each student should know and be able to do in each standard is outlined. This is not meant to be an exhaustive list nor a list that limits what is taught in the classroom. It is meant to be the key knowledge and skills that define the standard.

The Curriculum Framework serves as a guide for Standards of Learning assessment development. Assessment items may not and should not be a verbatim reflection of the information presented in the Curriculum Framework. Students are expected to continue to apply knowledge and skills from Standards of Learning presented in previous grades as they build mathematical expertise.

**MATHEMATICAL ANALYSIS
STANDARD MA.1**

The student will investigate and identify the characteristics of polynomial and rational functions and use these to sketch the graphs of the functions. This will include determining zeros, upper and lower bounds, y-intercepts, symmetry, asymptotes, intervals for which the function is increasing or decreasing, and maximum or minimum points. Graphing utilities will be used to investigate and verify these characteristics.

ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
<ul style="list-style-type: none"> • The graphs of polynomial and rational functions can be determined by exploring characteristics and components of the functions. 	<p>The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to</p> <ul style="list-style-type: none"> • Identify a polynomial function, given an equation or graph. • Identify rational functions, given an equation or graph. • Identify domain, range, zeros, upper and lower bounds, y-intercepts, symmetry, asymptotes, intervals for which the function is increasing or decreasing, points of discontinuity, end behavior, and maximum and minimum points, given a graph of a function. • Sketch the graph of a polynomial function. • Sketch the graph of a rational function. • Investigate and verify characteristics of a polynomial or rational function, using a graphing calculator.

**MATHEMATICAL ANALYSIS
STANDARD MA.2**

The student will apply compositions of functions and inverses of functions to real-world situations. Analytical methods and graphing utilities will be used to investigate and verify the domain and range of resulting functions.

ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
<ul style="list-style-type: none"> • In composition of functions, a function serves as input for another function. • A graph of a function and its inverse are symmetric about the line $y = x$. • $(f \circ f^{-1})(x) = (f^{-1} \circ f)(x) = x$ 	<p>The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to</p> <ul style="list-style-type: none"> • Find the composition of functions. • Find the inverse of a function algebraically and graphically. • Determine the domain and range of the composite functions. • Determine the domain and range of the inverse of a function. • Verify the accuracy of sketches of functions, using a graphing utility.

**MATHEMATICAL ANALYSIS
STANDARD MA.3**

The student will investigate and describe the continuity of functions, using graphs and algebraic methods.

ESSENTIAL UNDERSTANDINGS

ESSENTIAL KNOWLEDGE AND SKILLS

- Continuous and discontinuous functions can be identified by their equations or graphs.

The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to

- Describe continuity of a function.
- Investigate the continuity of absolute value, step, rational, and piece-wise-defined functions.
- Use transformations to sketch absolute value, step, and rational functions.
- Verify the accuracy of sketches of functions, using a graphing utility.

**MATHEMATICAL ANALYSIS
STANDARD MA.7**

The student will find the limit of an algebraic function, if it exists, as the variable approaches either a finite number or infinity. A graphing utility will be used to verify intuitive reasoning, algebraic methods, and numerical substitution.

ESSENTIAL UNDERSTANDINGS

- The limit of a function is the value approached by $f(x)$ as x approaches a given value or infinity.

ESSENTIAL KNOWLEDGE AND SKILLS

The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to

- Verify intuitive reasoning about the limit of a function, using a graphing utility.
- Find the limit of a function algebraically, and verify with a graphing utility.
- Find the limit of a function numerically, and verify with a graphing utility.
- Use limit notation when describing end behavior of a function.

**MATHEMATICAL ANALYSIS
STANDARD MA.9**

The student will investigate and identify the characteristics of exponential and logarithmic functions in order to graph these functions and solve equations and real-world problems. This will include the role of e, natural and common logarithms, laws of exponents and logarithms, and the solution of logarithmic and exponential equations.

ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
<ul style="list-style-type: none"> • Exponential and logarithmic functions are inverse functions. • Some examples of appropriate models or situations for exponential and logarithmic functions are: <ul style="list-style-type: none"> - Population growth; - Compound interest; - Depreciation/appreciation; - Richter scale; and - Radioactive decay. 	<p>The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to</p> <ul style="list-style-type: none"> • Identify exponential functions from an equation or a graph. • Identify logarithmic functions from an equation or a graph. • Define e, and know its approximate value. • Write logarithmic equations in exponential form and vice versa. • Identify common and natural logarithms. • Use laws of exponents and logarithms to solve equations and simplify expressions. • Model real-world problems, using exponential and logarithmic functions. • Graph exponential and logarithmic functions, using a graphing utility, and identify asymptotes, intercepts, domain, and range.

**MATHEMATICAL ANALYSIS
STANDARD MA.4**

The student will expand binomials having positive integral exponents through the use of the Binomial Theorem, the formula for combinations, and Pascal’s Triangle.

ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
<ul style="list-style-type: none"> • The Binomial Theorem provides a formula for calculating the product $(a + b)^n$ for any positive integer n. • Pascal’s Triangle is a triangular array of binomial coefficients. 	<p>The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to</p> <ul style="list-style-type: none"> • Expand binomials having positive integral exponents. • Use the Binomial Theorem, the formula for combinations, and Pascal’s Triangle to expand binomials.

**MATHEMATICAL ANALYSIS
STANDARD MA.5**

The student will find the sum (sigma notation included) of finite and infinite convergent series, which will lead to an intuitive approach to a limit.

ESSENTIAL UNDERSTANDINGS

- Examination of infinite sequences and series may lead to a limiting process.
- Arithmetic sequences have a common difference between any two consecutive terms.
- Geometric sequences have a common factor between any two consecutive terms.

