K.6 The student will model and solve single-step story and picture problems with sums to 10 and differences within 10, using concrete objects.

Understanding the Standard

- Students should experience a variety of problem types related to addition and subtraction.
- The problem types most appropriate for students at this level include:

KINDERGARTEN: COMMON ADDITION AND SUBTRACTION PROBLEM TYPES	
Join (Result Unknown)	Sue had 4 pennies. Josh gave her 2 more. How many pennies does Sue have altogether?
Separate (Result Unknown)	Sue had 8 pennies. She gave 5 pennies to Josh. How many pennies does Sue have now?
Part-Part-Whole (Whole Unknown)	Josh has 4 red balloons and 3 blue balloons. How many balloons does he have?
Part-Part-Whole (Both Parts Unknown)	Josh has 5 balloons. Some of them are red and some of them are blue. How many balloons can be blue and how many can be red?

- Join problems involve the process of combining or joining sets or quantities. Separate problems can be viewed as a taking away or separating process. Part-part-whole problems involve two quantities that are combined into one whole but no physical action is required. Comparison problems that ask *how many more* or *how many fewer* should be reserved for grades one and two.
- Operation symbols (+, -) are introduced in grade one.
- Single-step refers to the least number of steps necessary to solve a problem.
- Number relationships help students develop strategies for addition and subtraction. These strategies include:
  - Instant recognition of the amount in a set of objects (subitize) that are arranged in a familiar pattern such as the dots on number cubes; and
  - One more than, one less than, two more than, two less than.
- Counting on from the larger set to determine the sum of the combined sets is one strategy for determining a sum.

Essential Knowledge and Skills

The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to

- Model and solve various types of story and picture problems using 10 or fewer concrete objects. (Types of problems should include joining, separating, and part-part-whole scenarios.)

1.6 The student will create and solve single-step story and picture problems using addition and subtraction within 20.

Understanding the Standard	Essential Knowledge and Skills
• Addition and subtraction should be taught concurrently in order to develop understanding of the inverse relationship. • The problem-solving process is enhanced when students: – create their own story problems; – visualize the action in the story problem and draw a picture to show their thinking; and – model the problem using manipulatives, representations, or number sentences/equations. • The least number of steps necessary to solve a single-step problem is one. • In problem solving, emphasis should be placed on thinking and reasoning rather than on key words. Focusing on key words such as *in all, altogether, difference,* etc., encourages students to perform a particular operation rather than make sense of the context of the problem. A key-word focus prepares students to solve a limited set of problems and often leads to incorrect solutions as well as challenges in upcoming grades and courses. • Provide practice in the use and selection of strategies. Encourage students to develop efficient strategies. Examples of strategies for developing the basic addition and subtraction facts include: – counting on; – counting back; – "one more than," "two more than"; – "one less than," "two less than"; – "doubles" (e.g., 6 + 6 =__); – "near doubles" (e.g., 7 + 8 = (7 + 7) + 1 = or (8 + 8) – 1); – "make ten" (e.g., 7 + 4 can be thought of as 7 + 3 + 1 in order to make a 10); – "think addition for subtraction" (e.g., for 9 – 5 = __, think "5 and what number makes 9?"); – use of the commutative property (e.g., 14 +3 is the same as 3 + 14); – use of related facts (e.g., 14 + 3 = 17 , 3 + 14 = 17, 17 – 4 = 13, and 17 – 13 = 4); – use of the additive identity property (e.g., 14 + 0 = 14); and – use patterns to make sums (e.g., 0 + 15 = 15, 1 + 14 = 15, 2 + 13 = 15, etc.). • Students at this level are not expected to use the parentheses or to name the properties. • Students should develop fluency with facts to 10 and then use strategies and known facts to 10 to determine facts to 20.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Create and solve single-step oral or written story and picture problems, using addition and subtraction within 20. • Identify a number sentence to solve an oral or written story and picture problem, selecting from among addition and/or subtraction equations (e.g., number sentences). • Combine parts contained in larger numbers up to 20 by using related combinations (e.g., 9 + 7 can be thought of as 9 broken up into 2 and 7; using doubles, 7 + 7 = 14; 14 + 2 = 16 or 7 broken up into 1 and 6; making a ten, 1 + 9 = 10; 10 + 6 = 16). • Explain strategies used to solve addition and subtraction problems within 20 using spoken words, objects, pictorial models, and number sentences.

1.6 The student will create and solve single-step story and picture problems using addition and subtraction within 20.

Understanding the Standard	Essential Knowledge and Skills
• Flexibility with facts to 10 should be applied to facts to 20 (e.g., when adding 4 + 7, it is appropriate to think of 4 as 3 + 1 in order to combine 3 and 7 to make a 10 whereas adding 4 + 8, it is appropriate to think of 4 as 2 + 2 in order to combine 8 and 2 to make a 10). • Extensive research has been undertaken over the last several decades regarding different problem types. Many of these studies have been published in professional mathematics education publications using different labels and terminology to describe the varied problem types. • Students should have exposure to a variety of problem types related to addition and subtraction. Examples are represented in the chart below. It is important to note that Join Problems (with start unknown), Separate Problems (with start unknown), Compare Problems (with larger unknown – using "fewer") and Compare problems (with smaller unknown – using "more") are the most difficult and should be mastered in grade two.	

1.6 The student will create and solve single-step story and picture problems using addition and subtraction within 20.

Understanding the Standard

GRADE 1: COMMON ADDITION AND SUBTRACTION PROBLEM TYPES

Join (Result Unknown)	Join (Change Unknown)	Join (Start Unknown)
Sue had 9 pencils. Alex gave her 5 more pencils. How many pencils does Sue have altogether?	Sue had 9 pencils. Alex gave her some more pencils. Now Sue has 14 pencils. How many did Alex give her?	Sue had some pencils. Alex gave her 5 more. Now Sue has 14 pencils. How many pencils did Sue have to start with?
Separate (Result Unknown)	**Separate (Change Unknown)**	**Separate (Start Unknown)**
Brooke had 10 cookies. She gave 6 cookies to Joe. How many cookies does Brooke have now?	Brooke had 10 cookies. She gave some to Joe. She has 4 cookies left. How many cookies did Brooke give to Joe?	Brooke had some cookies. She gave 6 to Joe. Now she has 4 cookies left. How many cookies did Brooke start with?
Part-Part-Whole (Whole Unknown)	**Part-Part-Whole (One Part Unknown)**	**Part-Part-Whole (Both Parts Unknown)**
Lisa has 4 red markers and 8 blue markers. How many markers does she have?	Lisa has 12 markers. Four of the markers are red, and the rest are blue. How many blue markers does Lisa have?	Lisa has a pack of red and blue markers. She has 12 markers in all. How many markers could be red? How many could be blue?
Compare (Difference Unknown)	**Compare (Bigger Unknown)**	**Compare (Smaller Unknown)**
Ryan has 7 books and Chris has 2 books. How many more books does Ryan have than Chris? Ryan has 7 books. Chris has 2 books. How many fewer books does Chris have than Ryan?	Chris has 2 books. Ryan has 5 more books than Chris. How many books does Ryan have? Chris has 5 fewer books than Ryan. Chris has 2 books. How many books does Ryan have?	Ryan has 2 more books than Chris. Ryan has 7 books. How many books does Chris have? Chris has 5 fewer books than Ryan. Ryan has 7 books. How many books does Chris have?

Essential Knowledge and Skills

2.6 The student will

a) estimate sums and differences;

b) determine sums and differences, using various methods; and

c) create and solve single-step and two-step practical problems involving addition and subtraction.

Understanding the Standard	Essential Knowledge and Skills
• Addition and subtraction should be taught concurrently in order to develop understanding of the inverse relationship. • Grade two students should begin to explore the properties of addition as strategies for solving addition and subtraction problems using a variety of representations, including manipulatives and diagrams. • The properties of the operations are "rules" about how numbers work and how they relate to one another. Students at this level do not need to use the formal terms for these properties but should utilize these properties to further develop flexibility and fluency in solving problems. The following properties are most appropriate for exploration at this level: – The commutative property of addition states that changing the order of the addends does not affect the sum (e.g., 4 + 3 = 3 + 4). – The identity property of addition states that if zero is added to a given number, the sum is the same as the given number. – The associative property of addition states that the sum stays the same when the grouping of addends is changed (e.g., 4 + (6 + 7) = (4 + 6) + 7). • An equation (number sentence) is a mathematical statement representing two expressions that are equivalent. It consists of two expressions, one on each side of an 'equal' symbol (e.g., 5 + 3 = 8, 8 = 5 + 3, and 4 + 3 = 9 - 2). An equation can be represented using a balance scale, with equal amounts on each side (e.g., 3 + 5 = 6 + 2). • Rounding is one strategy used to estimate. • Estimation skills are valuable, time-saving tools particularly in practical situations when exact answers are not required or needed. • Estimation can be used to check the reasonableness of the sum or difference when an exact answer is required.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Estimate the sum of two whole numbers whose sum is 99 or less and recognize whether the estimation is reasonable (e.g., 27 + 41 is about 70, because 27 is about 30 and 41 is about 40, and 30 + 40 is 70). (a) • Estimate the difference between two whole numbers each 99 or less and recognize whether the estimate is reasonable. (a) • Determine the sum of two whole numbers whose sum is 99 or less, using various methods. (b) • Determine the difference of two whole numbers each 99 or less, using various methods. (b) • Create and solve single-step practical problems involving addition or subtraction. (c) • Create and solve two-step practical problems involving addition, subtraction, or both addition and subtraction. (c)

2.6 The student will

a) estimate sums and differences;

b) determine sums and differences, using various methods; and

c) create and solve single-step and two-step practical problems involving addition and subtraction.