ESSENTIAL KNOWLEDGE AND SKILLS

The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to

- Use and interpret the notation: Σ , n , n th, and a_n .
- Given the formula, find the n th term, a_n , for an arithmetic or geometric sequence.
- Given the formula, find the sum, S_n , if it exists, of an arithmetic or geometric series.
- Model and solve problems, using sequence and series information.
- Distinguish between a convergent and divergent series.
- Discuss convergent series in relation to the concept of a limit.

**MATHEMATICAL ANALYSIS
STANDARD MA.6**

The student will use mathematical induction to prove formulas and mathematical statements.

ESSENTIAL UNDERSTANDINGS

ESSENTIAL KNOWLEDGE AND SKILLS

- Mathematical induction is a method of proof that depends on a recursive process.
- Mathematical induction allows reasoning from specific true values of the variable to general values of the variable.

The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to

- Compare inductive and deductive reasoning.
- Prove formulas/statements, using mathematical induction.

**MATHEMATICAL ANALYSIS
STANDARD MA.10**

The student will investigate and identify the characteristics of the graphs of polar equations, using graphing utilities. This will include classification of polar equations, the effects of changes in the parameters in polar equations, conversion of complex numbers from rectangular form to polar form and vice versa, and the intersection of the graphs of polar equations.

ESSENTIAL UNDERSTANDINGS

- The real number system is represented geometrically on the number line, and the complex number system is represented geometrically on the plane where $a + bi$ corresponds to the point (a, b) in the plane.

ESSENTIAL KNOWLEDGE AND SKILLS

The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to

- Recognize polar equations (rose, cardioid, limaçon, lemniscate, spiral, and circle), given the graph or the equation.
- Determine the effects of changes in the parameters of polar equations on the graph, using a graphing utility.
- Convert complex numbers from rectangular form to polar form and vice versa.
- Find the intersection of the graphs of two polar equations, using a graphing utility.

**MATHEMATICAL ANALYSIS
STANDARD MA.12**

The student will use parametric equations to model and solve application problems.

ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
<ul style="list-style-type: none"> • Parametric equations are used to express two dependent variables, x and y, in terms of an independent variable (parameter), t. • Some curves cannot be represented as a function, $f(x)$. Parametric graphing enables the representation of these curves in terms of functions. 	<p>The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to</p> <ul style="list-style-type: none"> • Graph parametric equations, using a graphing utility. • Use parametric equations to model motion over time. • Determine solutions to parametric equations, using a graphing utility. • Compare and contrast traditional solution methods with parametric methods.

**MATHEMATICAL ANALYSIS
STANDARD MA.14**

The student will use matrices to organize data and will add and subtract matrices, multiply matrices, multiply matrices by a scalar, and use matrices to solve systems of equations.

ESSENTIAL UNDERSTANDINGS

- Matrices are a convenient shorthand for solving systems of equations.
- Matrices can model a variety of linear systems.
- Solutions of a linear system are values that satisfy every equation in the system.
- Matrices can be used to model and solve real-world problems.

ESSENTIAL KNOWLEDGE AND SKILLS

- The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to**
- Add, subtract, and multiply matrices and multiply matrices by a scalar.
 - Model problems with a system of no more than three linear equations.
 - Express a system of linear equations as a matrix equation.
 - Solve a matrix equation.
 - Find the inverse of a matrix.
 - Verify the commutative and associative properties for matrix addition and multiplication.

**MATHEMATICAL ANALYSIS
STANDARD MA.8**

The student will investigate and identify the characteristics of conic section equations in (h, k) and standard forms. Transformations in the coordinate plane will be used to graph conic sections.

ESSENTIAL UNDERSTANDINGS

- Matrices can be used to represent transformations of figures in the plane.

ESSENTIAL KNOWLEDGE AND SKILLS

The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to

- Given a translation or rotation matrix, find an equation for the transformed function or conic section.
- Investigate and verify graphs of transformed conic sections, using a graphing utility.

**MATHEMATICAL ANALYSIS
STANDARD MA.11**

The student will perform operations with vectors in the coordinate plane and solve real-world problems using vectors. This will include the following topics: operations of addition, subtraction, scalar multiplication, and inner (dot) product; norm of a vector; unit vector; graphing; properties; simple proofs; complex numbers (as vectors); and perpendicular components.

ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
<ul style="list-style-type: none"> • Every vector has an equal vector that has its initial point at the origin. • The magnitude and direction of a vector with the origin as the initial point are completely determined by the coordinates of its terminal point. 	<p>The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to</p> <ul style="list-style-type: none"> • Use vector notation. • Perform the operations of addition, subtraction, scalar multiplication, and inner (dot) product on vectors. • Graph vectors and resultant vectors. • Express complex numbers in vector notation. • Define <i>unit vector</i>, and find the unit vector in the same direction as a given vector. • Identify properties of vector addition, scalar multiplication, and dot product. • Find vector components. • Find the norm (magnitude) of a vector. • Use vectors in simple geometric proofs. • Solve real-world problems using vectors.

**MATHEMATICAL ANALYSIS
STANDARD MA.13**

The student will identify, create, and solve real-world problems involving triangles. Techniques will include using the trigonometric functions, the Pythagorean Theorem, the Law of Sines, and the Law of Cosines.

ESSENTIAL UNDERSTANDINGS

ESSENTIAL KNOWLEDGE AND SKILLS

- Real-world problems can be modeled using trigonometry and vectors.

The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to

- Solve and create problems, using trigonometric functions.
- Solve and create problems, using the Pythagorean Theorem.
- Solve and create problems, using the Law of Sines and the Law of Cosines.
- Solve real-world problems using vectors.