Understanding the Standard	Essential Knowledge and Skills
• Problem solving means engaging in a task for which a solution or a method of solution is not known in advance. Solving problems using data and graphs offers one way to connect mathematics to practical situations. • The problem-solving process is enhanced when students: – create their own story problems; and – model word problems, using manipulatives, drawings, or acting out the problem. • The least number of steps necessary to solve a single-step problem is one. • Using concrete materials (e.g., base-10 blocks, connecting cubes, beans and cups, etc.) to explore, model and stimulate discussion about a variety of problem situations helps students understand regrouping and enables them to move from the concrete to the abstract. Regrouping is used in addition and subtraction algorithms. • Conceptual understanding begins with concrete and contextual experiences. Next, students must make connections that serve as a bridge to the symbolic. Student-created representations, such as drawings, diagrams, tally marks, graphs, or written comments are strategies that help students make these connections. • In problem solving, emphasis should be placed on thinking and reasoning rather than on key words. Focusing on key words such as *in all, altogether, difference,* etc., encourages students to perform a particular operation rather than make sense of the context of the problem. A key-word focus prepares students to solve a limited set of problems and often leads to incorrect solutions as well as challenges in upcoming grades and courses. • Extensive research has been undertaken over the last several decades regarding different problem types. Many of these studies have been published in professional mathematics education publications using different labels and terminology to describe the varied problem types. • Students should experience a variety of problem types related to addition and subtraction. Problem type examples are included in the following chart:	

2.6 The student will

a) estimate sums and differences;

b) determine sums and differences, using various methods; and

c) create and solve single-step and two-step practical problems involving addition and subtraction.

Understanding the Standard

GRADE 2: COMMON ADDITION AND SUBTRACTION PROBLEM TYPES		
Join (Result Unknown)	**Join (Change Unknown)**	**Join (Start Unknown)**
Sue had 28 pencils. Alex gave her 14 more pencils. How many pencils does Sue have all together?	Sue had 28 pencils. Alex gave her some more pencils. Now Sue has 42 pencils. How many did Alex give her?	Sue had some pencils. Alex gave her 14 more. Now Sue has 42 pencils. How many pencils did Sue have to start with?
Separate (Result Unknown)	**Separate (Change Unknown)**	**Separate (Start Unknown)**
Brooke had 35 marbles. She gave 19 marbles to Joe. How many marbles does Brooke have now?	Brooke had 35 marbles. She gave some to Joe. She has 16 marbles left. How many marbles did Brooke give to Joe?	Brooke had some marbles. She gave 19 to Joe. Now she has 16 marbles left. How many marbles did Brooke start with?
Part-Part-Whole (Whole Unknown)	**Part-Part-Whole (One Part Unknown)**	**Part-Part-Whole (Both Parts Unknown)**
The teacher has 20 red markers and 25 blue markers. How many markers does he have?	The teacher has 45 markers. Twenty of the markers are red, and the rest are blue. How many blue markers does he have?	The teacher has a tub of red and blue markers. She has 45 markers in all. How many markers could be red? How many could be blue?
Compare (Difference Unknown)	**Compare (Bigger Unknown)**	**Compare (Smaller Unknown)**
Ryan has 20 books and Chris has 9 books. How many more books does Ryan have than Chris? Ryan has 20 books. Chris has 9 books. How many fewer books does Chris have than Ryan?	Chris has 9 books. Ryan has 11 more books than Chris. How many books does Ryan have? Chris has 11 fewer books than Ryan. Chris has 9 books. How many books does Ryan have?	Ryan has 11 more books than Chris. Ryan has 20 books. How many books does Chris have? Chris has 11 fewer books than Ryan. Ryan has 20 books. How many books does Chris have?

Essential Knowledge and Skills

2.6 The student will

a) estimate sums and differences;

b) determine sums and differences, using various methods; and

c) create and solve single-step and two-step practical problems involving addition and subtraction.

Understanding the Standard	Essential Knowledge and Skills

- Strategies for adding and subtracting two-digit numbers can include, but are not limited to, using concrete objects, a hundred chart, number line, and invented strategies.
- Mental computation helps build number sense in students. Strategies for mentally adding or subtracting two-digit numbers should be student-invented strategies. Some of these strategies may include:

Partial Sums

56 + 41 = __
50 + 40 = 90
6 + 1 = 7
90 + 7 = 97

Counting On

36 + 62 = __
36 + 60 = 96
96 + 2 = 98

87 – 25 = __
20 + 60 = 80
5 + 2 = 7
60 + 2 = 62

87 – 25 = __
25 + 60 = 85
85 + 2 = 87
60 + 2 = 62

87 – 25 = __
25 + 2 = 27
27 + 60 = 87
2 + 60 = 62

98 – 41 = __
98 – 40 = 58
58 – 1 = 57

56 – 21 = __
50 – 20 = 30
6 – 1 = 5
30 + 5 = 35

- The terms used in addition are

23 → *addend*
+ 46 → *addend*
69 → *sum*

- The terms often used in subtraction are

98 → *minuend*
– 41 → *subtrahend*
57 → *difference*

- At this level, students do not need to use the terms *addend, minuend, or subtrahend* for addition and subtraction as shown above.